The effect of the timing of selection on the mutation load and population size.

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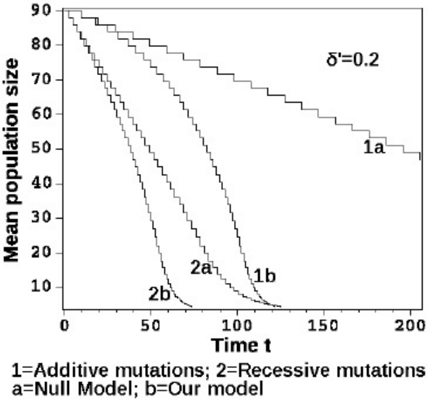
Small populations

→ Mutational meltdown
(Lande 1994, Lynch et al. 1995, Coron et al. 2013)



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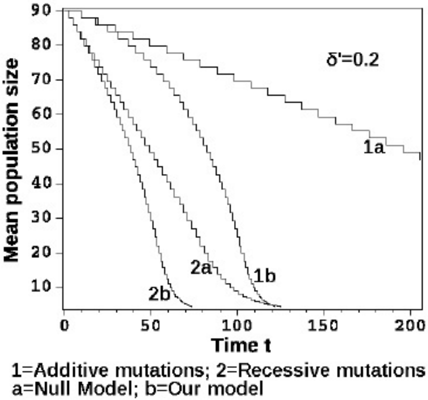

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"... we have emphasized that load has no direct relationship to population abundance or persistence. Instead, mutation load refers to the reduction in fitness of individuals, not populations, relative to a mutation-free reference genotype."

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$$\frac{dN_2}{dt} = N_2\left(\frac{w_2k_2}{k_2 + w_2N_2 + \alpha_2 w_1 N_1} - 1\right).$$

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A combination of ecological and genetic models makes it possible to predict the effects of mutation on population size. Although all disadvantageous mutants produce the same genetic load (as conventionally defined) different types of mutants may have different effects upon the numerical equilibrium.

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Estimating the genetic load L

Demography is neglected in models studying L

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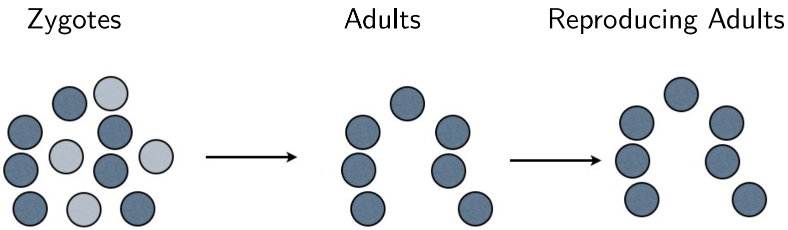

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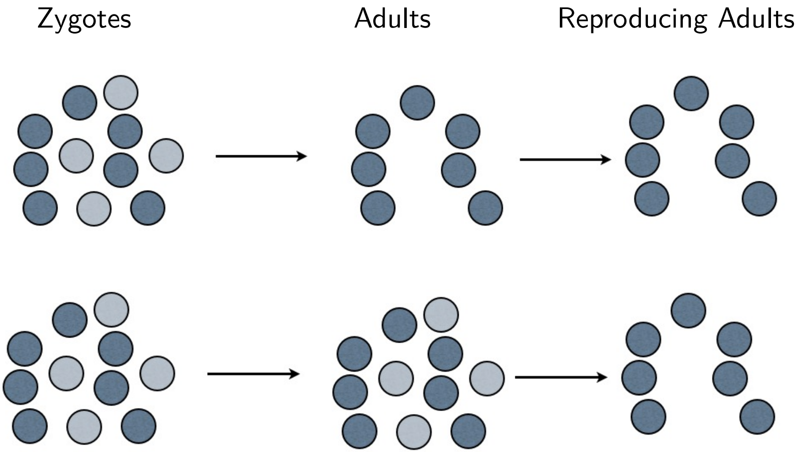

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General model

- Panmictic population (sexual reproduction)
- One bi-allelic locus (A, the Wild-type and a the mutant).

$$\frac{dN_t}{d_t} = \underbrace{R_t^N}_{R_t^X + R_t^Y + R_t^Z} - \underbrace{M_t^N}_{M_t^X + M_t^Y + M_t^Z}$$

$X = aa$

$Y = Aa$

$Z = AA$

where R_t^N is the total birth rate and M_t^N is the total death rate.

General model

$$R_t^X = b\left(\frac{X_t}{N_t}X_t + \frac{1}{2}\frac{Y_t}{N_t}X_t + \frac{1}{2}\frac{X_t}{N_t}Y_t + \frac{1}{4}\frac{Y_t}{N_t}Y_t\right)$$

$$R_t^Y = b\left(\frac{1}{2}\frac{Y_t}{N_t}Y_t + \frac{1}{2}\frac{X_t}{N_t}Y_t + \frac{1}{2}\frac{Z_t}{N_t}Y_t + \frac{1}{2}\frac{Y_t}{N_t}X_t + \frac{Z_t}{N_t}X_t + \frac{1}{2}\frac{Y_t}{N_t}Z_t + \frac{X_t}{N_t}Z_t\right)$$

$$R_t^Z = b\left(\frac{Z_t}{N_t}Z_t + \frac{1}{2}\frac{Y_t}{N_t}Z_t + \frac{1}{2}\frac{Z_t}{N_t}Y_t + \frac{1}{4}\frac{Y_t}{N_t}Y_t\right))$$

where b is the inherent birth rate.

$$M_t^X = dX_t\frac{N_t}{K}$$

$$M_t^Y = dY_t\frac{N_t}{K}$$

$$M_t^Z = dZ_t\frac{N_t}{K}$$

where K is the carrying capacity and d the inherent death rate.

General model

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General model

Ordinary Differential Equations:

$$\frac{dN_t}{d_t} = (R_t^X + R_t^Y + R_t^Z) - (M_t^X + M_t^Y + M_t^Z) = 0$$

Without Selection or Mutation

At equilibrium:

$\rightarrow$ Genotypic frequencies at Hardy-Weinberg equilibrium

$$\begin{array}{ccc} X & Y & Z \\ p^2 & 2pq & q^2 \end{array}$$

$\rightarrow N_{eq} = \frac{bK}{d}$

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Introducing mutation and selection

Mutation from $A \to a$ occurs at rate μ

Here we consider selection on:

- Model 1 : Reproduction
  - a) Mating success (quality of gametes)
  - b) Fecundity (quantity of gametes)
- Model 2 : Survival
  - a) Adult survival (before reproduction)
  - b) Zygote survival (before consuming any ressources)

Relative Fitnesses: $\begin{array}{ccc} X & Y & Z \\ (1-s) & (1-hs) & 1 \end{array}$

Model 1a : Mating success

$$\frac{dX_t}{dt} =$$

$$\frac{b}{N_t}\left(\widetilde{X}_t^2 + 2\widetilde{X}_t\widetilde{Z}_t\mu + \widetilde{Z}_t^2\mu^2 + \widetilde{X}_t\widetilde{Y}_t(1+\mu) + \widetilde{Y}_t\widetilde{Z}_t\mu(1+\mu) + \frac{1}{4}\widetilde{Y}_t^2(1+\mu)^2\right)$$

$$- dX_t\frac{N_t}{K}$$

$\widetilde{X}_t$	$\widetilde{Y}_t$	$\widetilde{Z}_t$
$(1-s)X_t$	$(1-hs)Y$	Z_t

Model 1b : Fecundity

$$\frac{dX_t}{dt} = \frac{b}{\widetilde{X}_t + \widetilde{Y}_t + \widetilde{Z}_t}\left(\widetilde{X}_t^2 + 2\widetilde{X}_t\widetilde{Z}_t\mu + \widetilde{Z}_t^2\mu^2 + \widetilde{X}_t\widetilde{Y}_t(1+\mu) + \widetilde{Y}_t\widetilde{Z}_t\mu(1+\mu) + \frac{1}{4}\widetilde{Y}_t^2(1+\mu)^2\right) - dX_t\frac{N_t}{K}$$

$\widetilde{X}_t$	$\widetilde{Y}_t$	$\widetilde{Z}_t$
$(1-s)X_t$	$(1-hs)Y$	Z_t

Model 2a : Adult survival

$$\frac{dX_t}{dt} =$$

$$\frac{b}{\widetilde{X}_t + \widetilde{Y}_t + \widetilde{Z}_t} \Big(\widetilde{X}_t^2 + 2\widetilde{X}_t \widetilde{Z}_t \mu + \widetilde{Z}_t^2 \mu^2 + \widetilde{X}_t \widetilde{Y}_t (1+\mu)$$

$$+ \widetilde{Y}_t \widetilde{Z}_t \mu (1+\mu) + \frac{1}{4} \widetilde{Y}_t^2 (1+\mu)^2 \Big)$$

$$- dX_t \frac{\widetilde{X}_t + \widetilde{Y}_t + \widetilde{Z}_t}{K}$$

$\widetilde{X}_t$	$\widetilde{Y}_t$	$\widetilde{Z}_t$
$(1-s)X_t$	$(1-hs)Y$	Z_t

Model 2b : Zygote survival

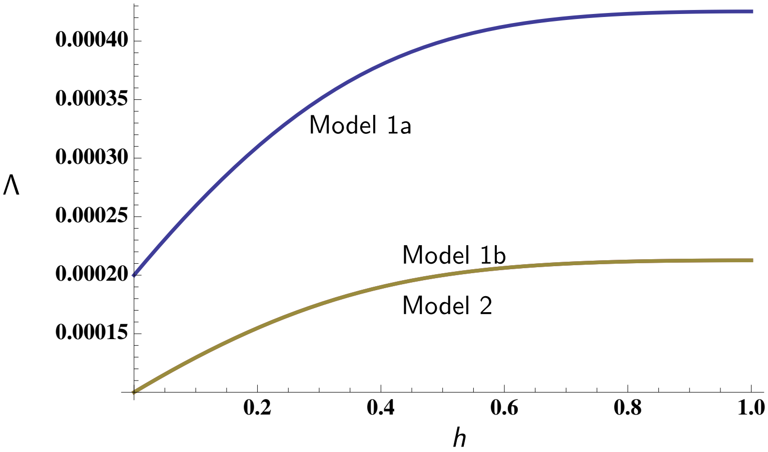
$$\frac{dX_t}{dt} = \frac{b(1-s)}{N_t}\Big(X_t^2 + 2X_tZ_t\mu + Z_t^2\mu^2 + X_tY_t(1+\mu) + Y_tZ_t\mu(1+\mu) + \frac{1}{4}Y_t^2(1+\mu)^2\Big) - dX_t\frac{N_t}{K}$$

Numerical Load Λ

Recessive mutations ($h = 0$)

Model	N_{mut}	Λ
Mating success	$N_{eq}(1-\mu)^2$	$2\mu - \mu^2$
Fecundity	$N_{eq}(1-\mu)$	μ
Adult survival	N_{eq}	–
Zygote survival	$N_{eq}(1-\mu)$	μ

Numerical Load Λ

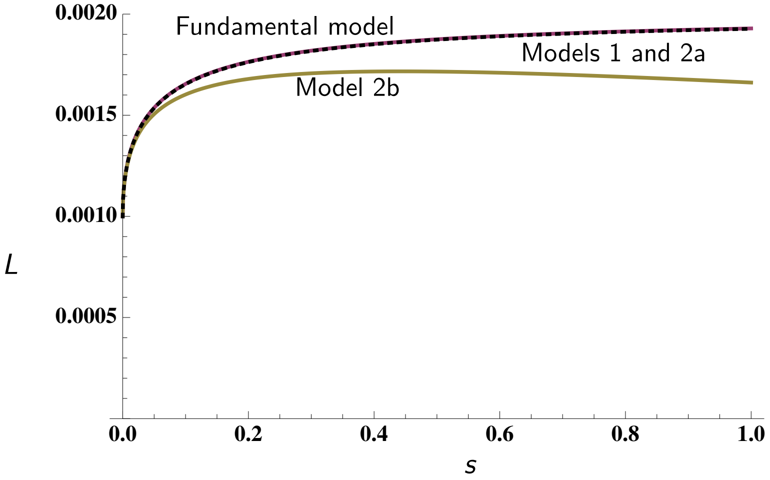


Genetic Load L

Recessive mutations ($h = 0$)

Model	Λ	L	μ_{fix}
Mating success	$2\mu - \mu^2$	μ	s
Fecundity	μ	μ	s
Adult survival	–	μ	s
Zygote survival	μ	$\frac{\mu(1-s)}{1-\mu}$	s
Fundamental model	–	$\frac{\mu}{1+\mu}$	$\frac{s}{1-s}$

Genetic Load L



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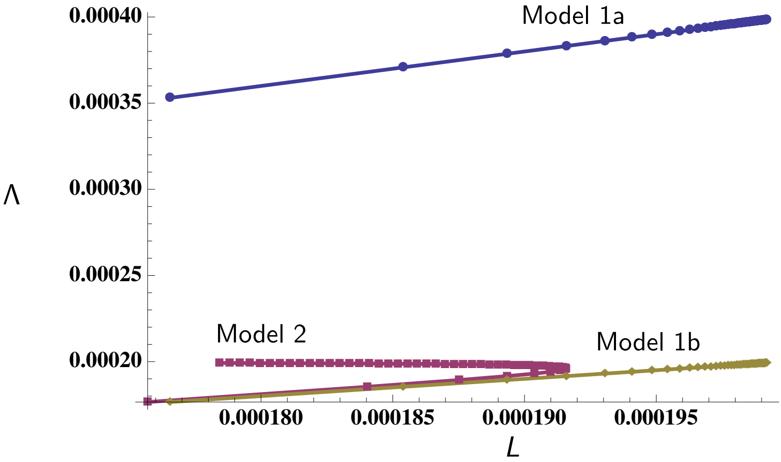
The interaction between Λ and L



Numerical Load Λ

Model 1a: Mating succes

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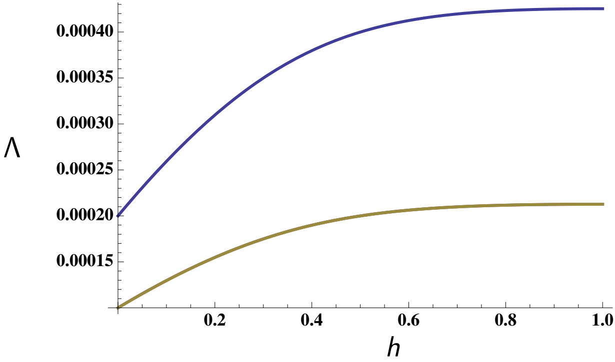
$$\begin{array}{ccc} \widetilde{X}_t & \widetilde{Y}_t & \widetilde{Z}_t \\ (1-s)X_t & (1-hs)Y & Z_t \end{array}$$

General conclusions

Population size can be affected by load

$\rightarrow$ but how important is this effect?



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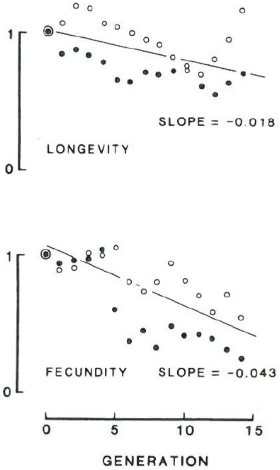
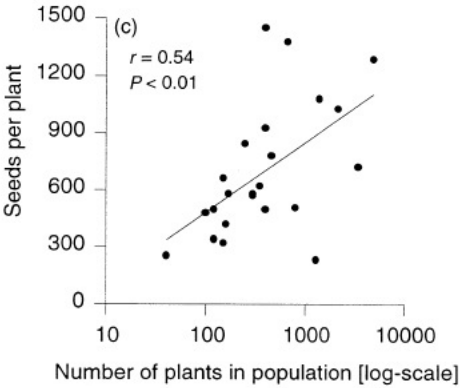
Timing of selection

	Model 1	Model 2
L	μ	$\frac{\mu(1-s)}{1-\mu}$



Bell 1988

Population size: Cause or Consequence?



Fischer and Matthies 1998
Gentianella germanica