Computation of the sign and absolute value of a matrix. Application to the numerical simulation of multifluid systems

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We are interested in the numerical solving of systems of PDEs, arising from the modeling of multiphase flows through averaging techniques. In the finite volume framework, the Roe Scheme requires a fast and robust computation of the sign or absolute value of a given matrix A^{Roe}. In the case of two or more fluids, the matrix A^{Roe} has a non trivial eigen structure, and interfacial forces can bring new differential terms. For these reasons an explicit algebraic diagonalisation is not tractable. Moreover, It is well known that A^{Roe} can present multiple eigenvalues and Jordan blocs, and numerical diagonalisation techniques are thus ill conditioned.

We present two new algorithms avoiding the diagonalisation : an iteration scheme, which turns out to be an exact scheme, and an interpolation scheme which is much faster. We take advantage of the fact that in most systems, the characteristic polynomial of A^{Roe} can be fastly computed which gives us an easy access to the eigenvalues. However, the iteration scheme can be used with only estimates of the eigenvalues, given for example by Gershgorin techniques.

Stability questions are adressed concerning Jordan blocs, complex eigenvalues and highly ocillatory interpolating polynomials. We finally show results of two-phase flows numerical simulations, using two-fluid and multifield models, taking into account the interfacial pressure term and the virtual mass force.