

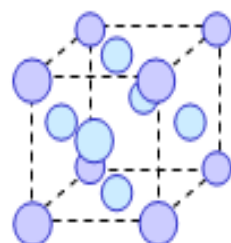
Precipitation
in a multicomponent alloy:
combining atomic
and mesoscopic simulations

Emmanuel Clouet, M. Nastar and A. Barbu

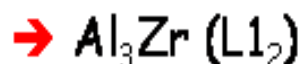
SRMP, CEA Saclay
91191 Gif-sur-Yvette
France

Introduction : Al-Zr-Sc Alloy

Zr and Sc precipitation in Al alloys



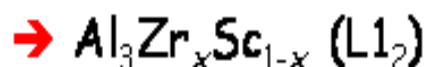
● Al ● Zr/Sc



$$x_{\text{Zr}}^{\text{eq}} \leq 0.27 \text{ at.}\%$$



$$x_{\text{Sc}}^{\text{eq}} \leq 0.24 \text{ at.}\%$$



Model system for ordering alloys

Interaction between solute atoms:

- repulsive for 1st n. n.
- attractive for 2nd n. n.
- no interaction beyond



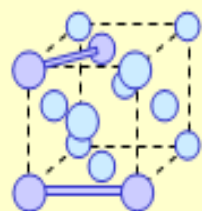
Validate mesoscopic modeling of precipitation by confronting them with atomic simulations

Introduction : Modeling Approach

Ab-initio calculations
Experimental data

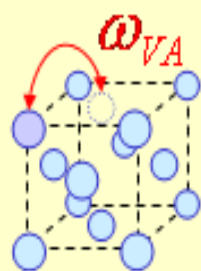


Atomic model (KMC)



Rigid lattice

Vacancy - atom
exchange



Mesoscopic models
(cluster dynamics)

Difficulties :

- validity of hypothesis
- predictions for ternary alloys



No direct
comparison
(super-saturations)

Experimental
characterization of
precipitation kinetics



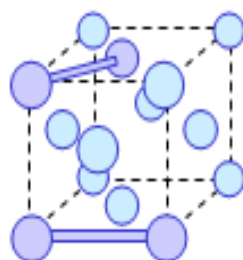
Comparison
(reachable super-saturations
and annealing times)

Thermodynamics :

Rigid lattice with pair interactions (Ising model)

➤ 1st nearest neighbours

➤ 2^{nds} nearest neighbours

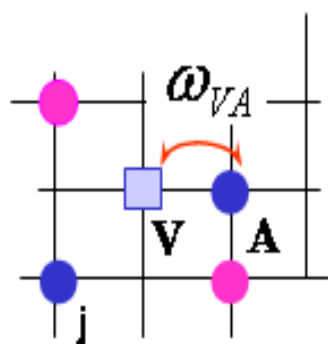


$$E = \frac{1}{2} \sum_{\substack{i \neq j \\ \alpha, \beta}} \epsilon_{\alpha\beta}^{(1)} n_i^\alpha n_j^\beta + \frac{1}{2} \sum_{i \neq j} \epsilon_{\alpha\beta}^{(2)} n_i^\alpha n_j^\beta$$

Atomic Model

Kinetics :

Atom-vacancy exchange thermally activated



$$\omega_{VA} = \nu_A \exp\left(-\frac{E_{VA}^{act}}{k_B T}\right)$$

$$E_{VA}^{act} = E_A^s - \sum_j \sum_\beta \epsilon_{V\beta} n_j^\beta - \sum_j \sum_\beta \epsilon_{A\beta} n_j^\beta$$

Energy profile diagram showing the energy barrier for atom-vacancy exchange. The energy barrier is labeled E_{VA}^{act} . Below the diagram, two configurations are shown: V-A (blue square and blue circle) and A-V (blue circle and blue square).

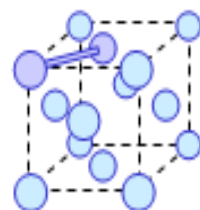
KMC using resident time algorithm :

At each time step :

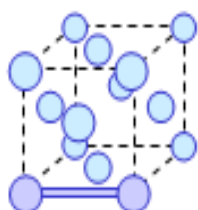
- calculate all transition probabilities ω_i of the vacancy (only 12)
- choose the next configuration according to this distribution of probabilities (random number)
- increment time with $\delta t = 1 / \sum_i \omega_i$

Atomic Model : Parameters

- Formation free energies $\Delta F(\text{Al}_3\text{Zr})$ et $\Delta F(\text{Al}_3\text{Sc})$
 ➔ **ab initio** calculations (energy + vibration entropy)



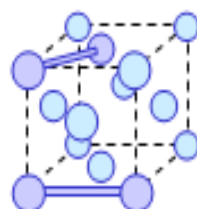
$$\begin{matrix} \varepsilon_{\text{Al-Zr}}^1(T) \\ \varepsilon_{\text{Al-Sc}}^1(T) \end{matrix}$$



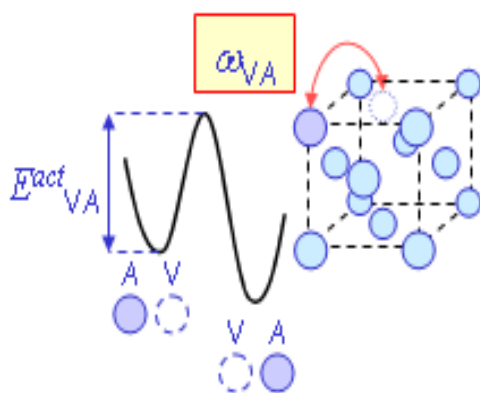
$$\begin{matrix} \varepsilon_{\text{Al-Zr}}^2(T) \\ \varepsilon_{\text{Al-Sc}}^2(T) \end{matrix}$$

- Solubility limits
 - ➔ Zr in Al : **ab initio** / **experimental** †
 - ➔ Sc in Al : **experimental data**

- Formation energies of ternary ordered compounds
 ➔ **ab initio** calculations (energy)



$$\begin{matrix} \varepsilon_{\text{Sc-Zr}}^1 \\ \varepsilon_{\text{Sc-Zr}}^2 \end{matrix}$$

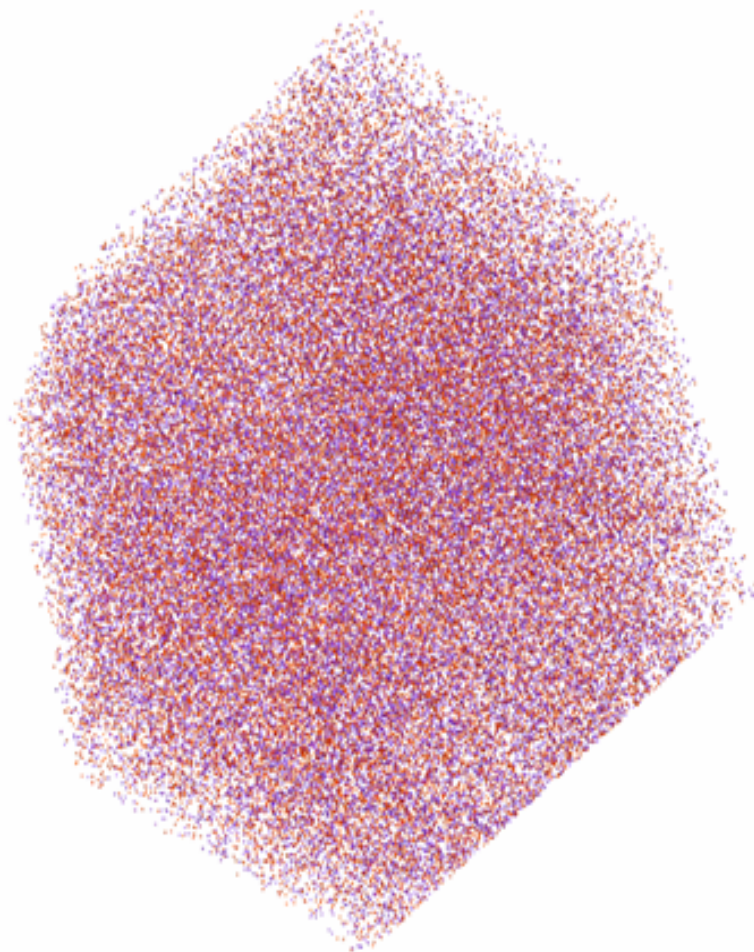


- Vacancy thermodynamics ($E_{\text{V}}^{\text{for}}$, $E_{\text{ZrV}}^{\text{bin}}$, $E_{\text{ScV}}^{\text{bin}}$)
- Diffusion coefficients $D_{\text{Al}^*}(T)$, $D_{\text{Sc}^*}(T)$ et $D_{\text{Zr}^*}(T)$
 ➔ **experimental data**

Atomic Model : Simulations

Precipitation in Al-Zr-Sc alloy

0.0000 s



Aluminum solid solution

$$c_{\text{Zr}}^0 = 0.5 \text{ at.}\%$$

$$c_{\text{Sc}}^0 = 0.5 \text{ at.}\%$$

$$T = 550^\circ\text{C}$$

200³ lattice sites

1 vacancy

- Zr
- Sc

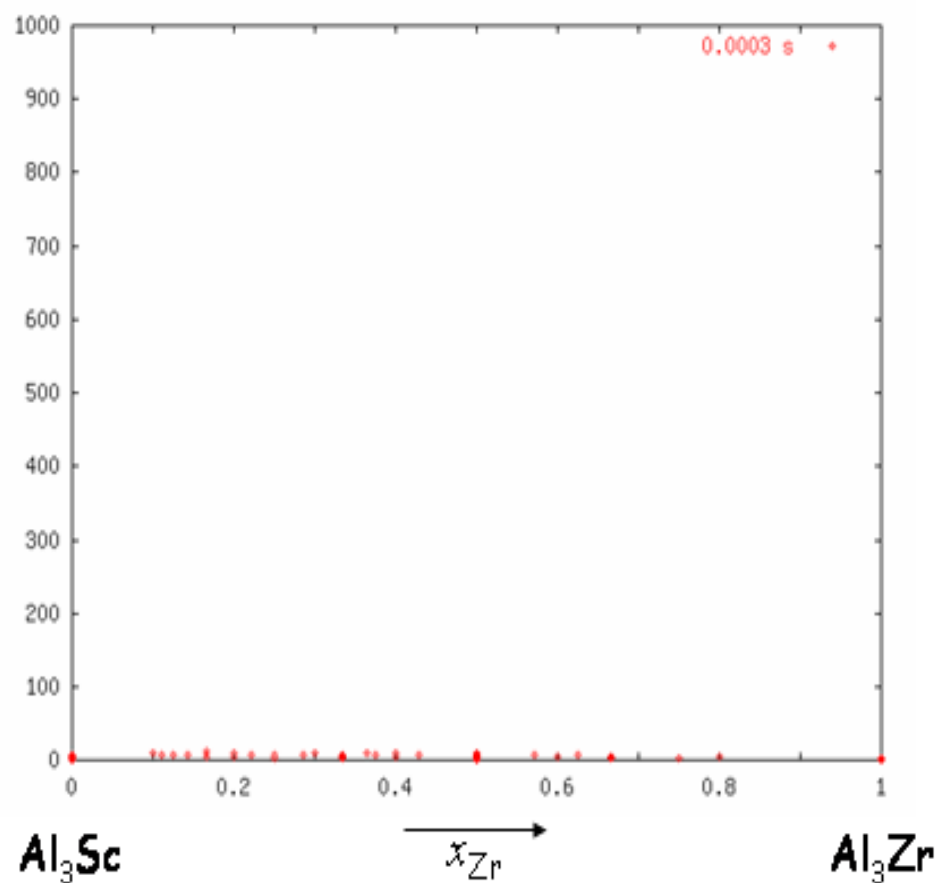
Explanation :

$$D_{\text{Sc}} \gg D_{\text{Zr}}$$

Atomic Model : Simulations

Precipitation in Al-Zr-Sc alloy

Precipitates' size (atoms)



Aluminum solid solution

$$c_{Zr}^0 = 0.5 \text{ at.}\%$$

$$c_{Sc}^0 = 0.5 \text{ at.}\%$$

$$T = 550^\circ\text{C}$$

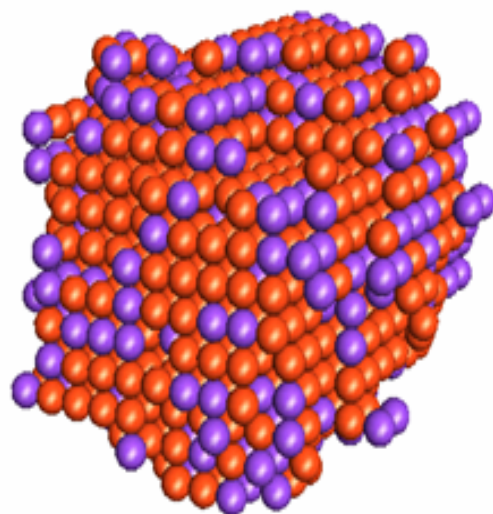
200^3 lattice sites

1 vacancy

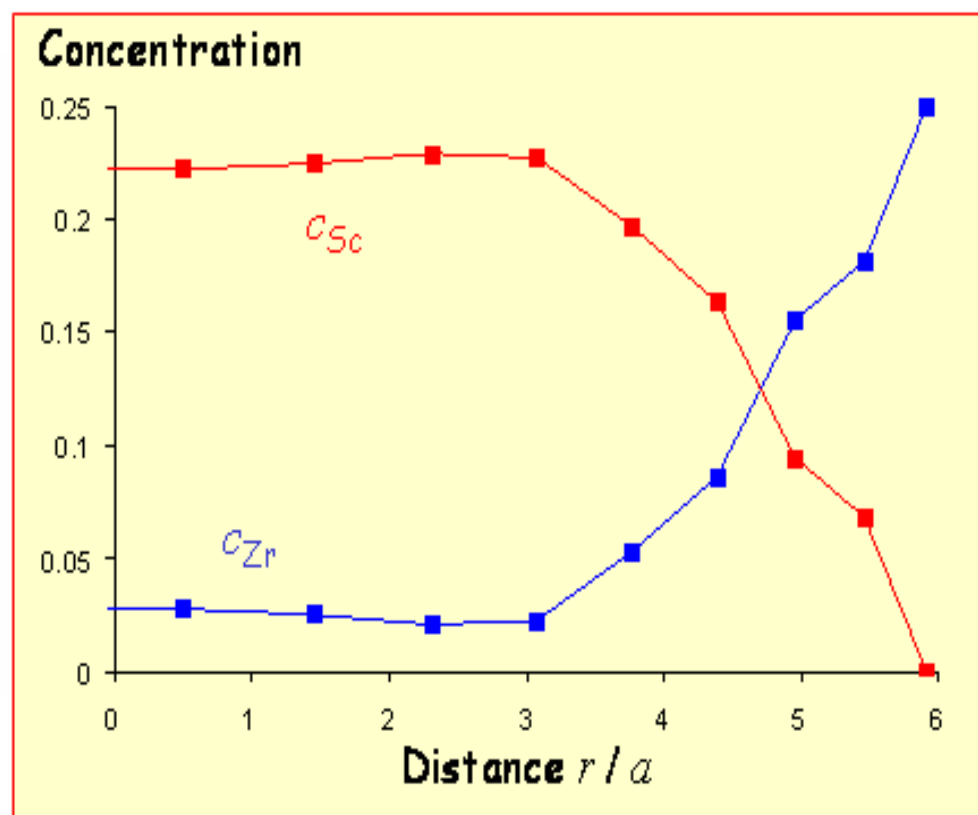
$$D_{Sc} \gg D_{Zr}$$

Atomic Model : Simulations

Precipitation in Al-Zr-Sc alloy



● Zr ● Sc



In agreement with experimental observations (3D atom probe¹ / HREM²)

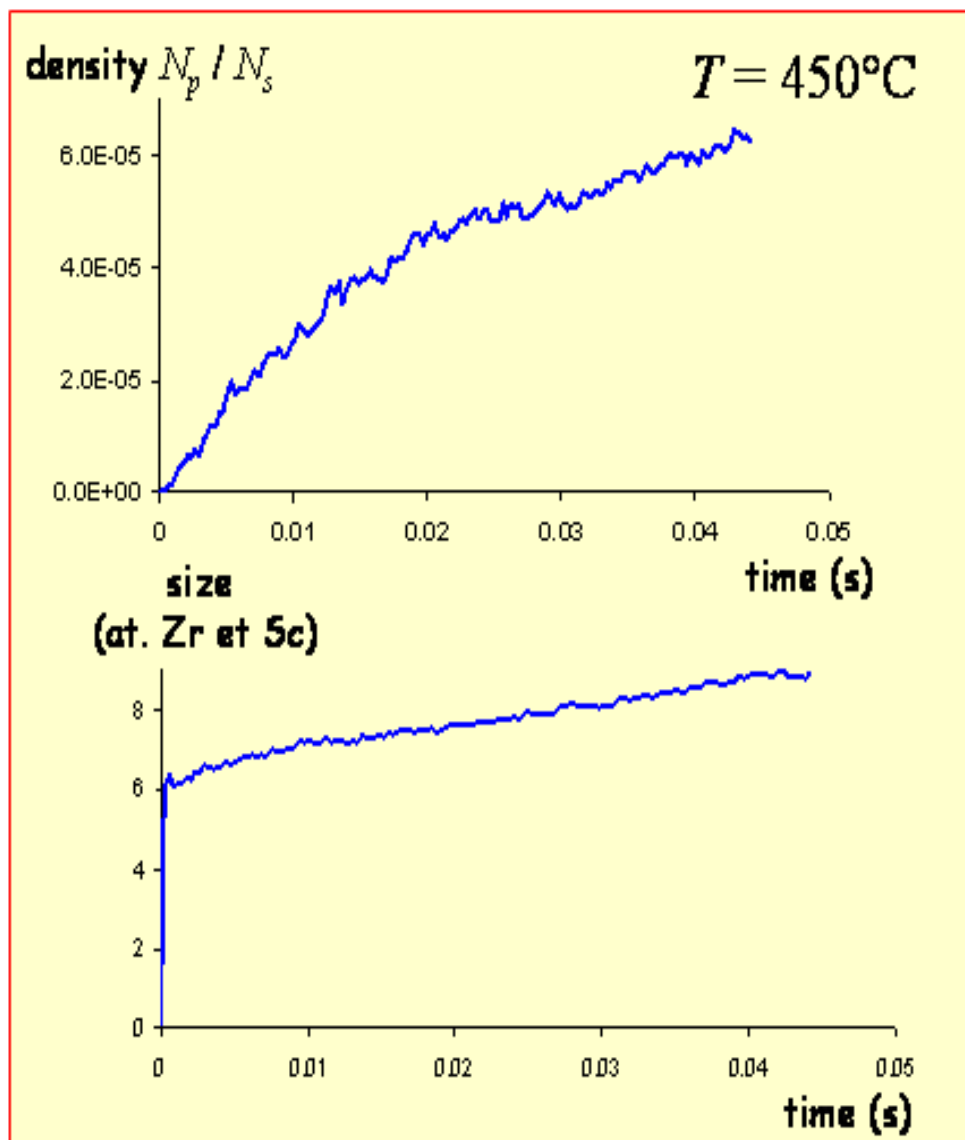
¹ B. Forbord, W. Lefebvre, F. Danoix, H. Hallem and K. Marthinsen, *Scr. Mater.* **51** (2004) p.333

² A. Tolley, V. Radmilovic and U. Dahmen, *Scr. Mater.* **52** (2005) p.621

Atomic Model : Simulations

C_{Sc} constant

0% Zr - 0.5% Sc



Atomic Model : Simulations

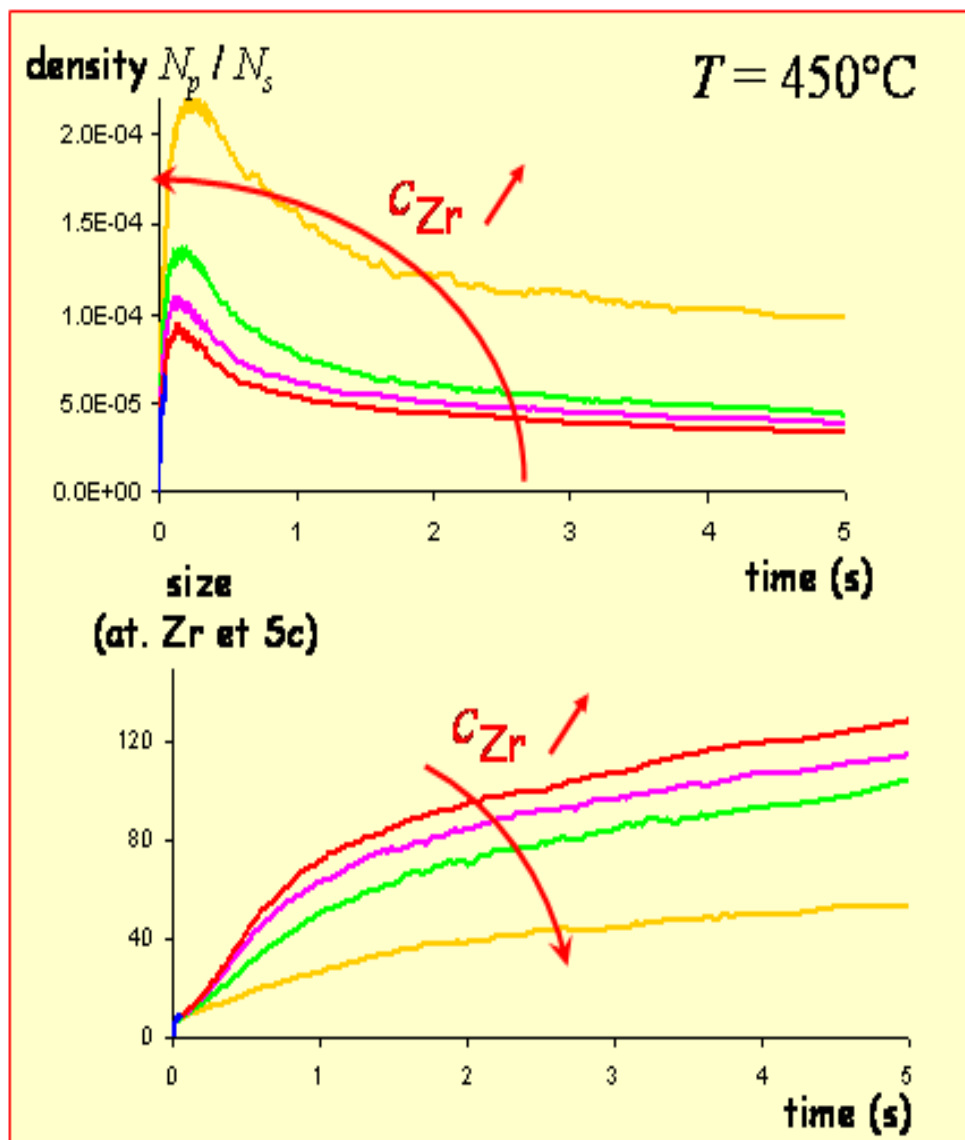
c_{Sc} constante

- 0% Zr - 0.5% Sc
- 0.05% Zr - 0.5% Sc
- 0.1% Zr - 0.5% Sc
- 0.2% Zr - 0.5% Sc
- 0.5% Zr - 0.5% Sc

Zr Addition :

- increases precipitates' density
- decreases precipitates' size

In agreement with
experimental
observations



Atomic Model : Simulations

Precipitation in Al-Zr-Sc alloy

- Thermodynamic factor ($\omega_{ZrSc}^2 \leq 0$)
 - Formation of nuclei containing Zr and Sc atoms
- Kinetic factor ($D_{Sc} \gg D_{Zr}$)
 - Nuclei growth by Sc absorption
- Decreasing of Sc concentration in the solid solution
 - Precipitates' external shelves richer in Zr

Zr addition to Al-Sc alloy

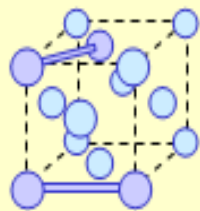
- Nucleation's increase
 - Higher precipitates' density
- No influence on the growth stage
 - Smaller precipitates (less Sc available for growth after nucleation)

Modeling Approach

Ab-initio calculations
Experimental data

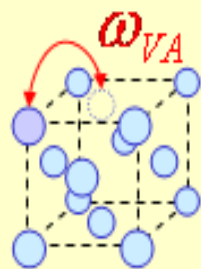


Atomic model (KMC)



Rigid lattice

Vacancy - atom
exchange



Mesoscopic models
(cluster dynamics)

Difficulties :

- validity of hypothesis
- predictions for ternary alloys



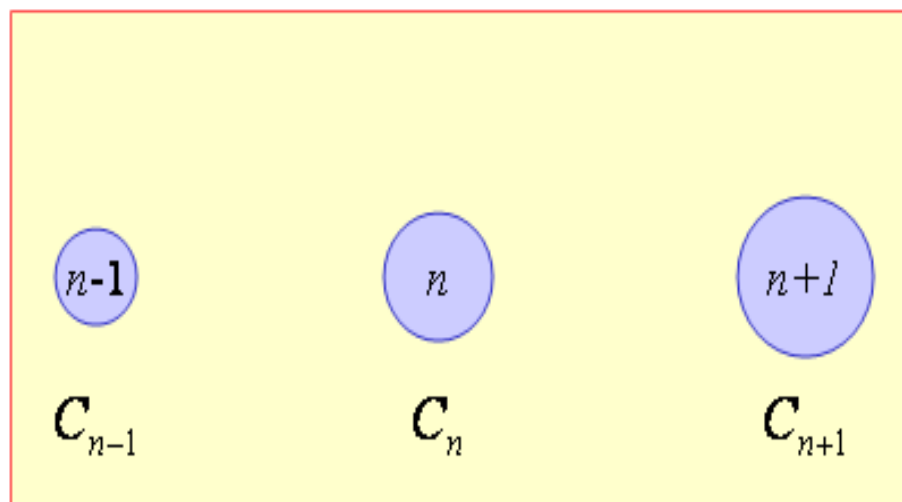
No direct
comparison
(super-saturations)

Experimental
characterization of
precipitation kinetics



Comparison
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and annealing times)

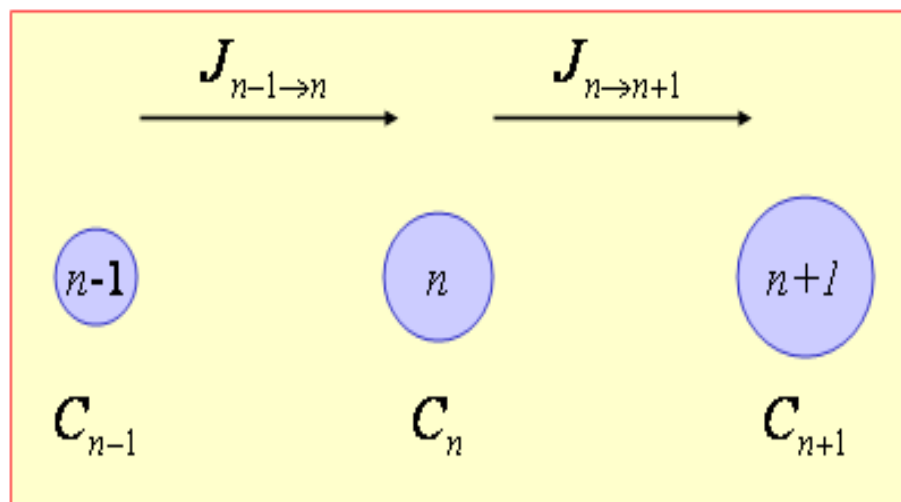
Cluster Dynamics : Theory



Cluster population described by its size distribution C_n

➔ No information on cluster position

Cluster Dynamics : Theory



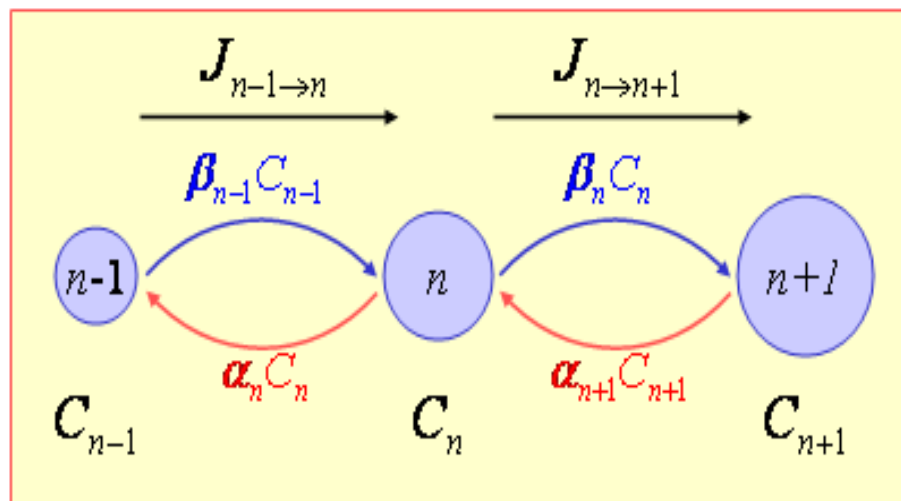
Time evolution of the distribution

$$\frac{dC_n}{dt} = J_{n-1 \rightarrow n} - J_{n \rightarrow n+1}, \quad \forall n \geq 2$$

$$\frac{dC_1}{dt} = -2J_{1 \rightarrow 2} - \sum_{n \geq 2} J_{n \rightarrow n+1}$$

➔ Flux only between adjacent classes

Cluster Dynamics : Theory



Time evolution of the distribution

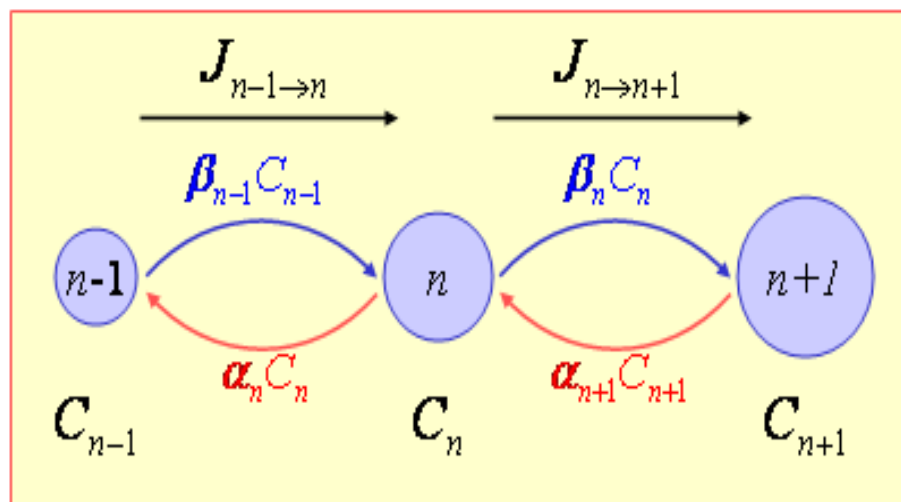
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$$\frac{dC_1}{dt} = -2J_{1 \rightarrow 2} - \sum_{n \geq 2} J_{n \rightarrow n+1}$$

$$J_{n \rightarrow n+1} = \beta_n C_n - \alpha_{n+1} C_{n+1}$$

- Monomer **absorption**
- Monomer **emission**

Cluster Dynamics : Theory



- **Absorption** controlled by the long-range diffusion of monomers

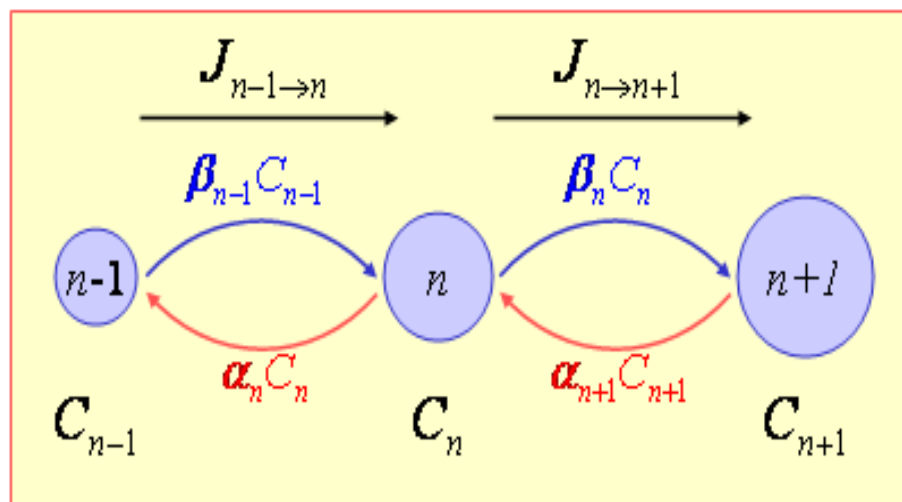
$$\beta_n = 4\pi R_n D_1 C_1$$

- **Emission** : intrinsic property of the cluster
(do not depend of the embedding solid solution)

Fluxes equilibrium in a saturated solid solution

$$\rightarrow \alpha_{n+1} = \frac{\beta_n \bar{C}_n}{\bar{C}_{n+1}} = 4\pi R_n D_1 \frac{\bar{C}_1 \bar{C}_n}{\bar{C}_{n+1}} = f(D_1, \sigma_n)$$

Cluster Dynamics : Theory



- **Absorption** controlled by the long-range diffusion of monomers

$$\beta_n = 4\pi R_n D_1 C_1$$

- **Emission** : intrinsic property of the cluster

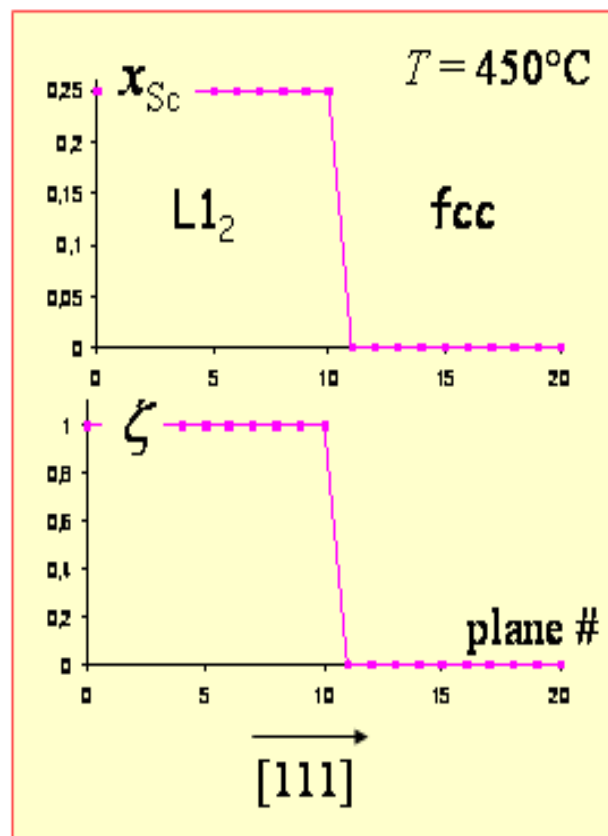
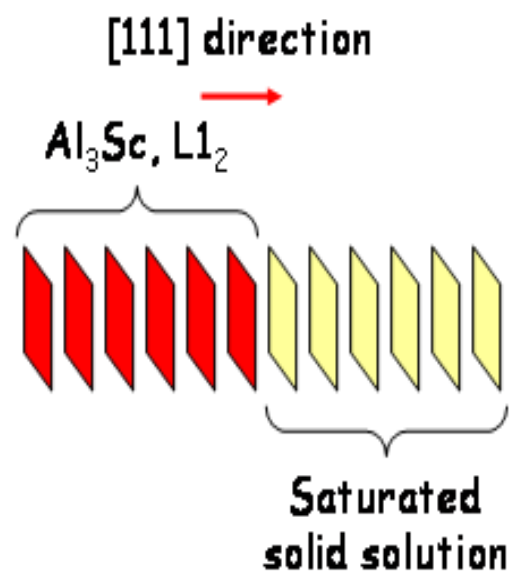
$$\alpha_n = f(D_1, \sigma_n)$$

➔ No information on the nucleation driving force
assumed thermodynamics = lattice gas

➔ Only 2 parameters : D_1 and σ_n

Interface Free Energy

Al₃Sc, L1₂ / Al fcc: planar interface

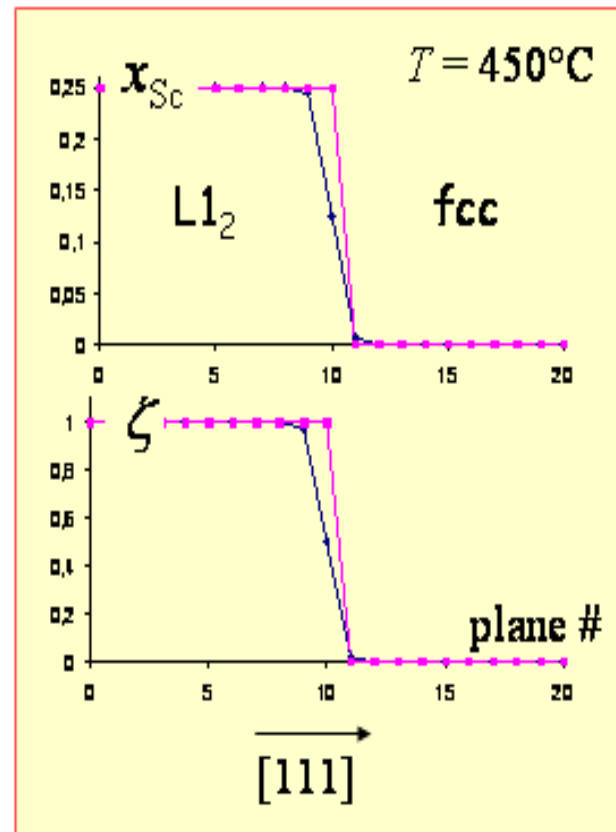
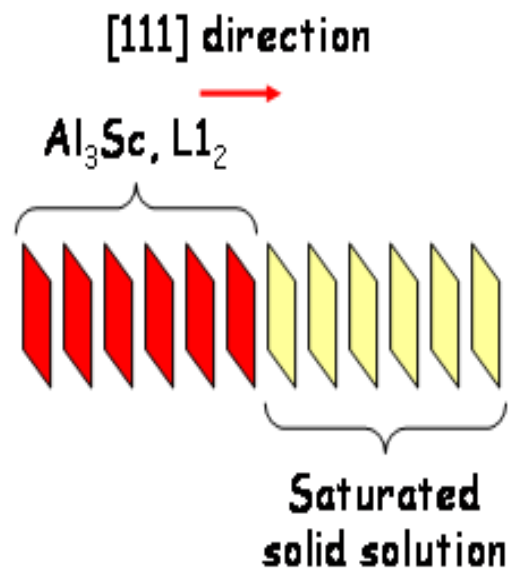


Interface Free Energy

Al₃Sc, L1₂ / Al fcc: planar interface

Relaxation within Bragg-Williams approximation:

- concentration
- order parameter



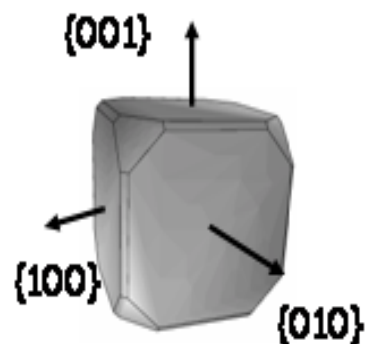
Interface Free Energy

Interface free energy

Planar interfaces

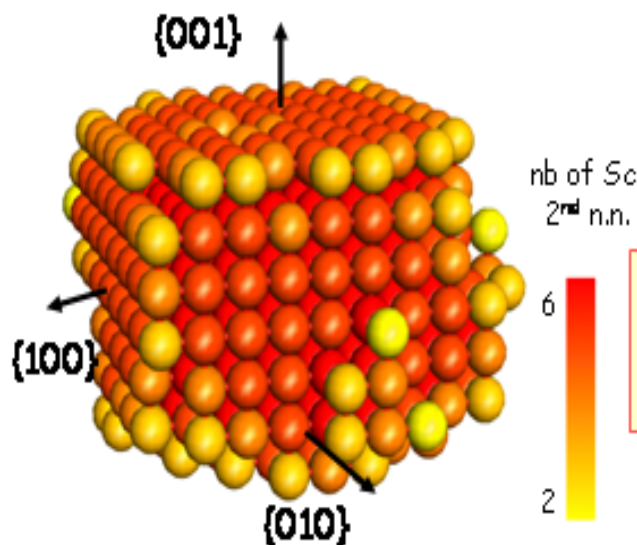
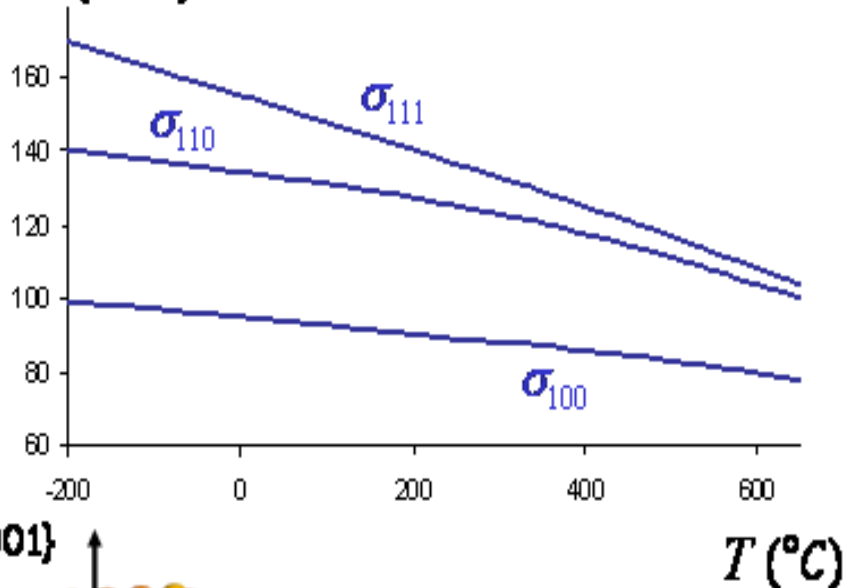
$$\sigma_{100} < \sigma_{110} < \sigma_{111}$$

➔ Facetted precipitates



Al_3Sc at 450°C

$a^2 \sigma$ (meV)



Al_3Sc precipitate
(KMC simulation,
 450°C , $C_{\text{Sc}}^0 = 0.5\%$)

Interface Free Energy

Interface free energy

Planar interfaces

$$\sigma_{100} < \sigma_{110} < \sigma_{111}$$



Wulff
construction

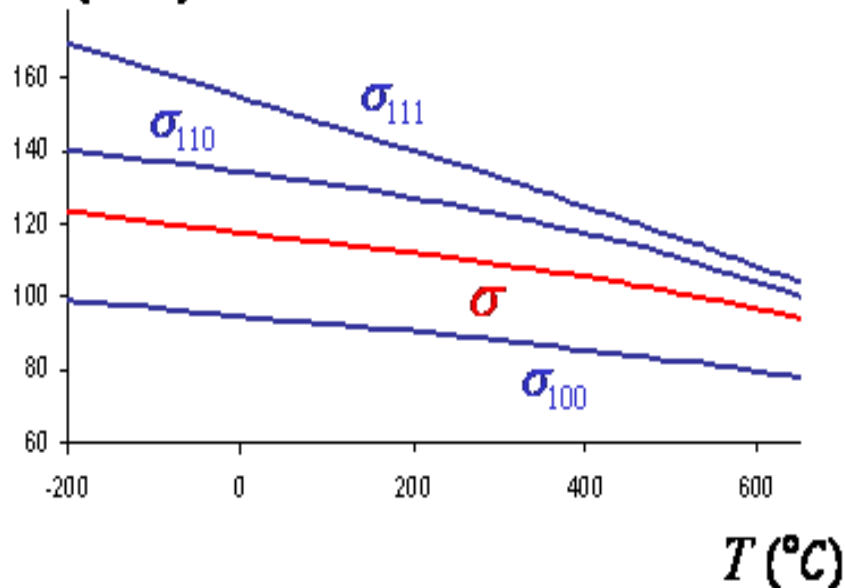


σ

Spherical precipitate



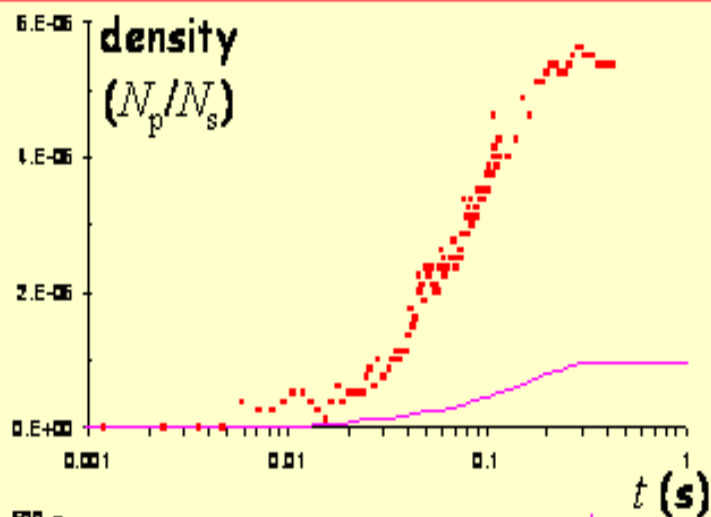
$a^2 \sigma$ (meV)



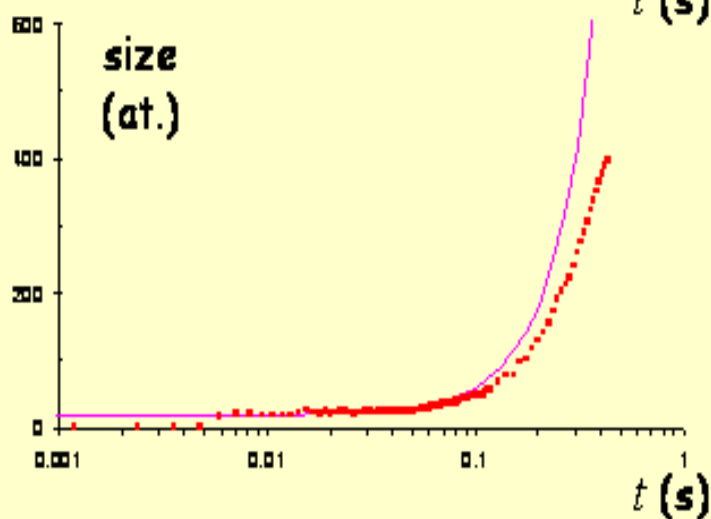
$$\sigma_n = \sigma, \quad \forall n$$

Cluster Dynamics : Simulations

Al + 0.4% Sc - T = 500°C



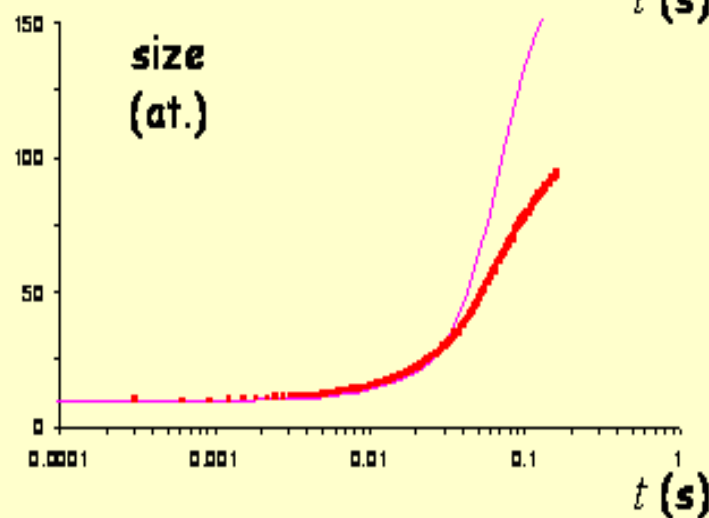
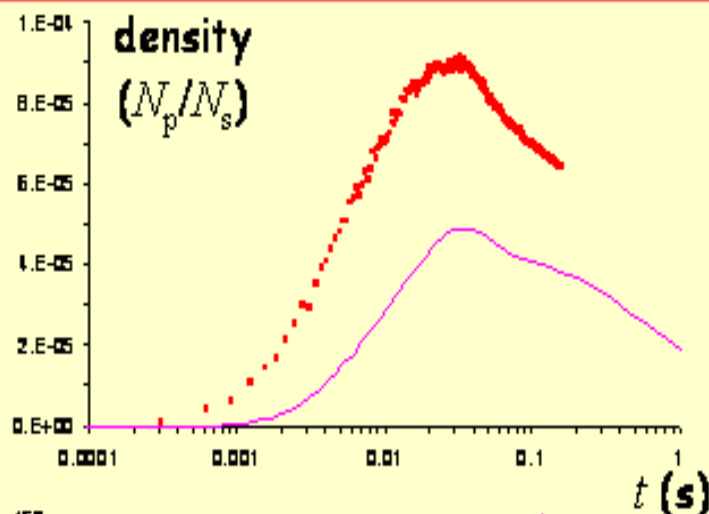
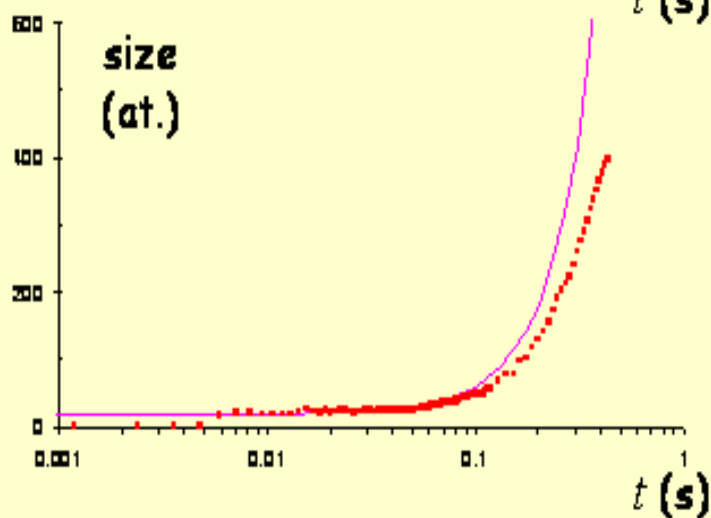
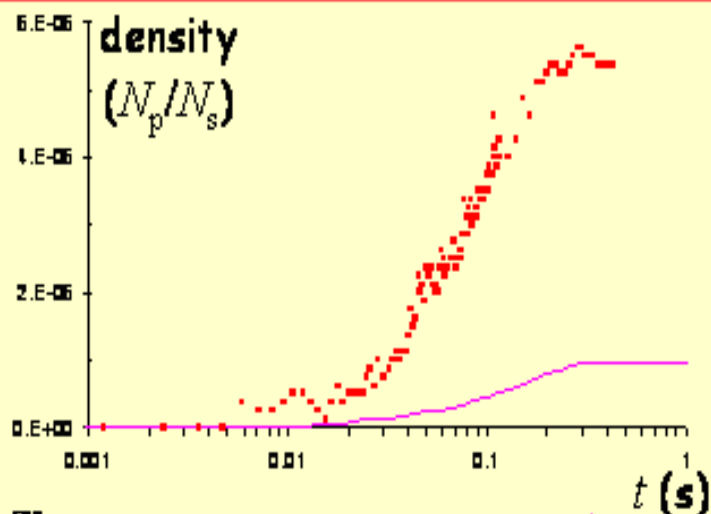
■ Kinetic Monte Carlo
— Cluster Dynamics



Cluster Dynamics : Simulations

Al + 0.4% Sc - T = 500°C

Al + 0.75% Sc - T = 500°C



Interface Free Energy

Dependence with size of σ_n

Counting of all reachable configurations of the cluster for

- each size n ($n < 10$)
- each energy class α

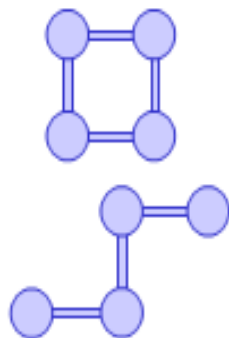


Cluster interface free energy

$$G_n = -kT \ln \left(\sum_{\alpha} D_{n,\alpha} \exp(-H_{n,\alpha} / kT) \right)$$

Example : $n = 4$

2 energy classes



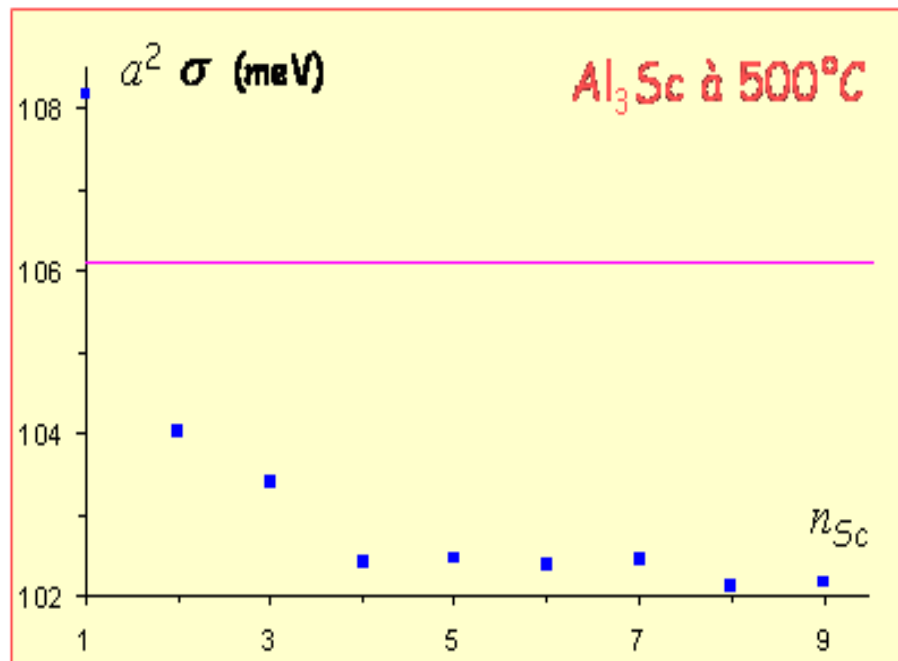
$$H_{4,1} = 16 \omega^{(2)} \quad D_{4,1} = 3$$

$$H_{4,2} = 18 \omega^{(2)} \quad D_{4,2} = 83$$



σ_n

Interface Free Energy



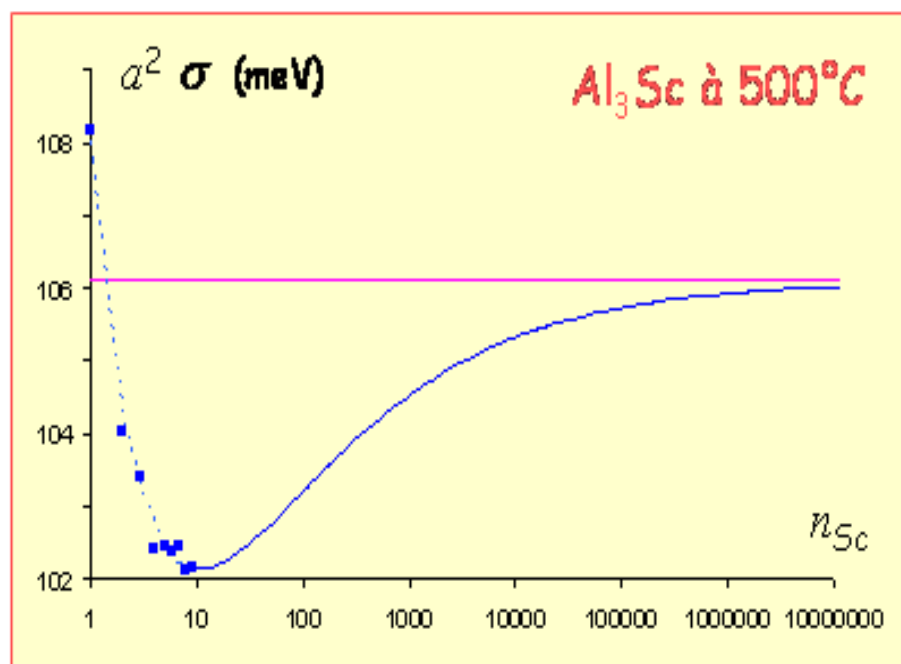
Constant interface free energy

$$\sigma_n = \sigma \text{ (Bragg-Williams + Wulff)}$$

Interface energy depending on cluster size

- Direct calculation

Interface Free Energy



Constant interface free energy

$$\sigma_n = \sigma \text{ (Bragg-Williams + Wulff)}$$

Interface energy depending on cluster size

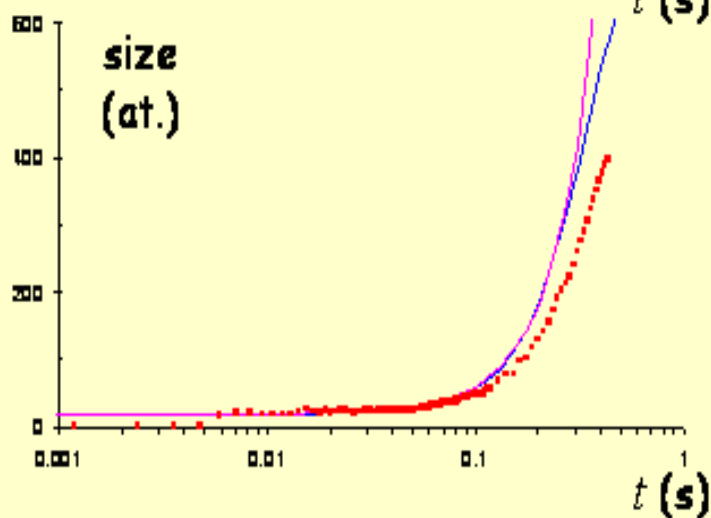
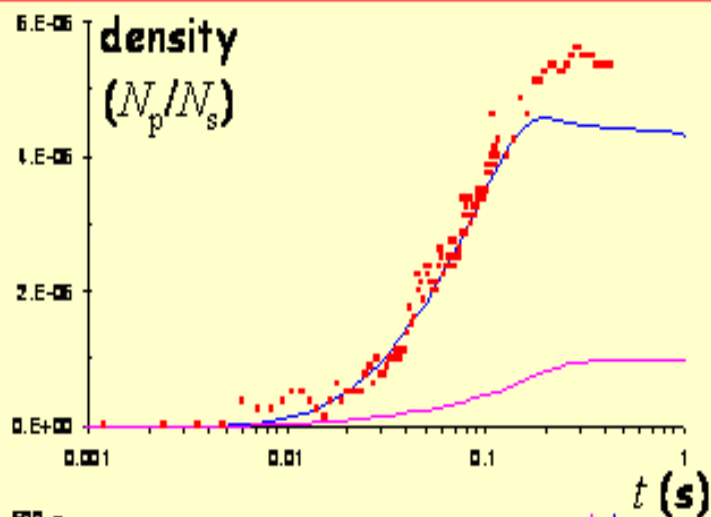
- Direct calculation

- ⌒ Fitting of

$$\sigma_n = \sigma (1 + cn^{-1/3} + dn^{-2/3})$$

Cluster Dynamics : Simulations

Al + 0.4% Sc - T = 500°C



■ Kinetic Monte Carlo

Cluster Dynamics

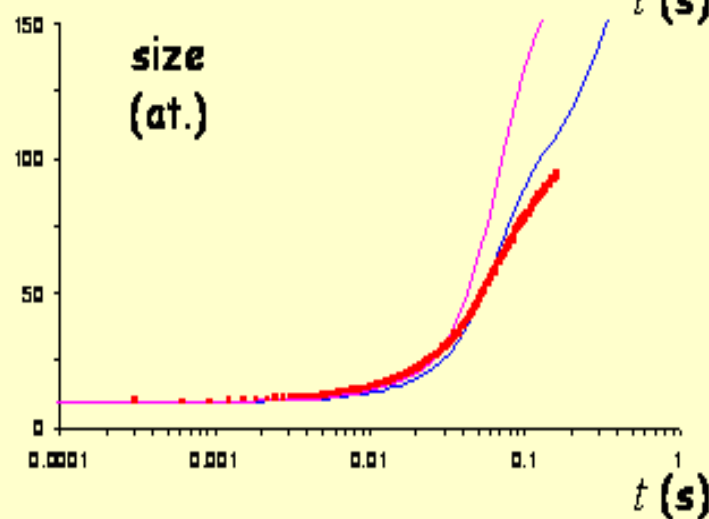
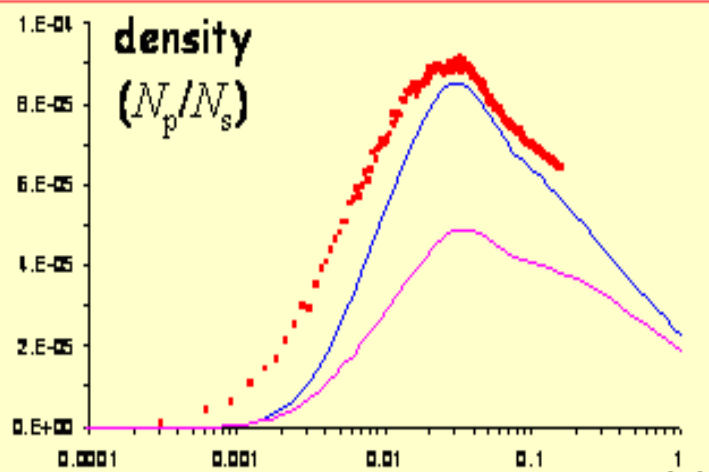
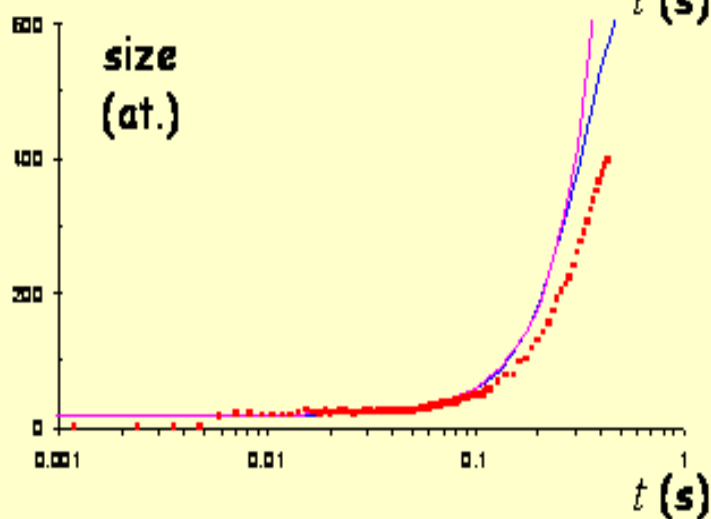
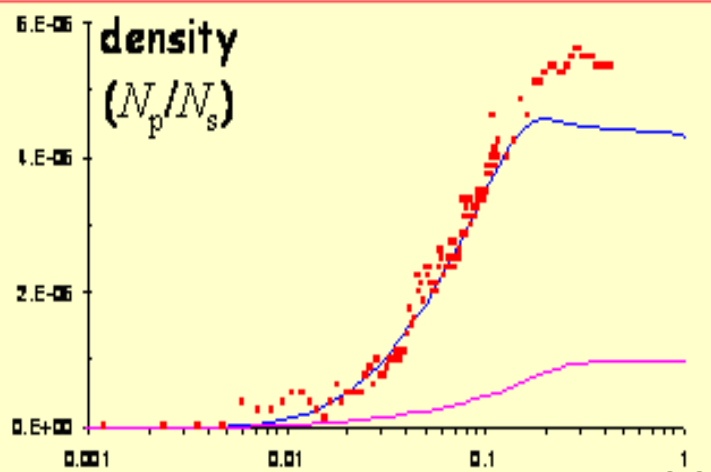
— σ_n constant

— $\sigma_n = f(n)$

Cluster Dynamics : Simulations

Al + 0.4% Sc - T = 500°C

Al + 0.75% Sc - T = 500°C

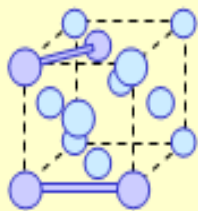


Modeling Approach

Ab-initio calculations
Experimental data

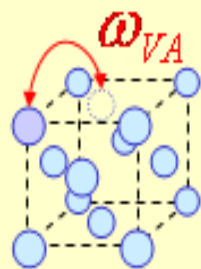


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(cluster dynamics)

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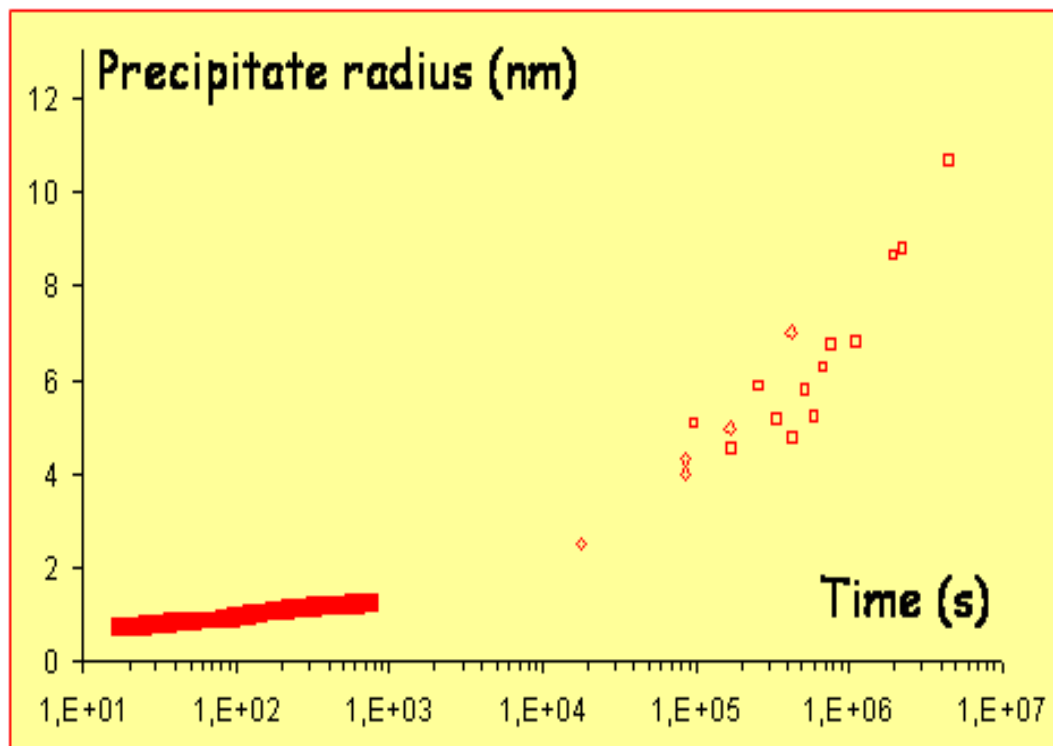
Experimental
characterization of
precipitation kinetics



Comparison
(reachable super-saturations
and annealing times)

Confrontation with Experimental Data

Al + 0.18 % Sc at 350°C : precipitate growth



Experimental data

Ref. 1 □

Ref. 2 ◇

KMC ■

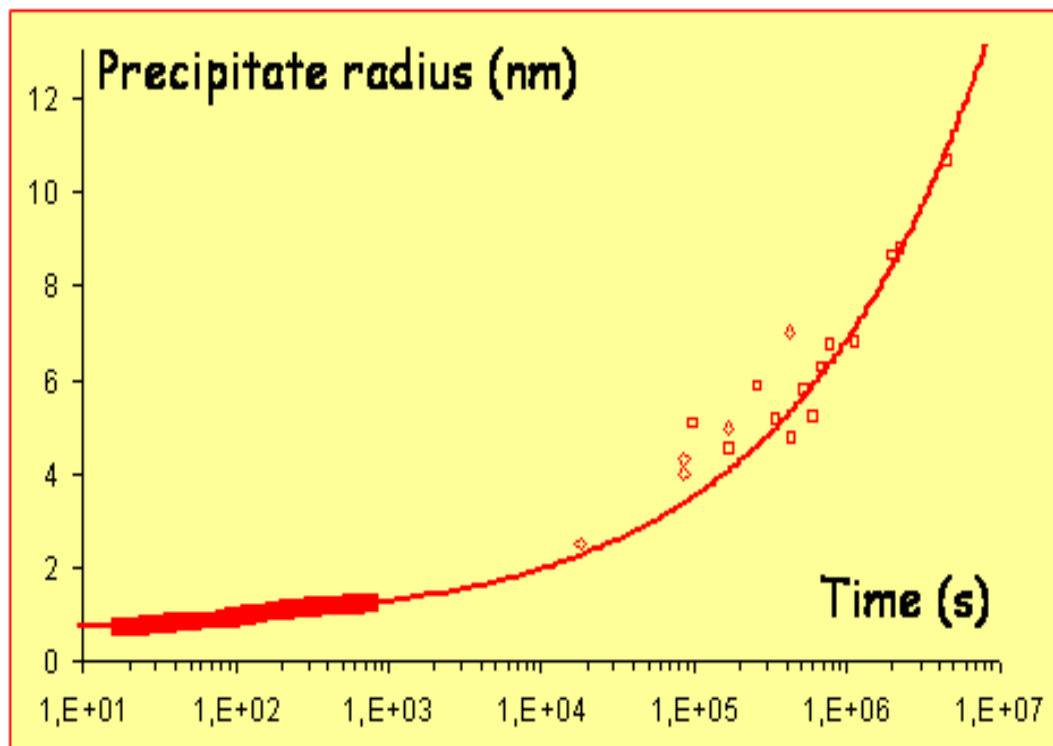
($R^{cut} \sim 0.75$ nm)

¹ G. M. Novotny and A. J. Ardell, *Precipitation of Al_3Sc in binary Al-Sc alloys*, Mater. Sci. Eng. **A318** (2001) 144-154

² E. Marquis and D. Seidman, *Nanoscale structural evolution of Al_3Sc precipitates in Al(Sc) alloys*, Acta Mater. **49** (2001) 1909

Confrontation with Experimental Data

Al + 0.18 % Sc at 350°C : precipitate growth



Experimental data

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Cluster dynamics

($R^{cut} \sim 0.75$ nm)

KMC

($R^{cut} \sim 0.75$ nm)

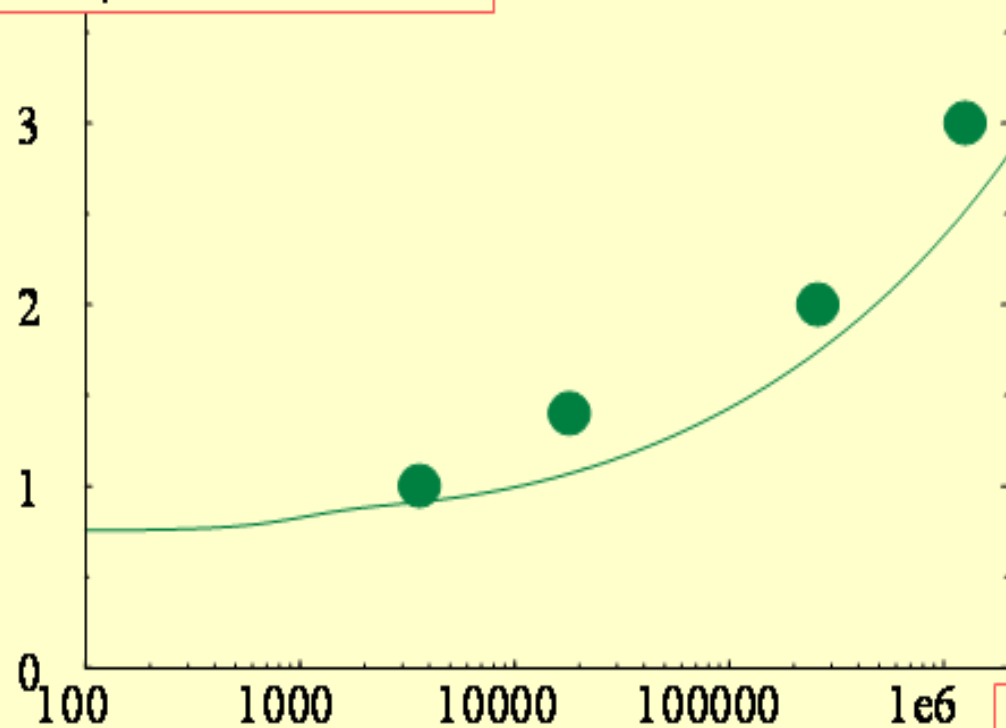
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Confrontation with Experimental Data

Al + 0.18 % Sc : precipitate growth

Precipitate radius (nm)



Exp. data.²



Cluster dynamics



($R^{cut} \sim 0.75$ nm)

300°C

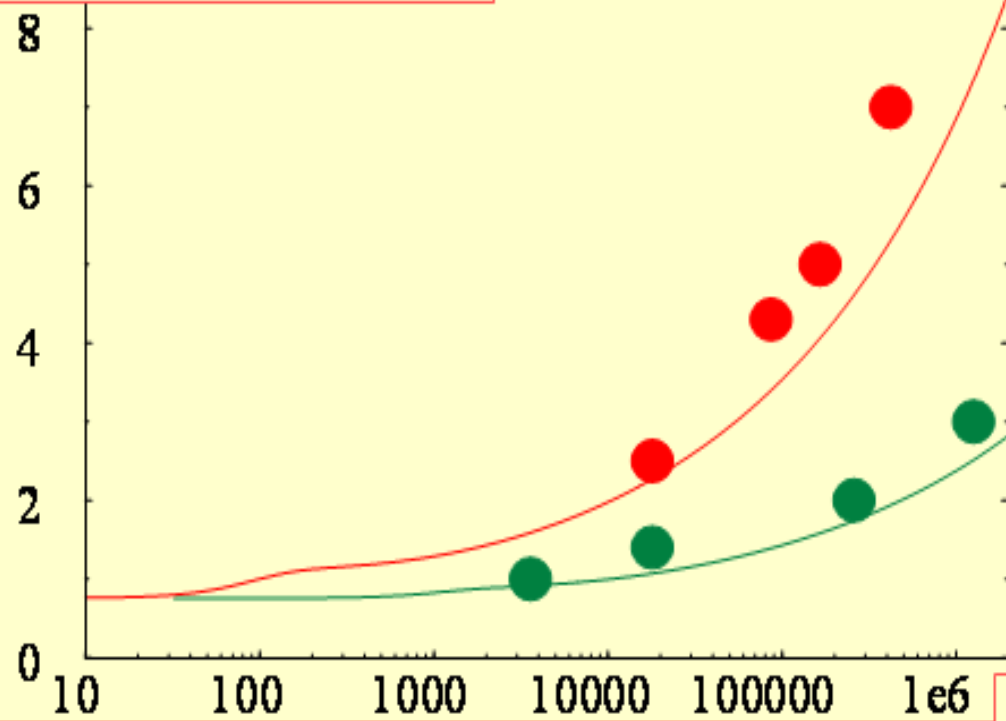
Time (s)

² E. Marquis and D. Seidman, *Nanoscale structural evolution of Al_3Sc precipitates in Al(Sc) alloys*, *Acta Mater.* **49** (2001) 1909

Confrontation with Experimental Data

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Precipitate radius (nm)



Exp. data.²



Cluster dynamics



($R^{cut} \sim 0.75$ nm)

350°C

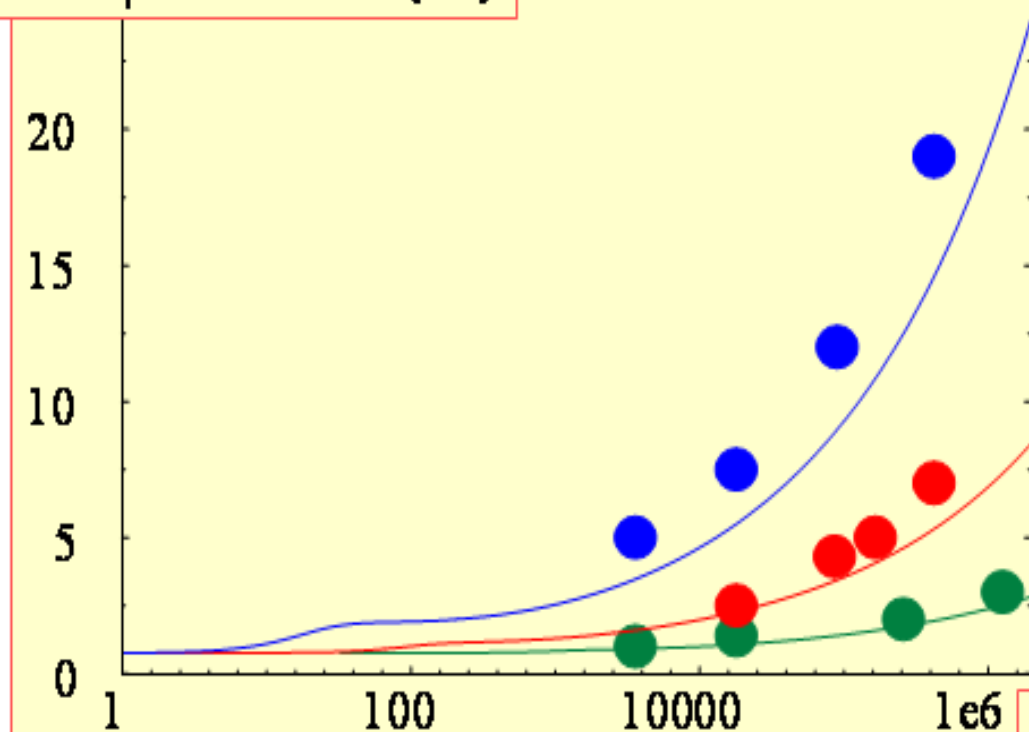
300°C

² E. Marquis and D. Seidman, *Nanoscale structural evolution of Al₃Sc precipitates in Al(Sc) alloys*, *Acta Mater.* **49** (2001) 1909

Confrontation with Experimental Data

Al + 0.18 % Sc : precipitate growth

Precipitate radius (nm)



Time (s)

Exp. data.²



Cluster dynamics



($R^{cut} \sim 0.75$ nm)

400°C

350°C

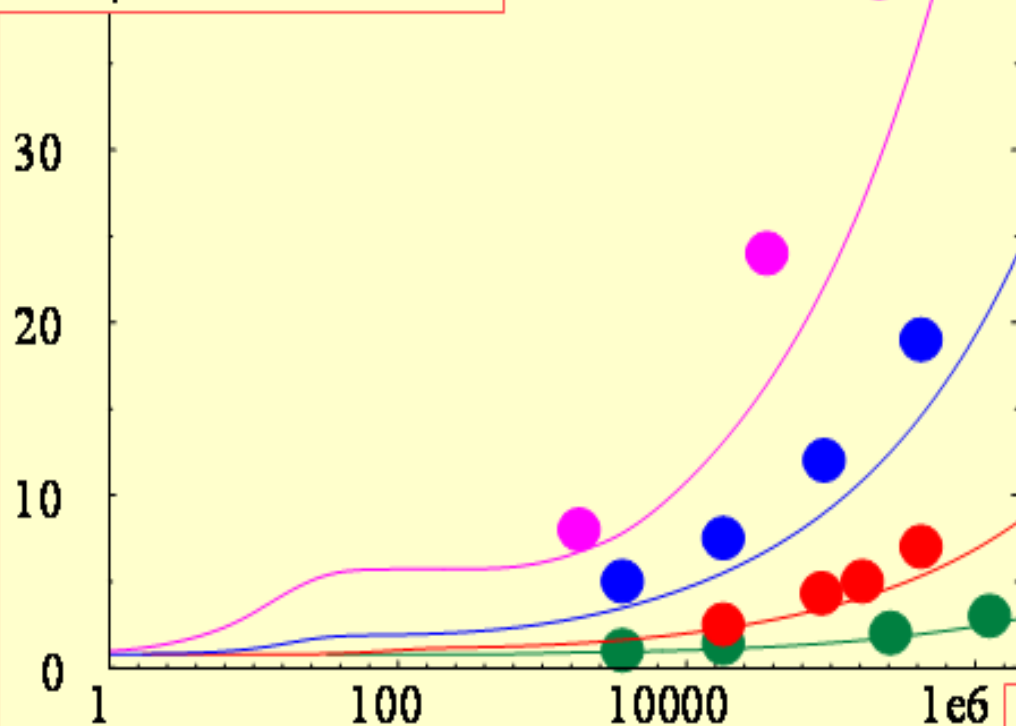
300°C

² E. Marquis and D. Seidman, *Nanoscale structural evolution of Al₃Sc precipitates in Al(Sc) alloys*, Acta Mater. **49** (2001) 1909

Confrontation with Experimental Data

Al + 0.18 % Sc : precipitate growth

Precipitate radius (nm)



Exp. data.²



Cluster dynamics



($R^{cut} \sim 0.75$ nm)

450°C

400°C

350°C

300°C

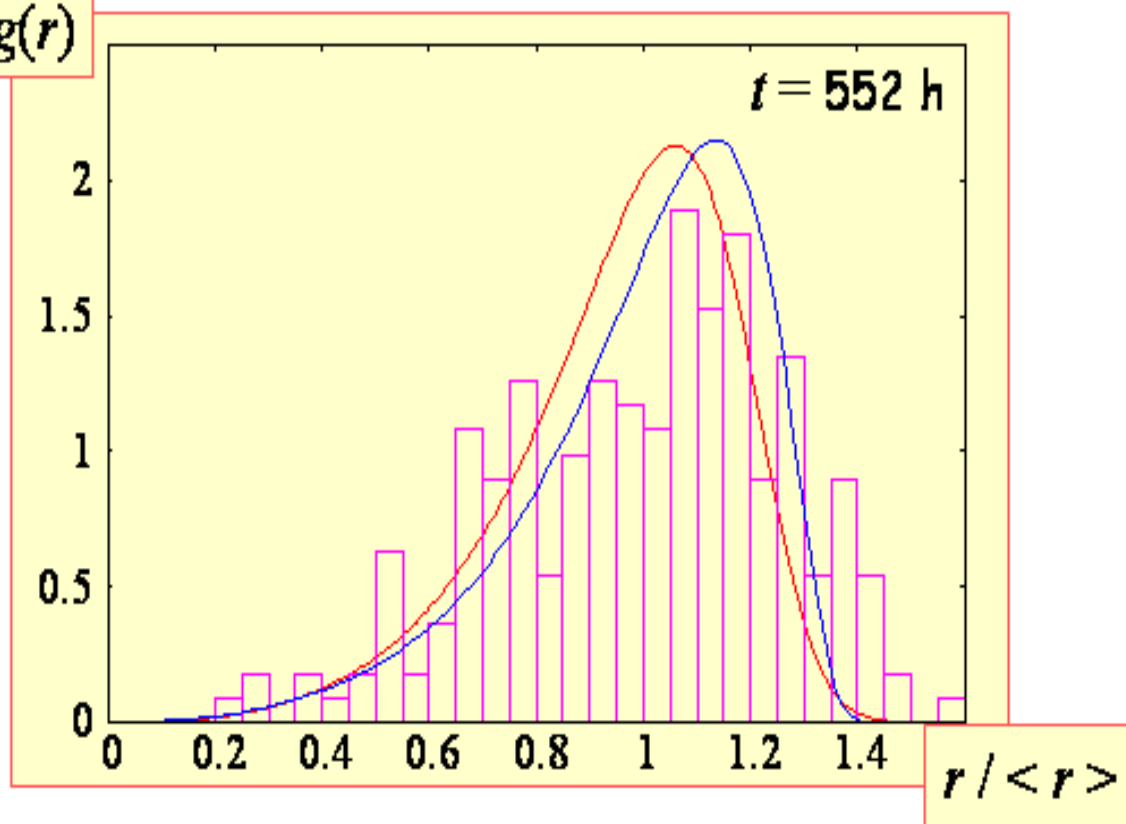
Time (s)

² E. Marquis and D. Seidman, *Nanoscale structural evolution of Al₃Sc precipitates in Al(Sc) alloys*, Acta Mater. **49** (2001) 1909

Confrontation with Experimental Data

Al + 0.18 % Sc at 350°C : size distribution

$g(r)$



Exp. data¹

Cluster dynamics

LSW

¹ G. M. Novotny and A. J. Ardell, *Precipitation of Al_3Sc in binary Al-Sc alloys*, Mater. Sci. Eng. **A31B** (2001) 144-154

Conclusions

✓ **Binaries** Al-Zr and Al-Sc

Good agreement between :

- KMC / cluster dynamics
- cluster dynamics / experimental data

➡ **Quantitative multiscale modelling of the precipitation kinetics**

✓ Precipitation in **ternary** alloy Al-Zr-Sc

- Nucleation of a compound $\text{Al}_3\text{Zr}_x\text{Sc}_{1-x}$ predicted by KMC
- Growth mainly controlled by Sc

➡ **Need to generalize mesoscopic model to ternary alloys in order to increase range of supersaturations and annealing times that can be predicted**