Diffusion and Cascading Behavior in Random Networks

Emilie Coupechoux (LIP6)
Marc Lelarge (INRIA-ENS)

ANR MANEGE

January 30, 2013.

(1) Diffusion Model

inspired from game theory
and statistical physics.

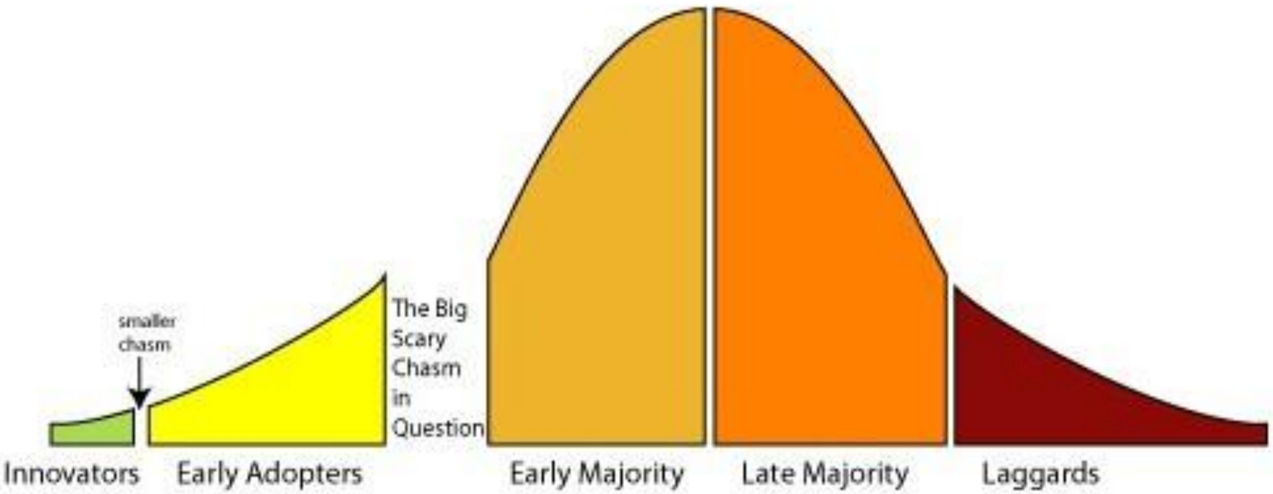
(2) Results

from a mathematical analysis.

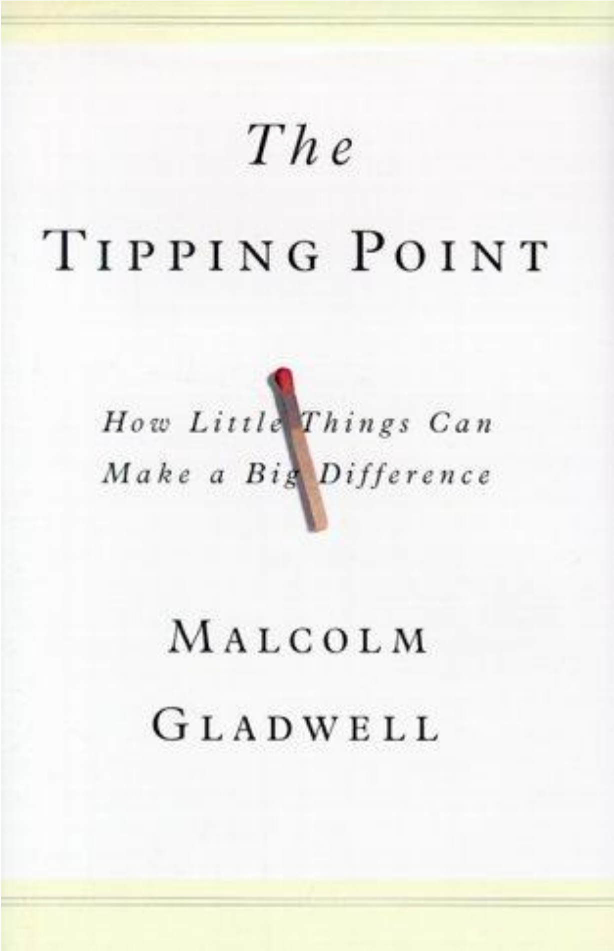
(3) Adding Clustering

Joint work with Emilie Coupechoux

(0) Context



Crossing the Chasm
(Moore 1991)



(1) Diffusion Model

(2) Results

(3) Adding Clustering

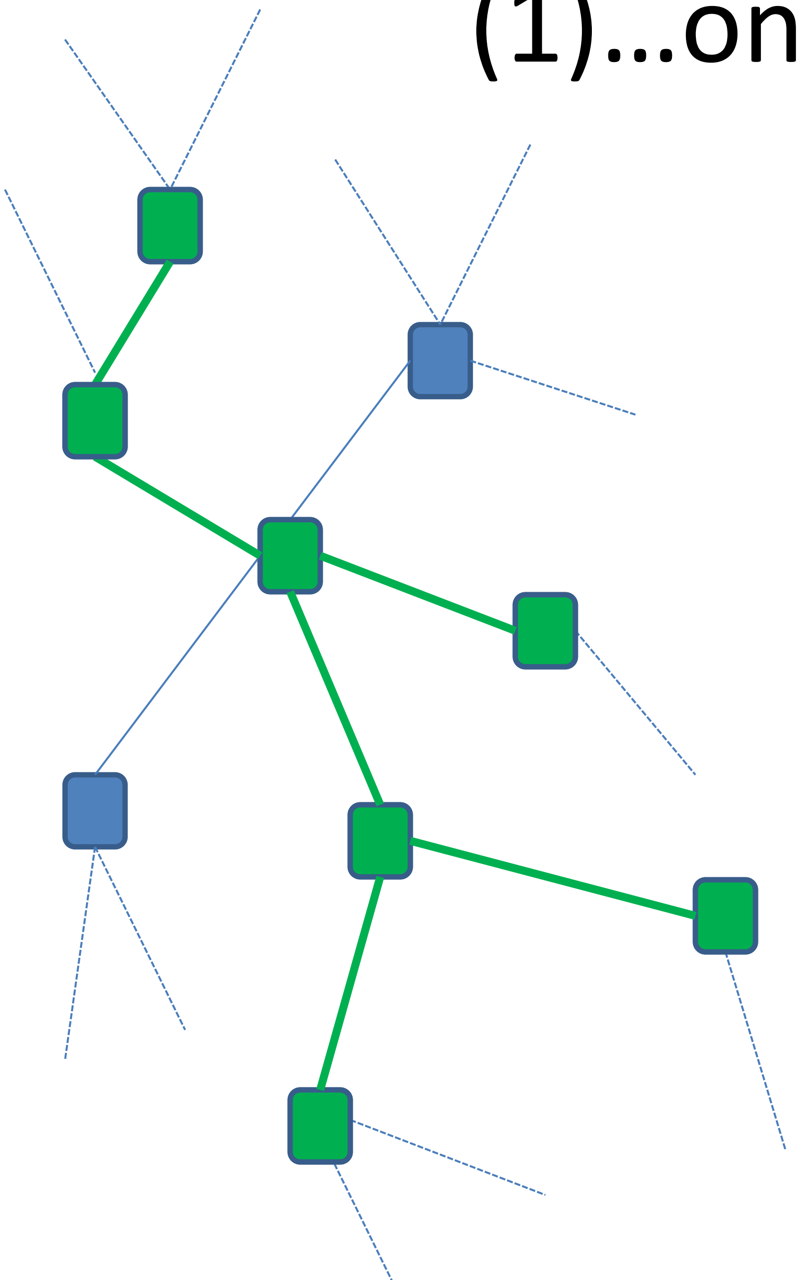
(1) Coordination game…

- Both receive payoff q.

- Both receive payoff 1-q>q.

- Both receive nothing.

(1)…on a network.



- Everybody start with icq everybody, everywhere
- Total payoff = sum of the payoffs with each neighbor.
- A seed of nodes switches to Google talk BETA

(Blume 95,
Morris 00)

(1) Threshold Model

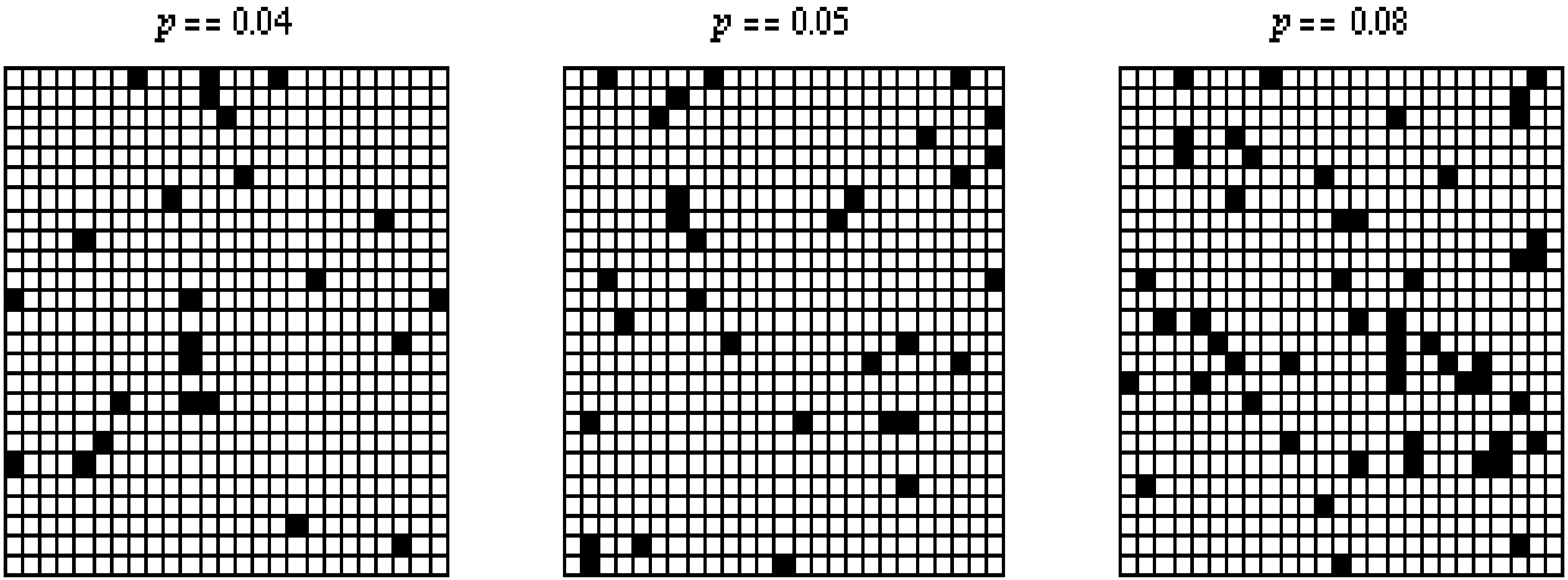
- State of agent i is represented by

$$X_i = \begin{cases} 0 & \text{if ICQ} \\ 1 & \text{if Google Talk} \end{cases}$$

- Switch from icq to Google talk if:

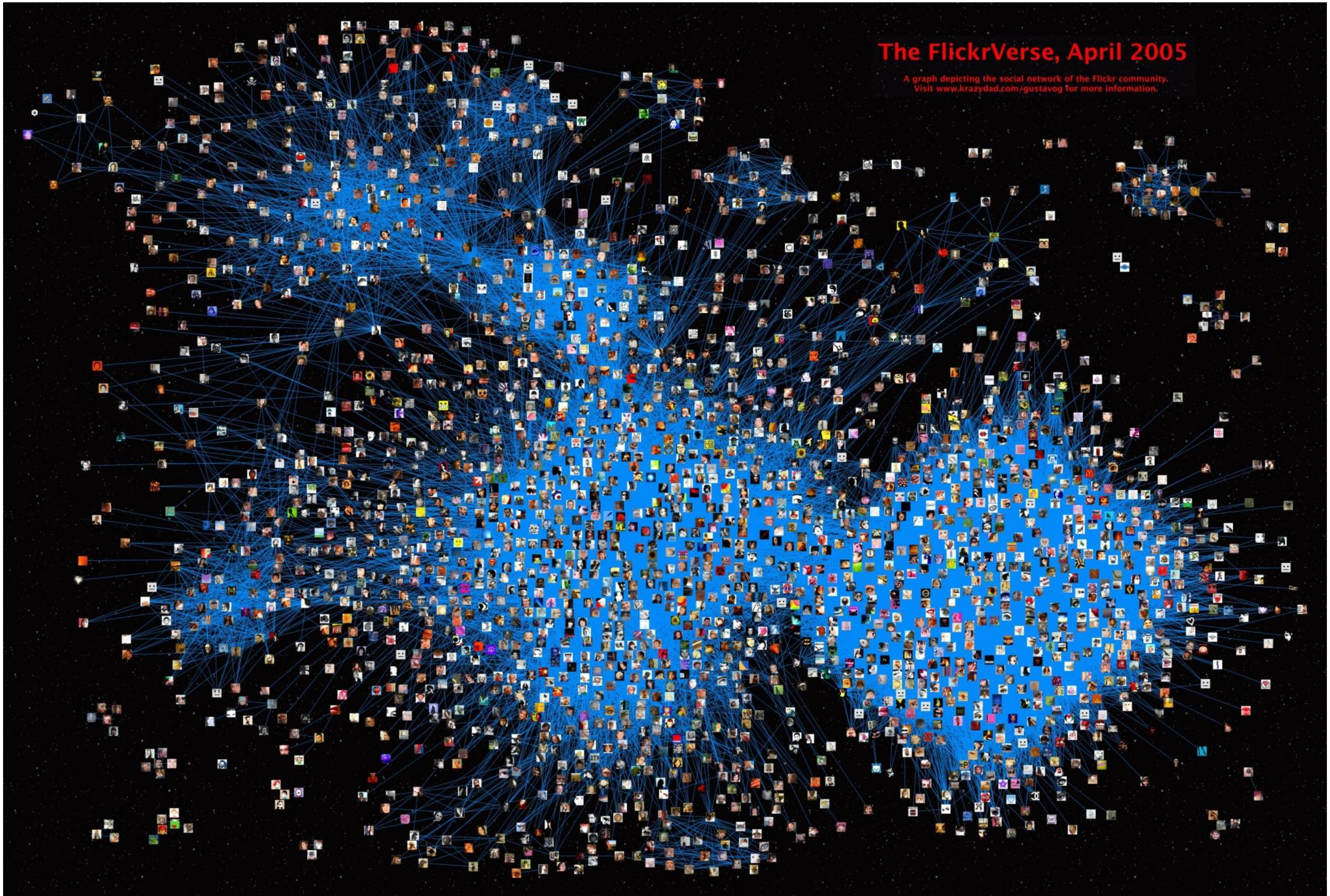
$$\sum_{j \sim i} X_j \geq q d_i$$

(1) Model for the network?



Statistical physics: bootstrap percolation.

(1) Model for the network?



(1) Random Graphs

- Random graphs with given degree sequence introduced by (Molloy and Reed, 95).
- Examples:
  - Erdös-Réyni graphs, G(n,λ/n).
  - Graphs with power law degree distribution.
- We are interested in large population asymptotics.
- Average degree is λ.
- No clustering: C=0.

(1) Diffusion Model

q = relative threshold

λ = average degree

(2) Results

(3) Adding Clustering

(1) Diffusion Model

q = relative threshold

λ = average degree

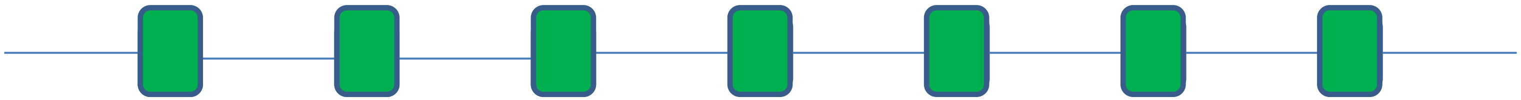
(2) Results

(3) Adding Clustering

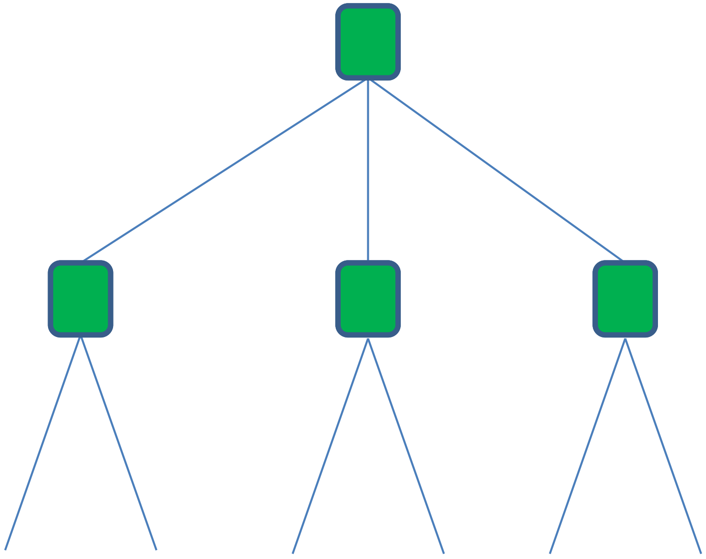
(2) Contagion (Morris 00)

- Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?
- Contagion threshold: q_c = largest q for which contagious dynamics are possible.
- Example: interaction on the line

$$q_c = \frac{1}{2}$$

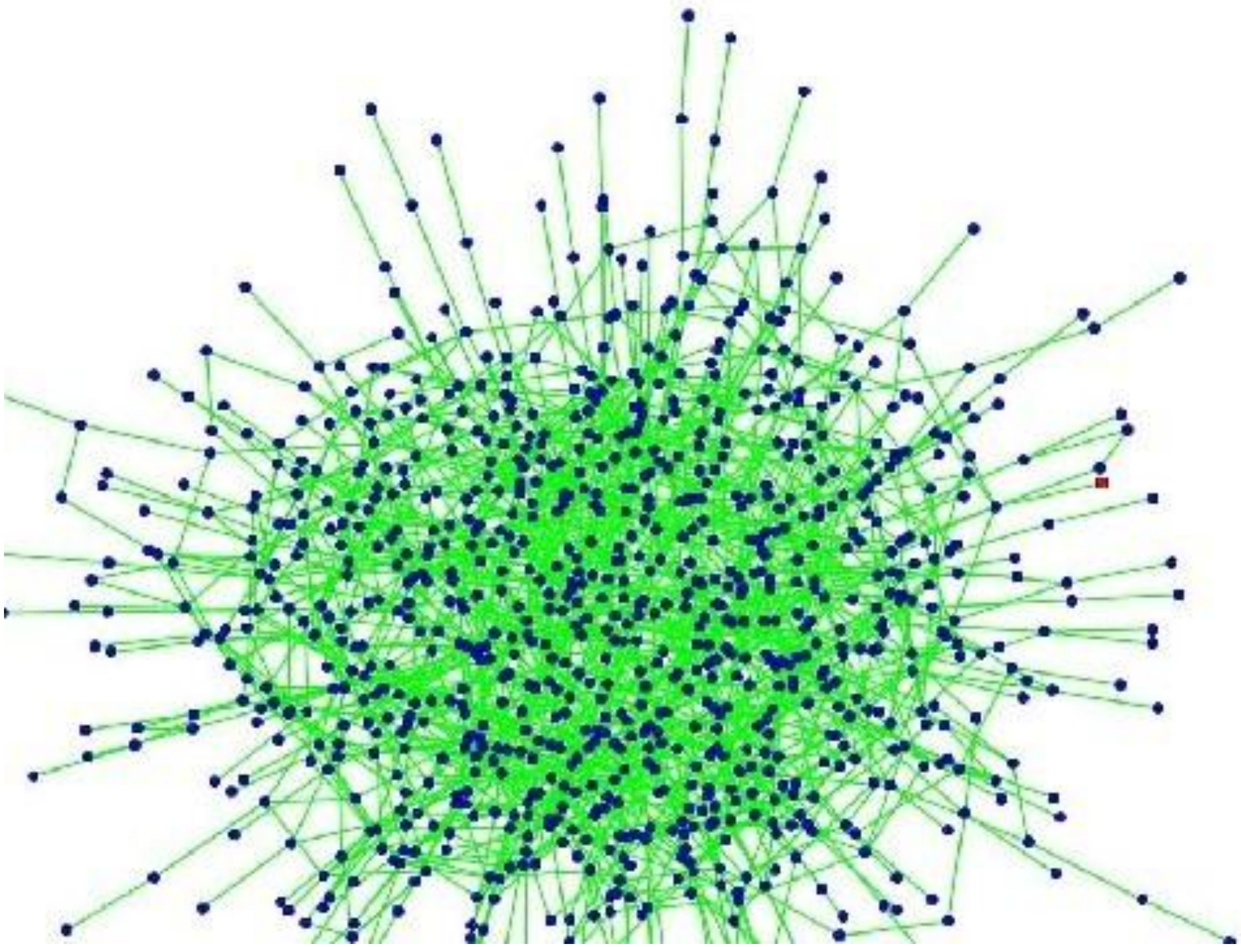


(2)Another example: d-regular trees



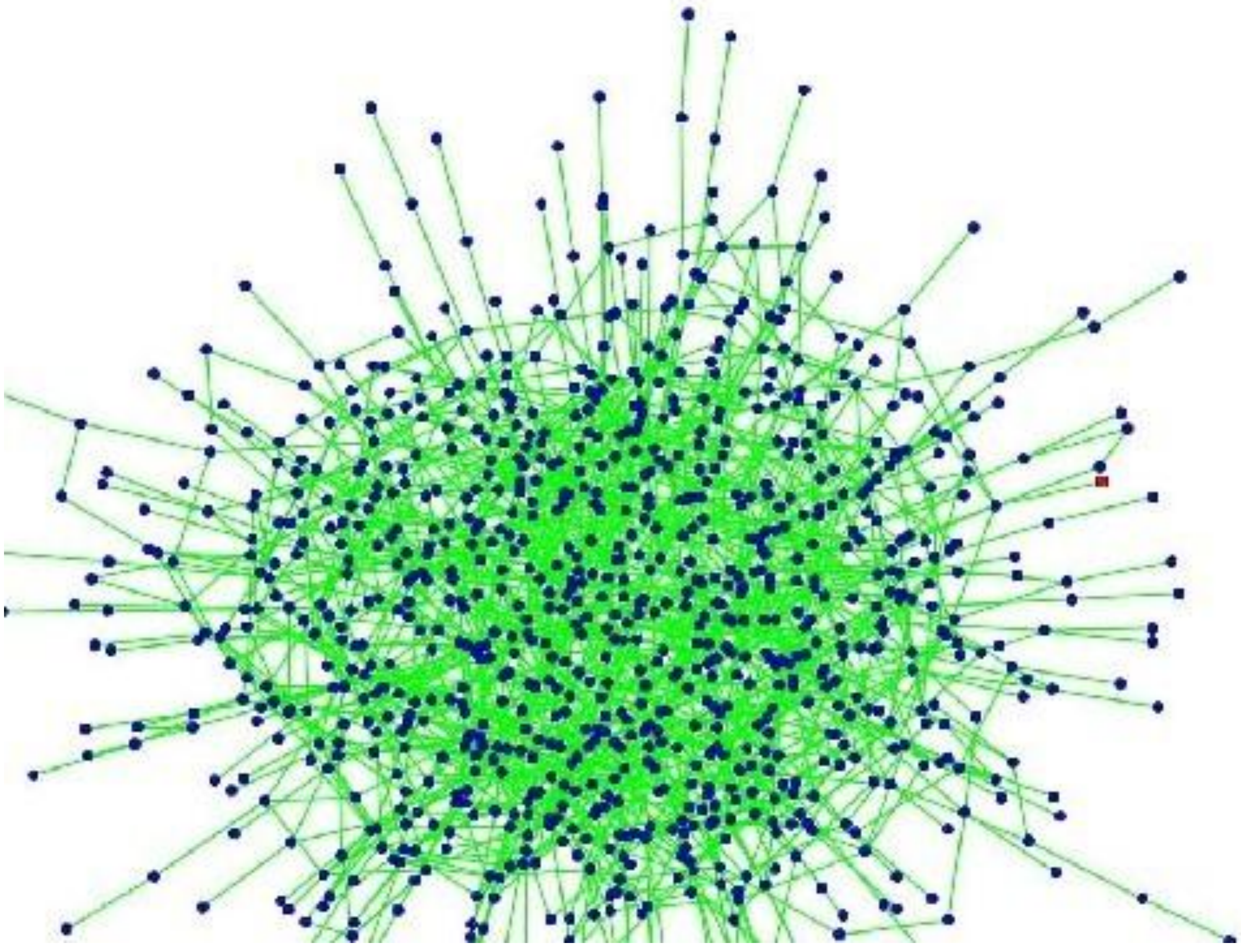
$$q_c = \frac{1}{d}$$

(2) Some experiments



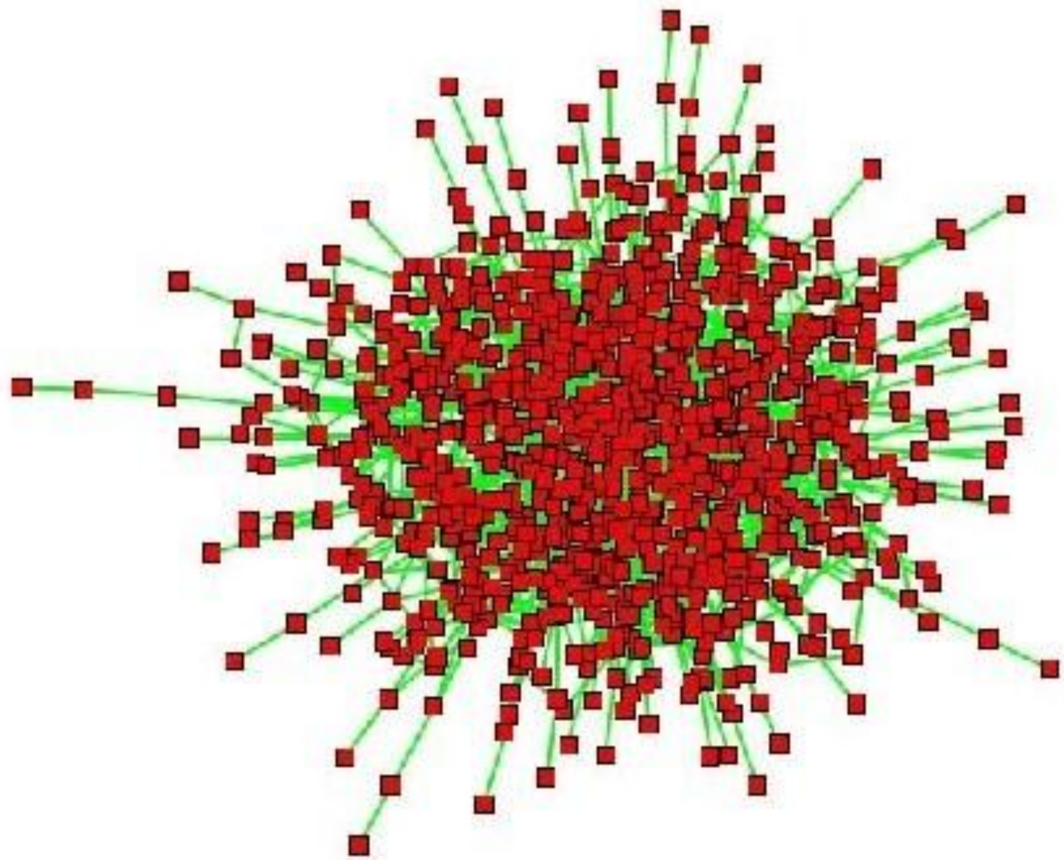
Seed = one node, λ=3 and q=0.24
(source: the Technoverse blog)

(2) Some experiments



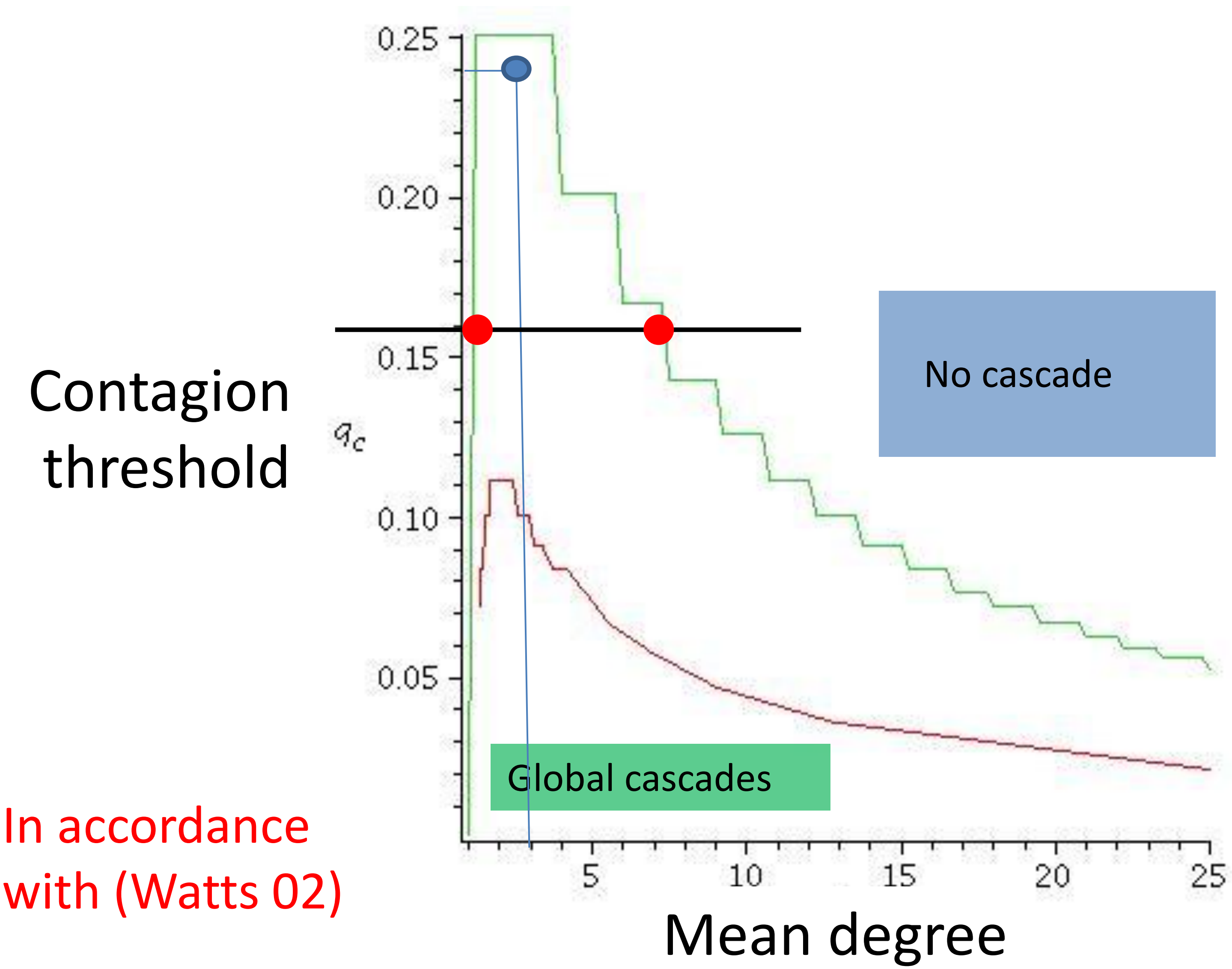
Seed = one node, λ=3 and 1/q>4
(source: the Technoverse blog)

(2) Some experiments



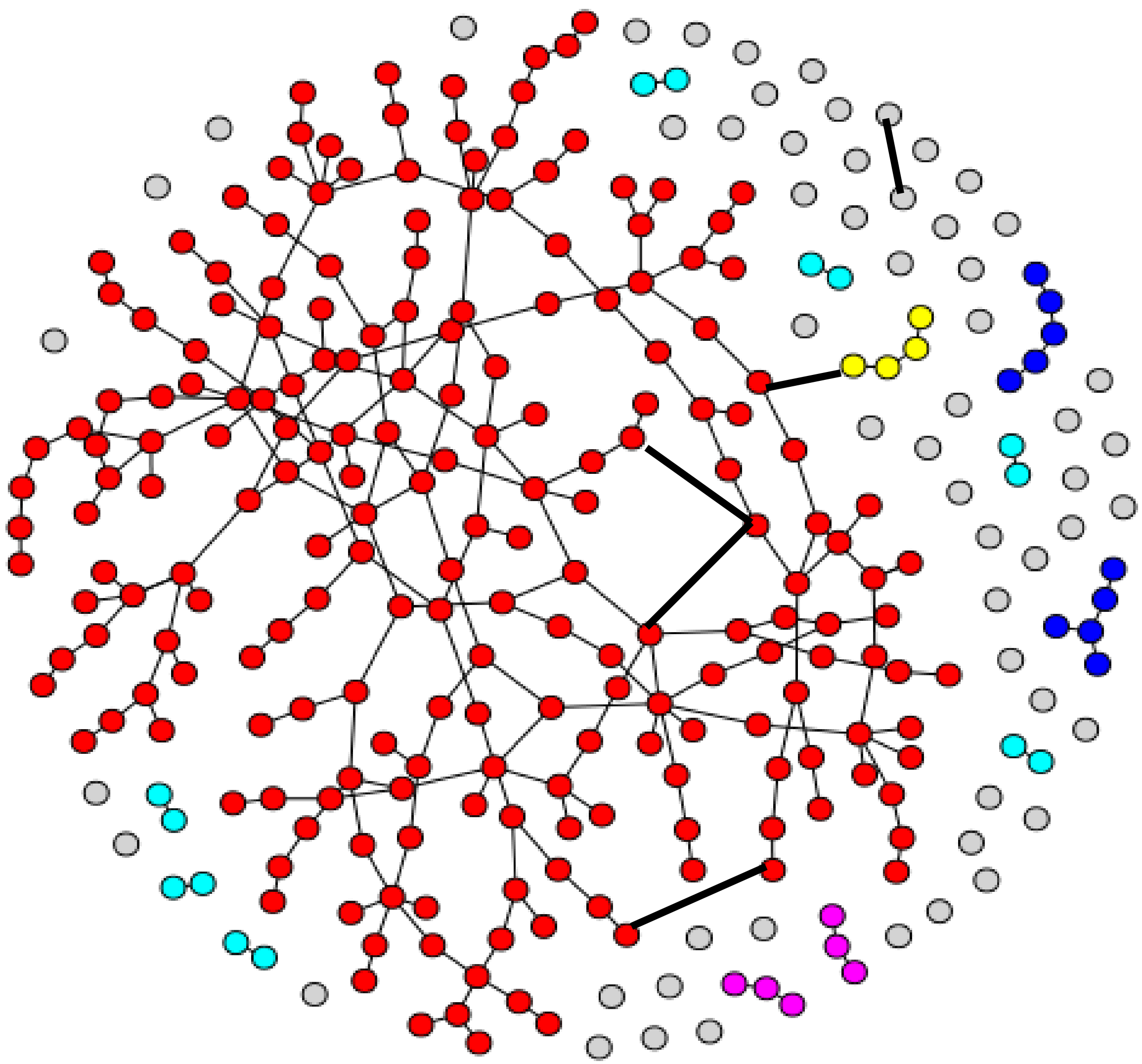
Seed = one node, λ=3 and q=0.24 (or 1/q>4)
(source: the Technoverse blog)

(2) Contagion threshold



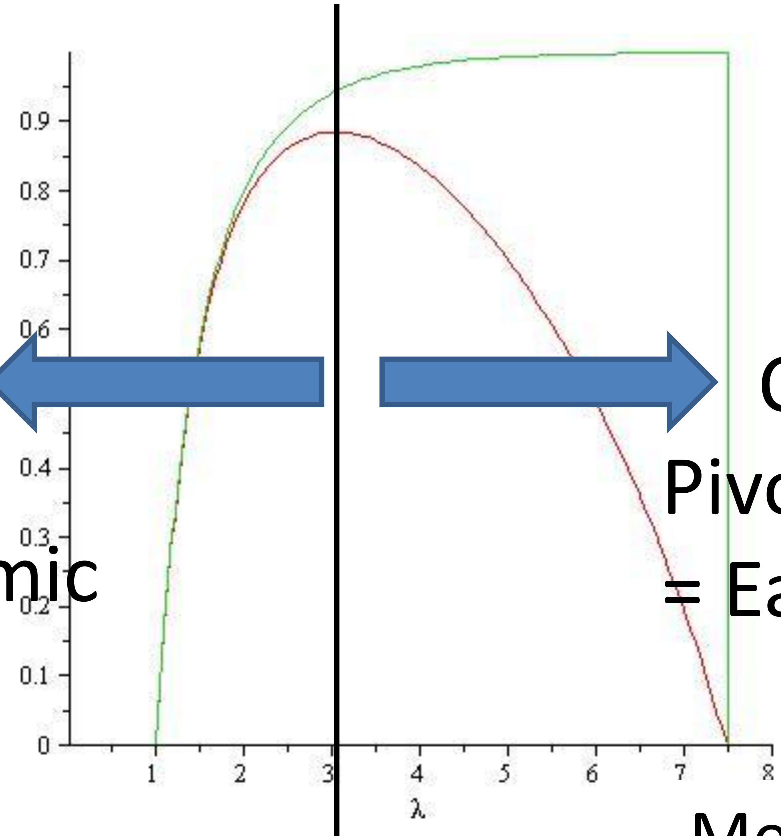
In accordance with (Watts 02)

(2) A new Phase Transition



(2) Pivotal players

- Giant component of players requiring only one neighbor to switch: deg <1/q.



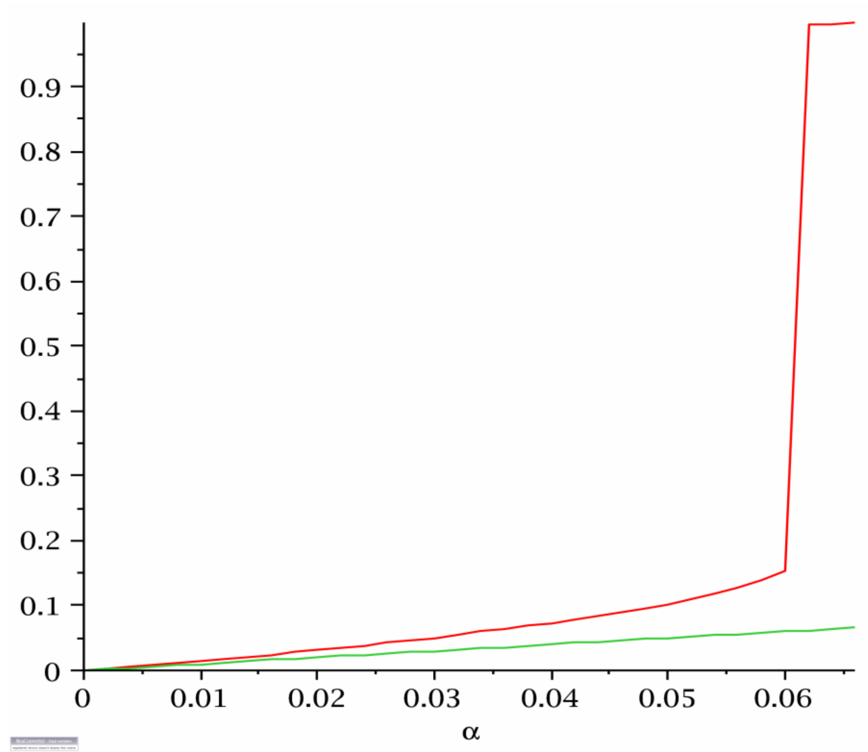
Tipping point: Diffusion like standard epidemic

Chasm : Pivotal players = Early adopters

Mean degree

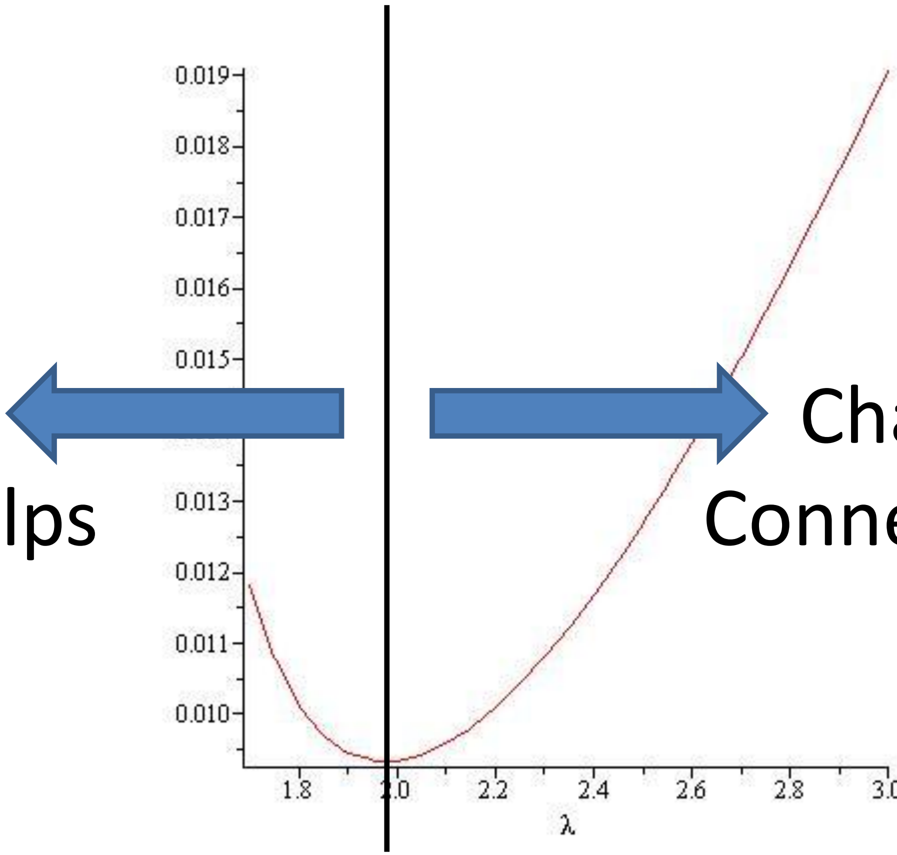
(2) q above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic → only one final state.



(2)Minimal size of the seed, q>1/4

Tipping point:
Connectivity helps

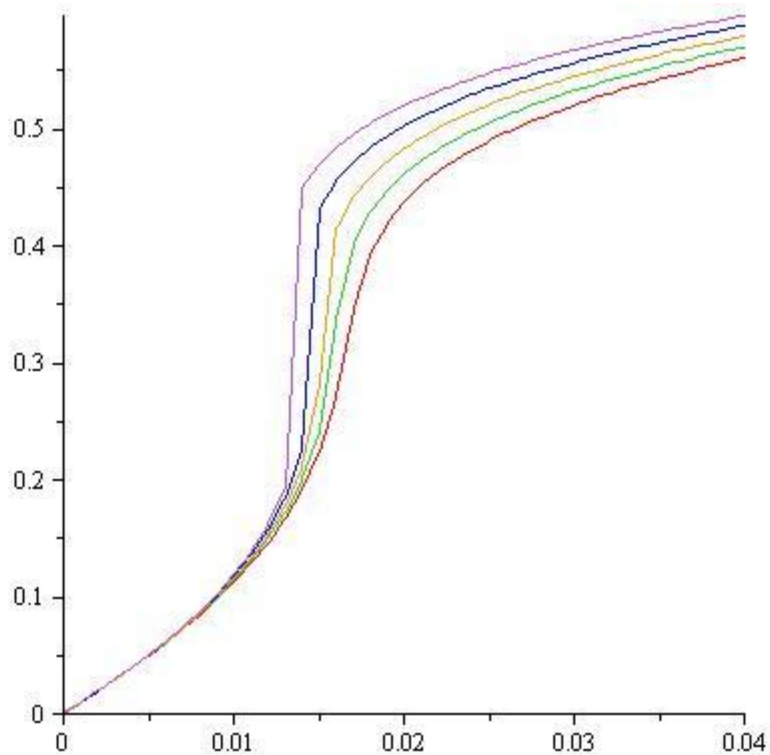


Chasm :
Connectivity hurts

Mean degree

(2) q>1/4, low connectivity

Size of the contagion

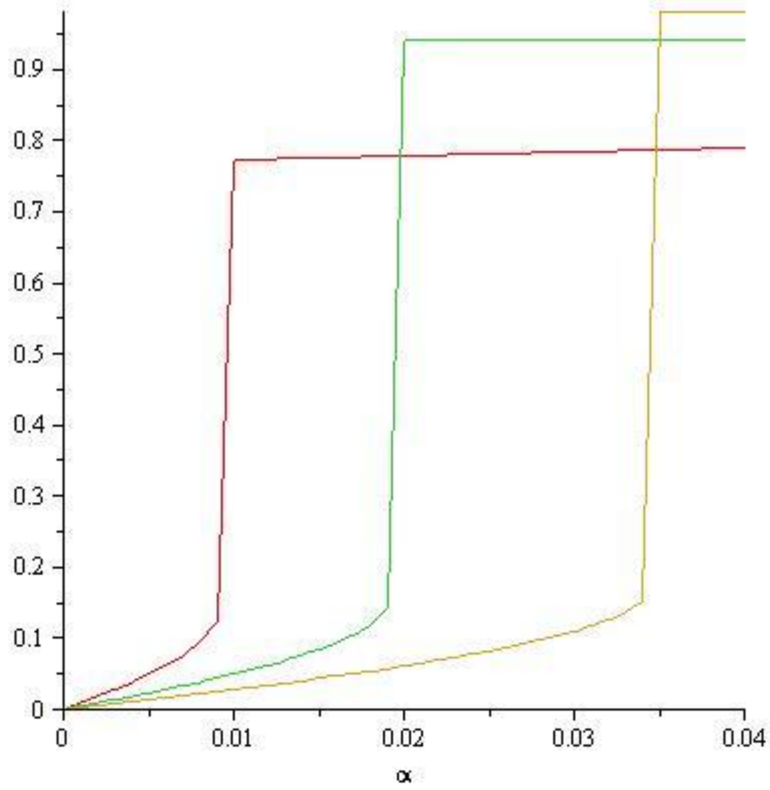


Size of the seed

Connectivity helps the diffusion.

(2) q>1/4, high connectivity

Size of the contagion



Size of the seed

Connectivity inhibits the global cascade,
but once it occurs, it facilitates its diffusion.

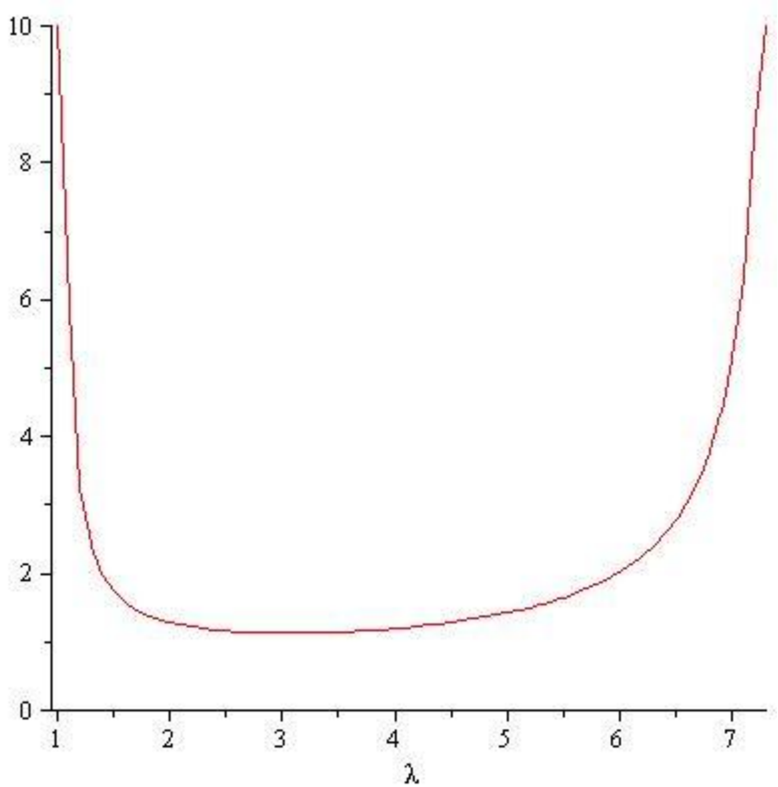
(2) Equilibria for $q<q_c$

- Trivial equilibria: all A / all B
- Initial seed applies best-response, hence can switches back. If the dynamic converges, it is an equilibrium.
- Robustness of all A equilibrium?
- Initial seed = 2 pivotal neighbors

  –> pivotal equilibrium

(2) Strength of Equilibria for $q<q_c$

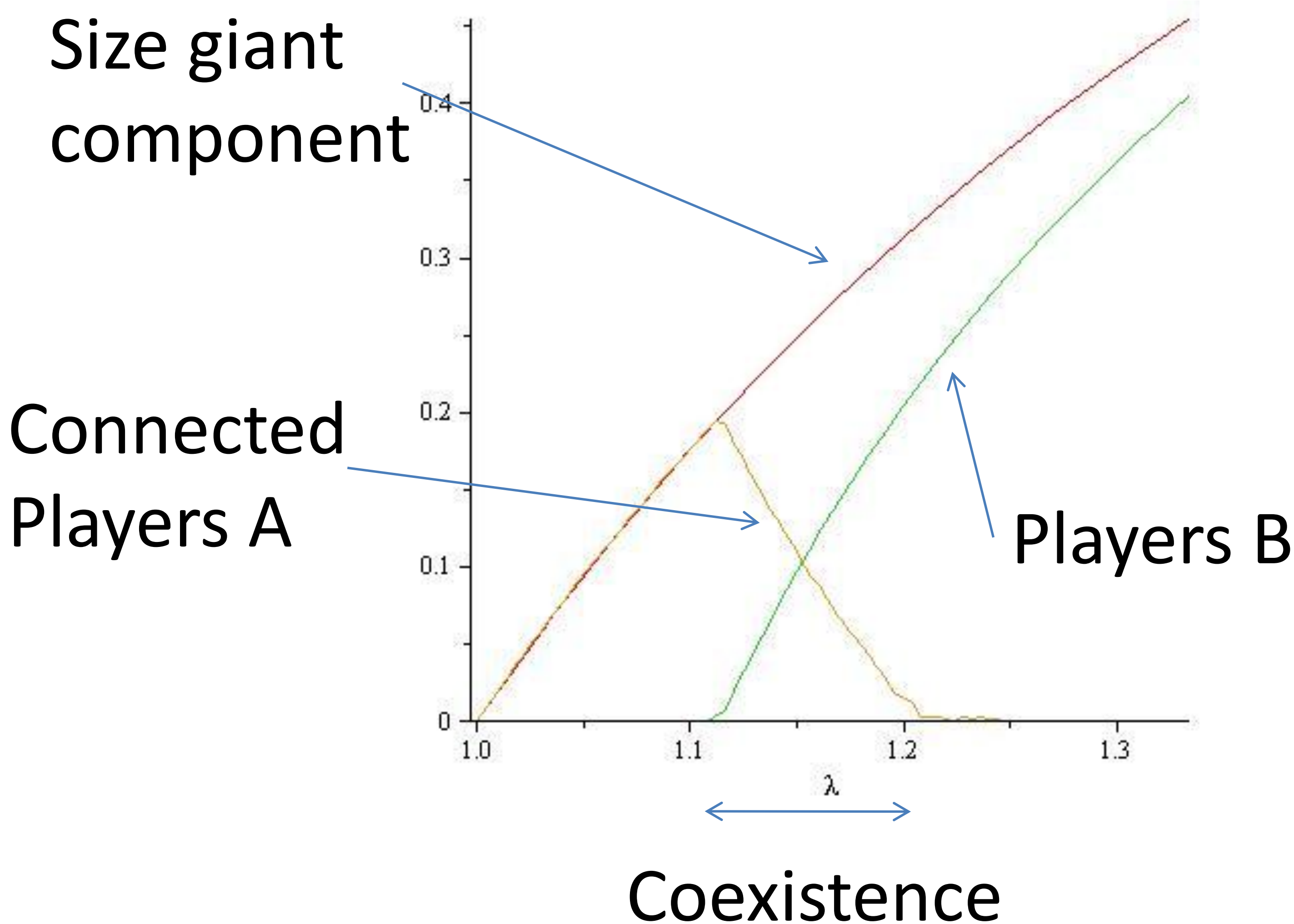
Mean number of trials to switch from all A to pivotal equilibrium



Mean degree

In Contrast with (Montanari , Saberi 10) Their results for $q \approx 1/2$

(2) Coexistence for $q<q_c$



(1) Diffusion Model

(2) Results

(3) Adding Clustering

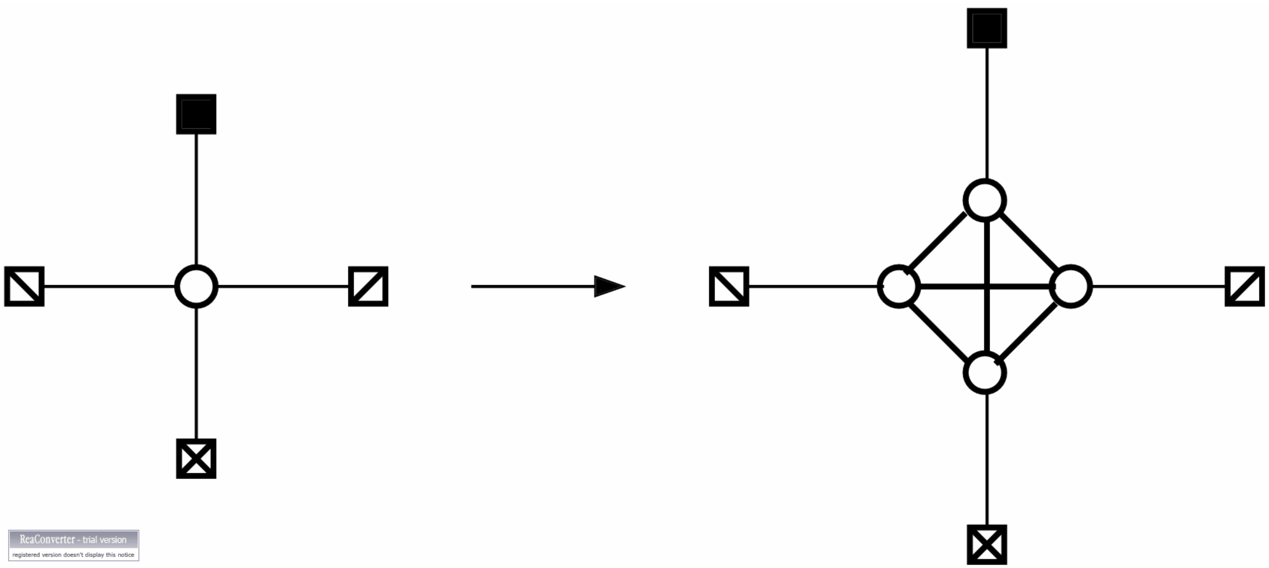
joint work with Emilie Coupechoux

(3) Simple model with tunable clustering

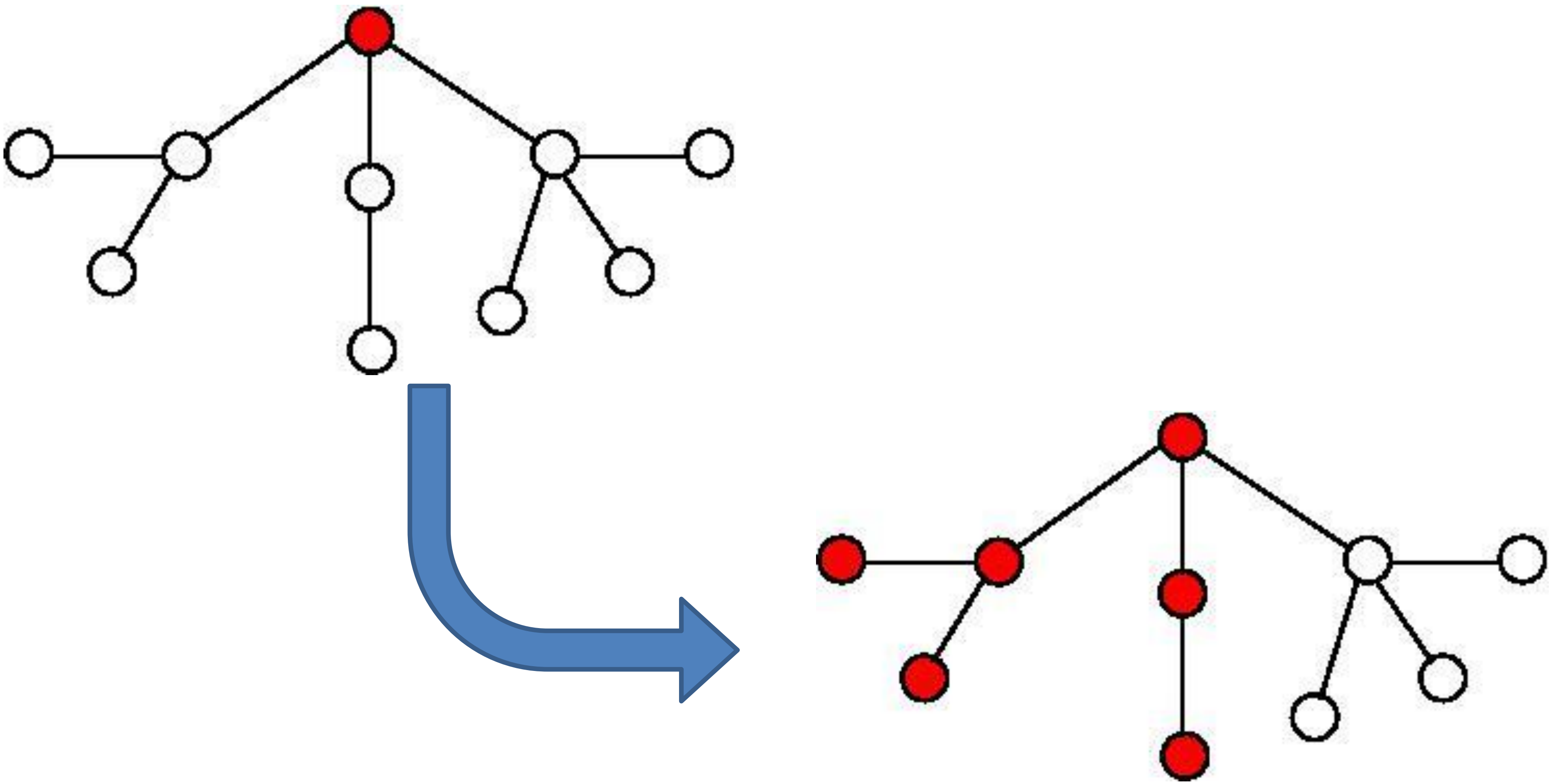
- Clustering coefficient:

$$C = \frac{3 \text{ number of triangles}}{\text{number of connected triples}}$$

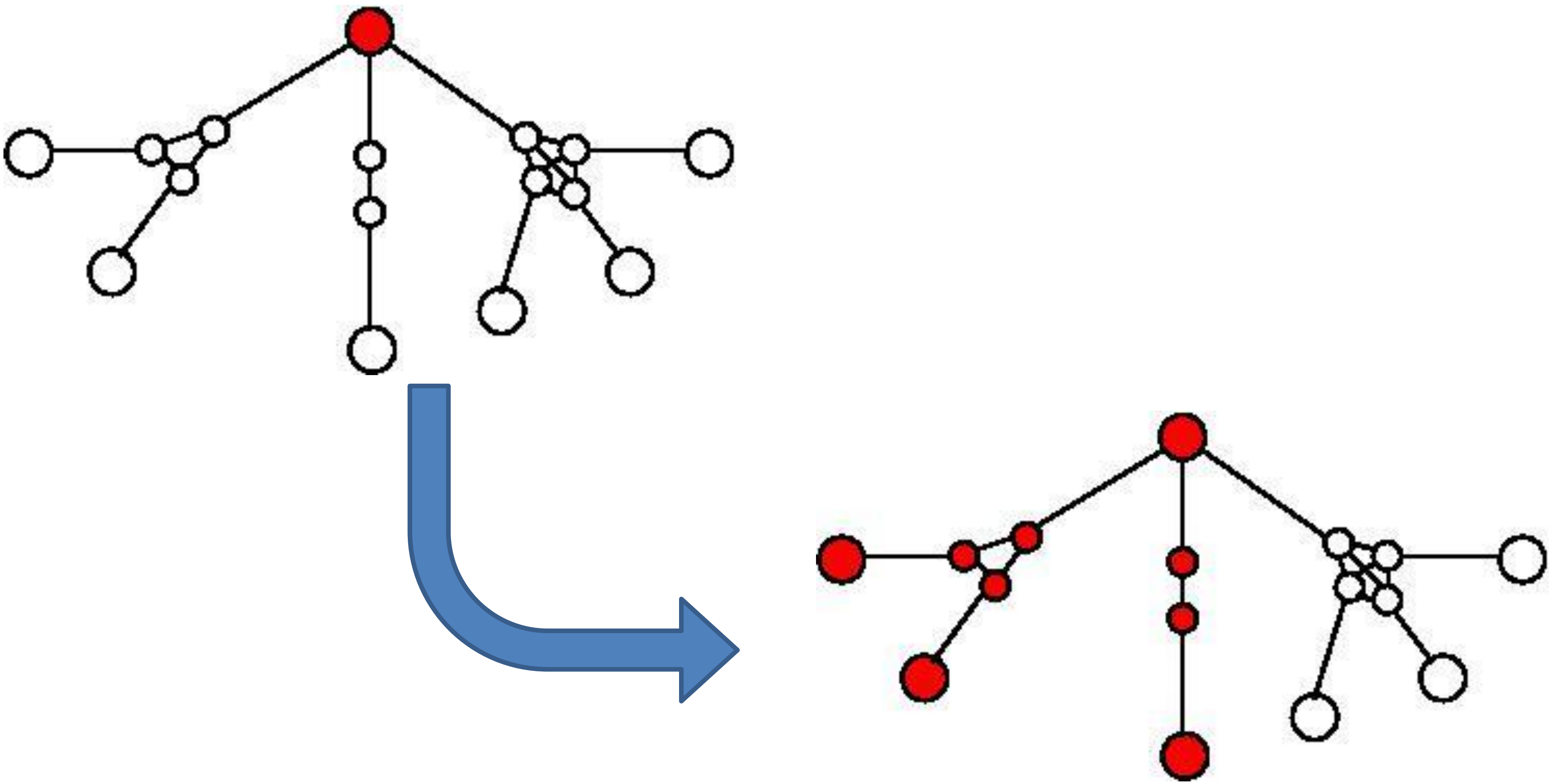
- Adding cliques (Trapman 07)



(3) Pivotal players are the same!



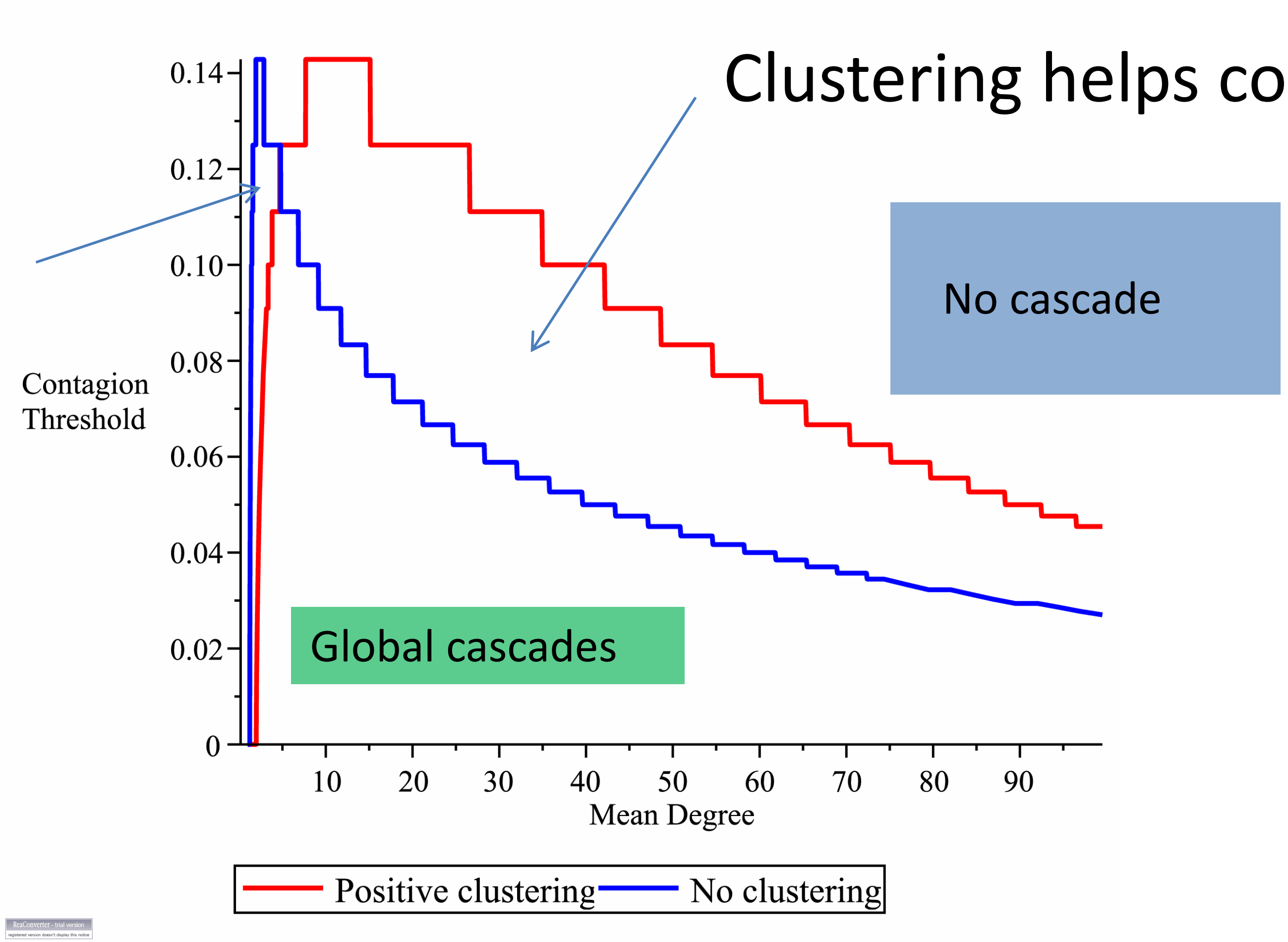
(3) Pivotal players are the same!



(3) Contagion threshold with clustering

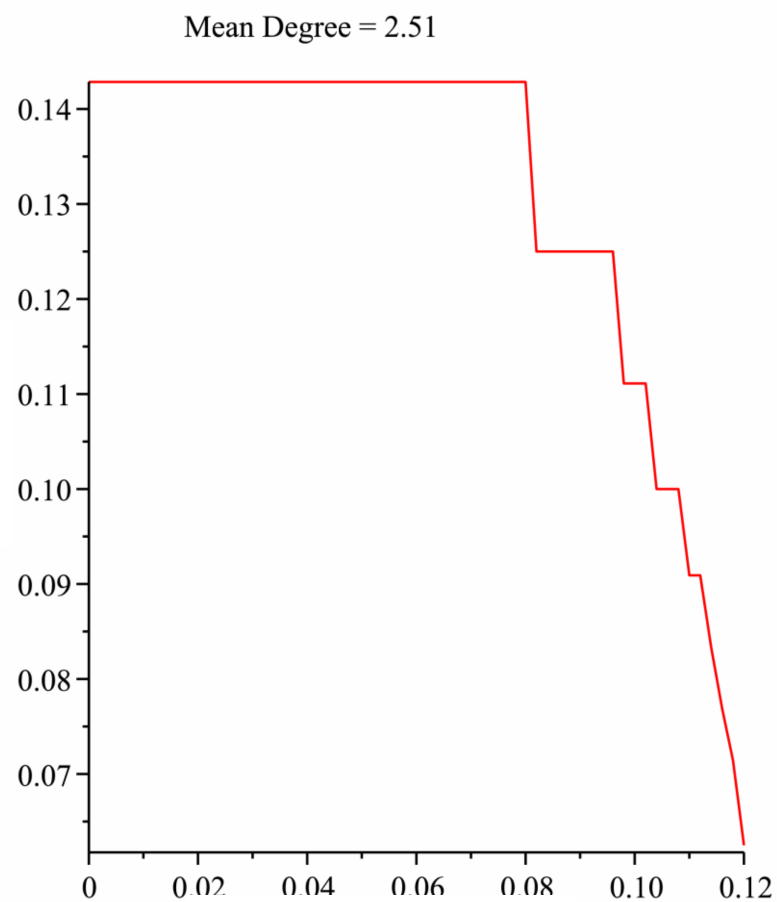
Clustering inhibits contagion

Clustering helps contagion



(3) Low connectivity: clustering hurts contagion

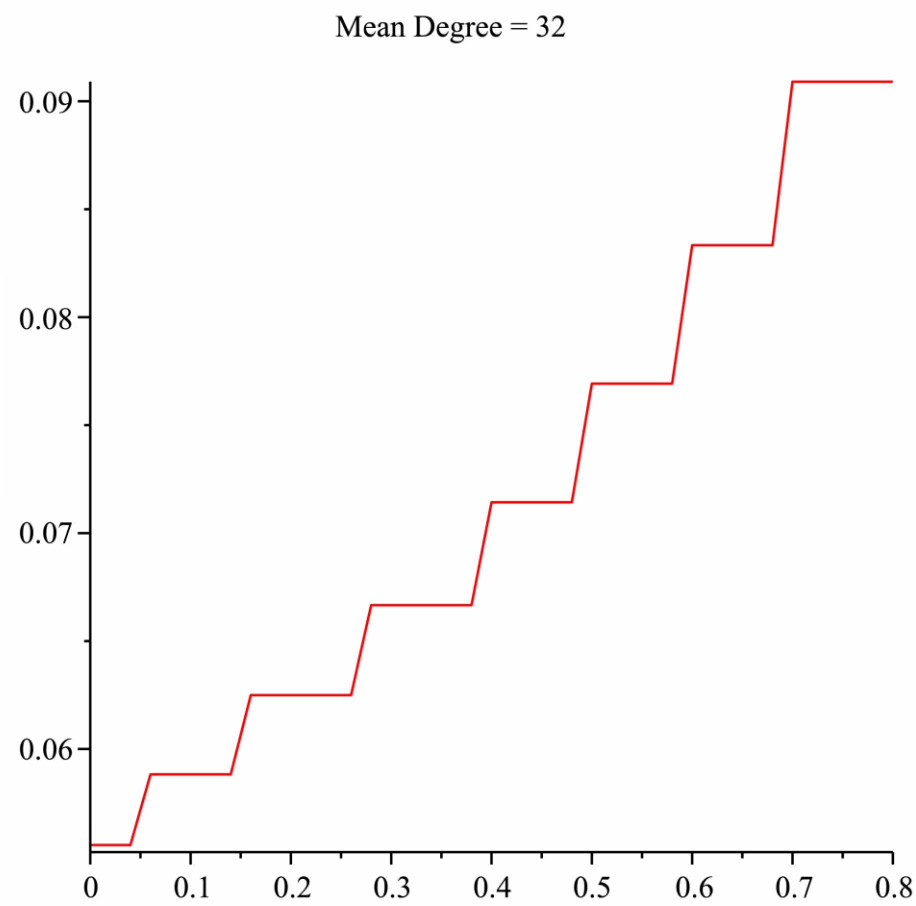
Contagion threshold



Clustering

(3) High connectivity: clustering helps contagion

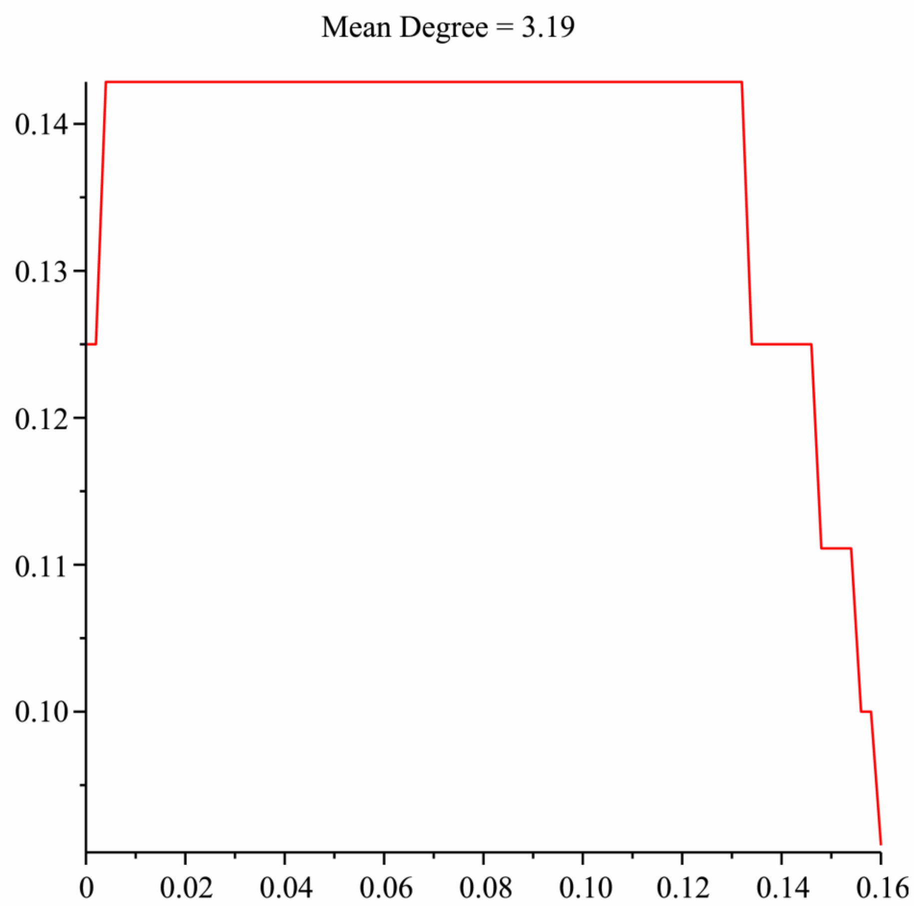
Contagion threshold



Clustering

(3) Intermediate regime: non-monotone effect of clustering

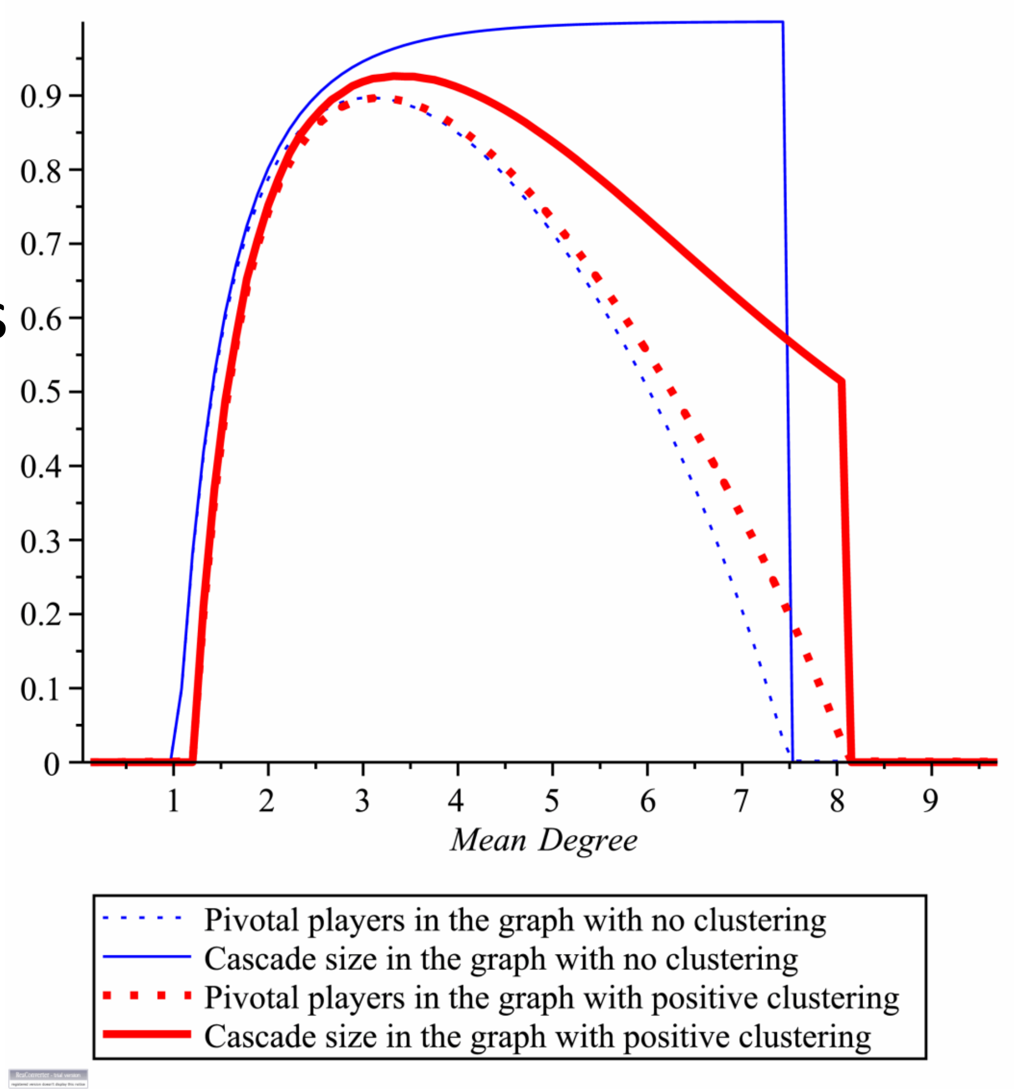
Contagion threshold



Clustering

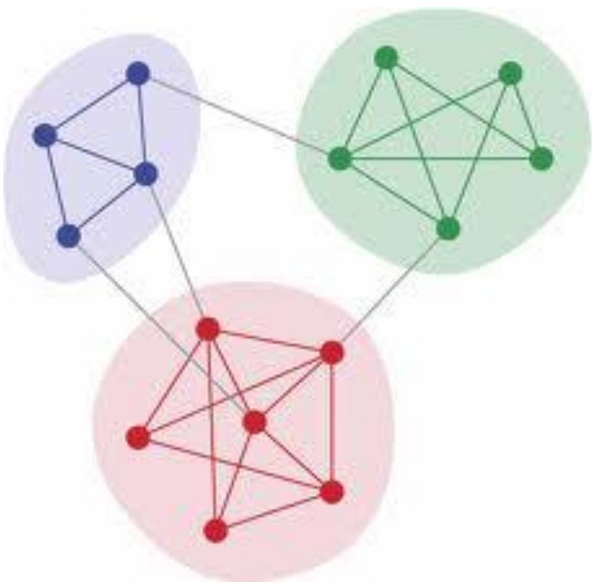
(3) Effect of clustering on the cascade size

Fraction of pivotal players and size of the cascade

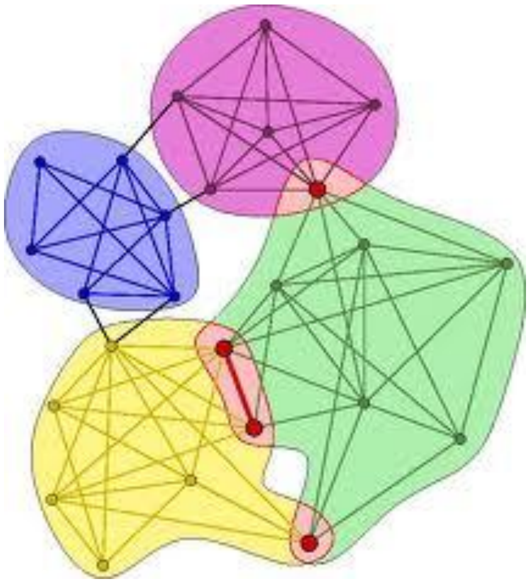


(3) Another model

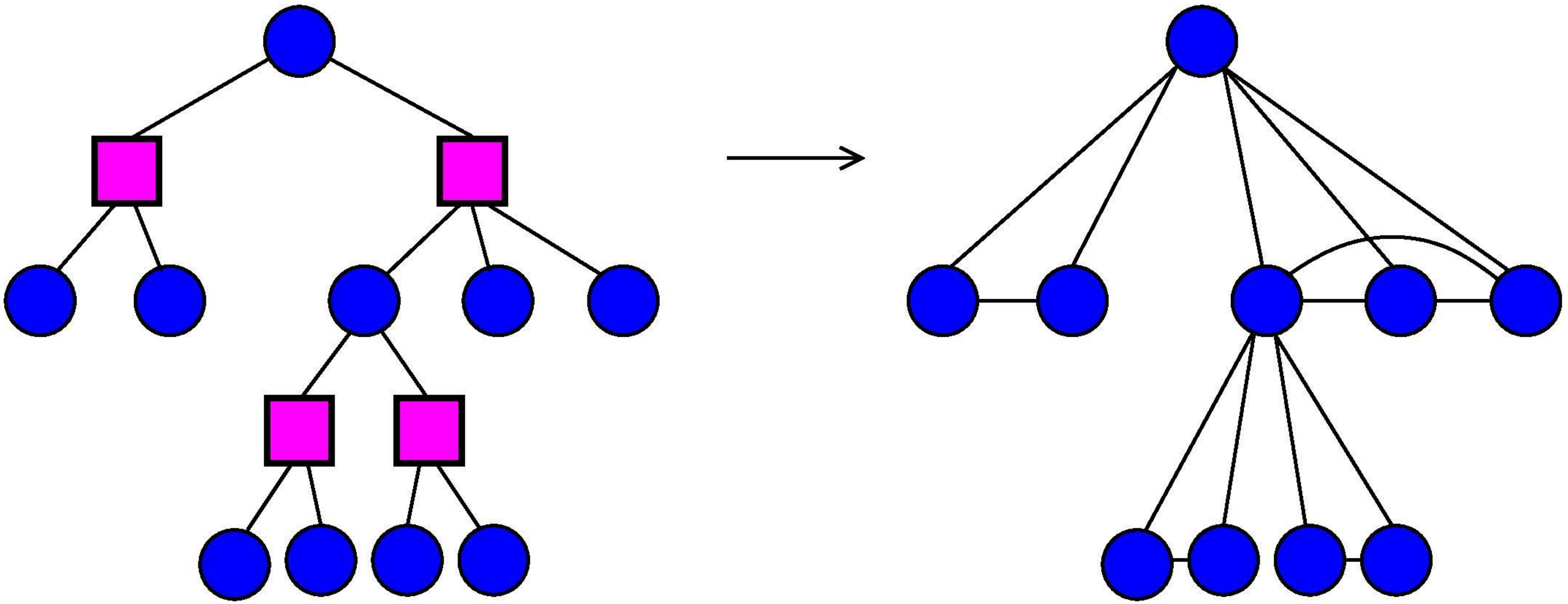
Separate communities
(Trapman 07)



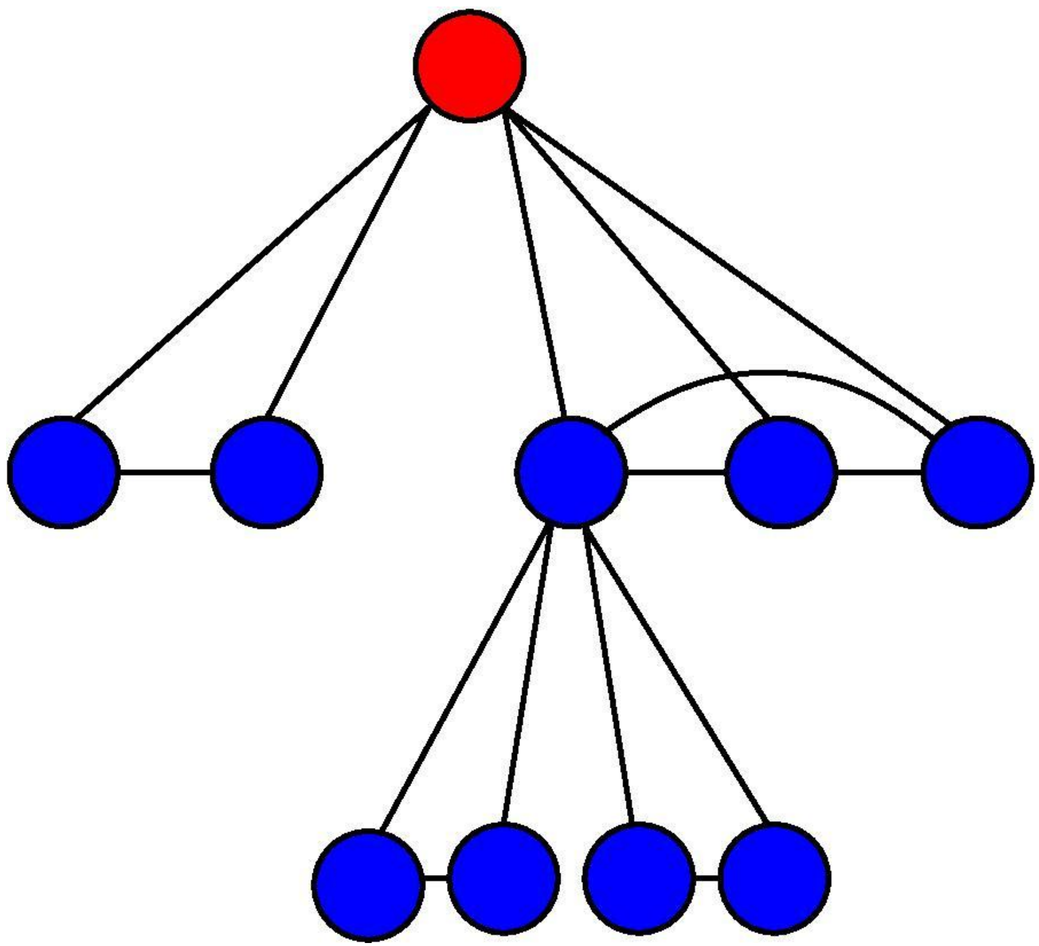
Overlapping communities
(Newman 03)



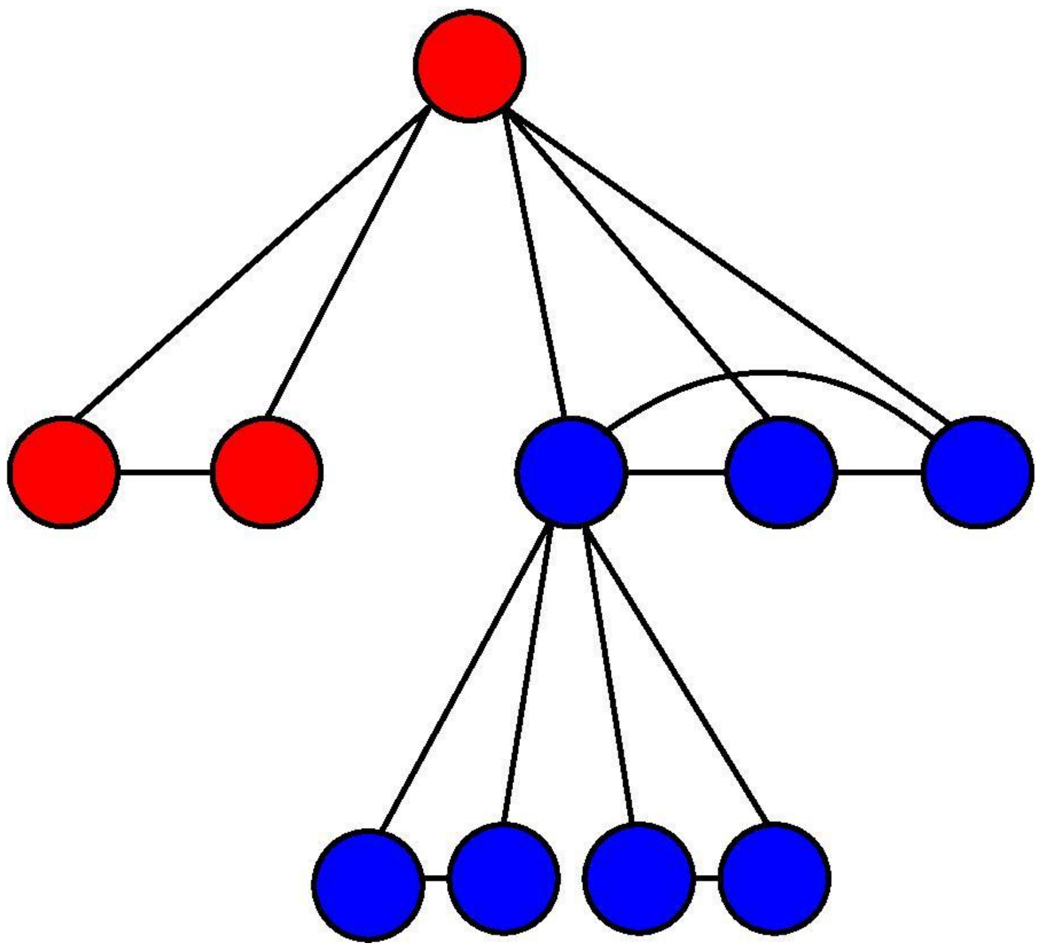
(3) Local Structure



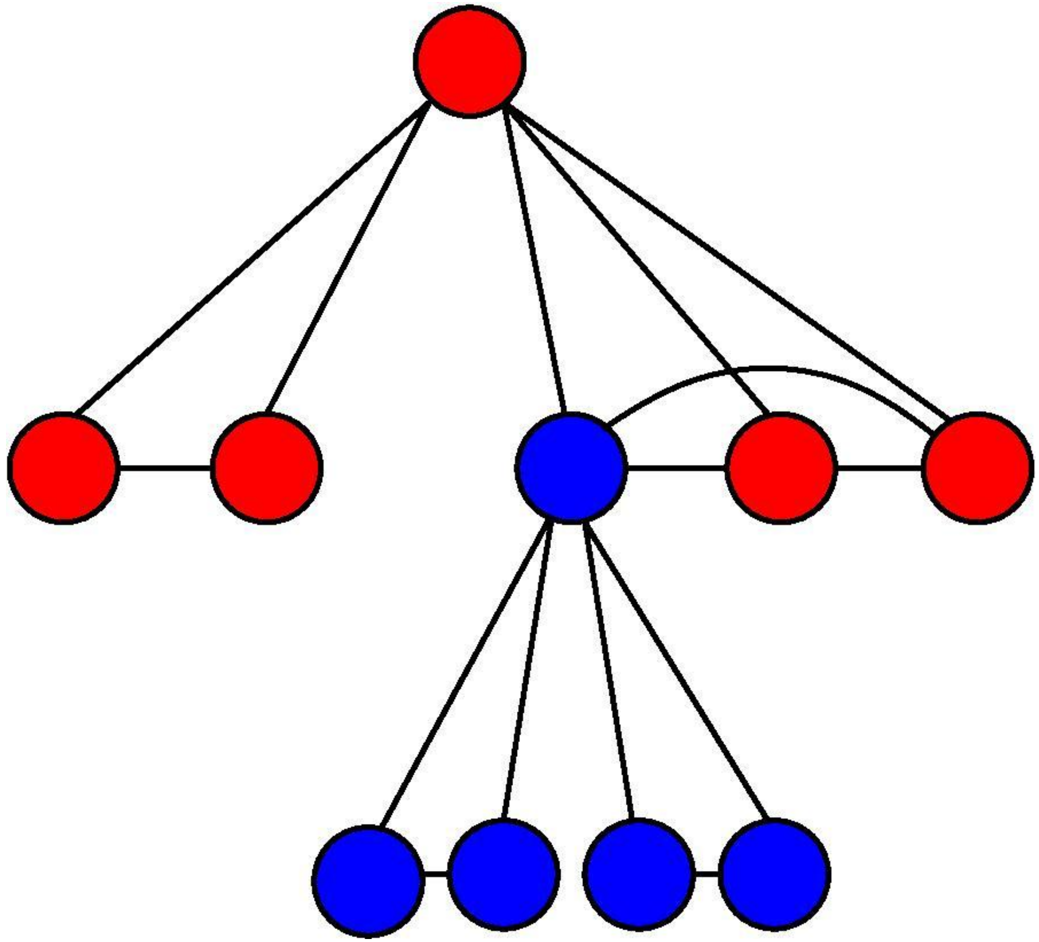
(3) Diffusion with overlapping communities



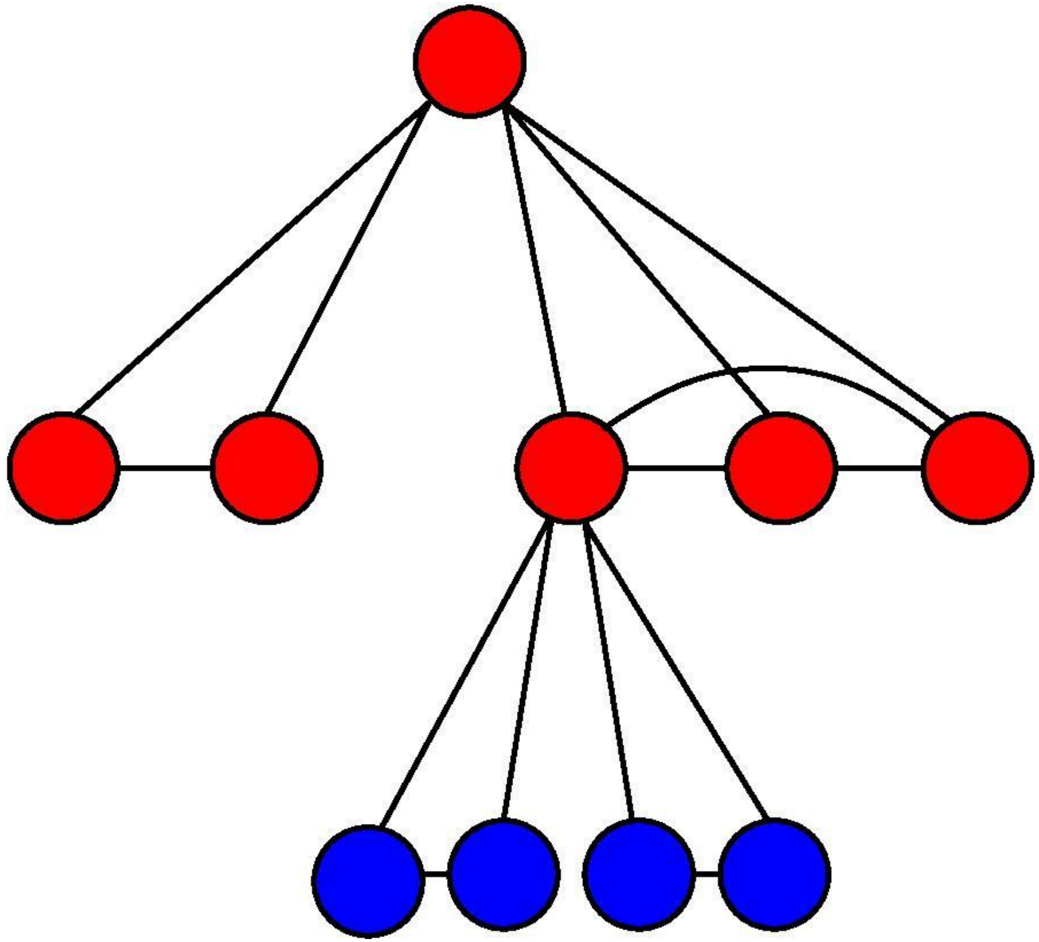
(3) Diffusion with overlapping communities



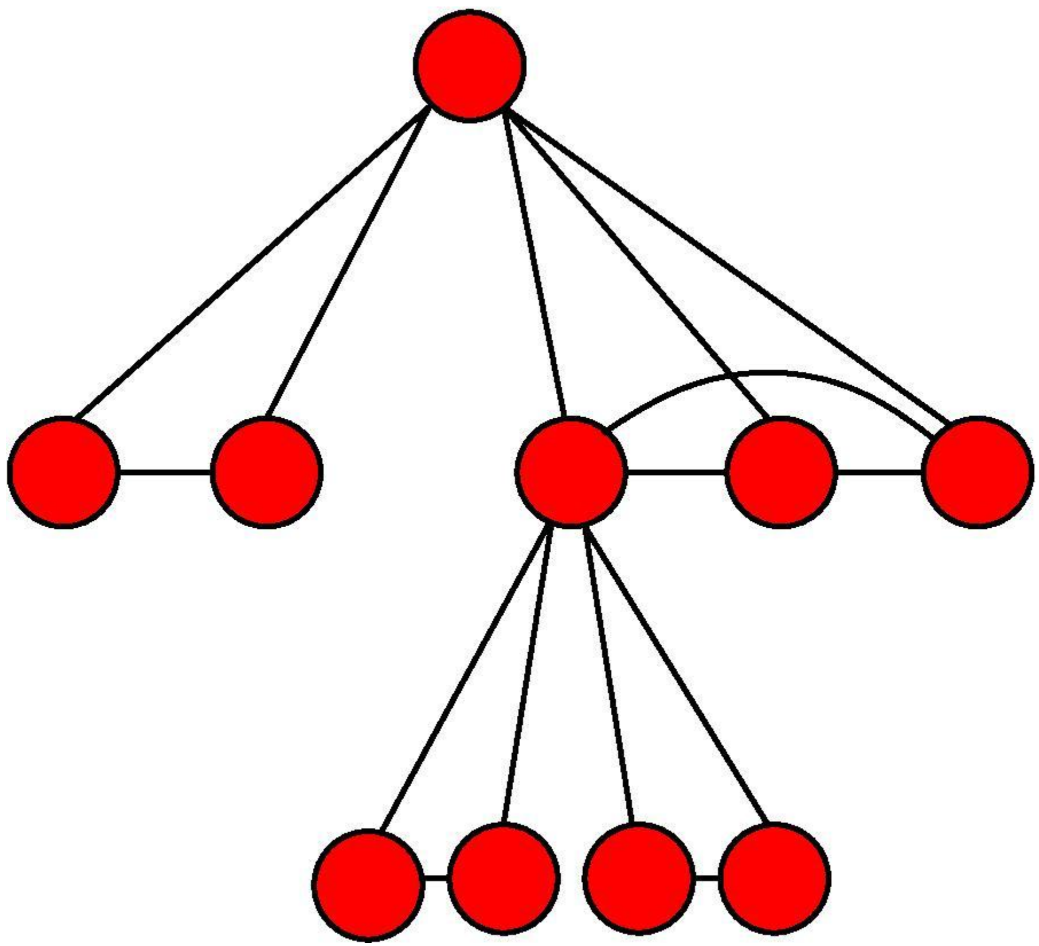
(3) Diffusion with overlapping communities



(3) Diffusion with overlapping communities



(3) Diffusion with overlapping communities



Conclusion

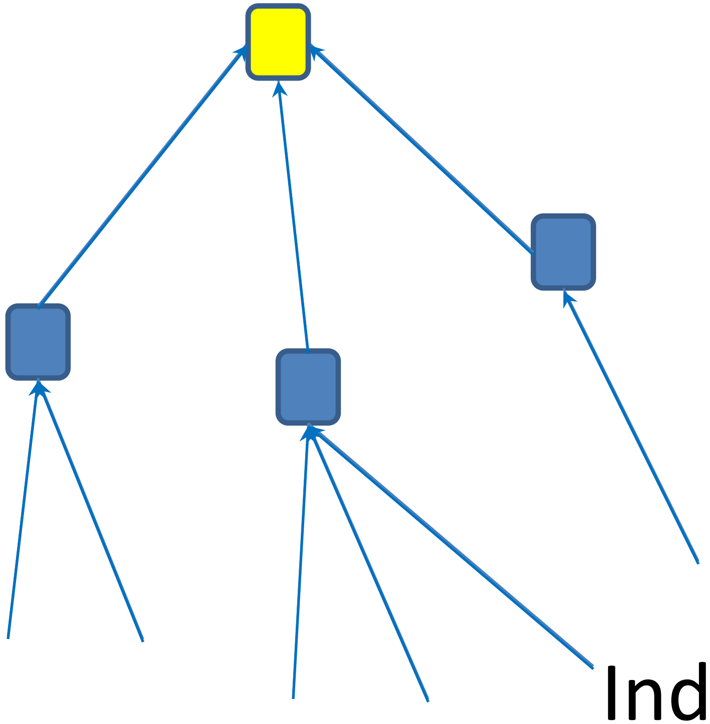
- Simple tractable model:
  – Threshold rule
  – Random network : heterogeneity of population
  – Tunable degree/clustering
- 1 notion: Pivotal Players and 2 regimes:
  – Low connectivity: tipping point / clustering hurts
  – High connectivity: chasm / clustering helps activation
- More results in the papers:
  – heterogeneity of thresholds, active/inactive links, rigorous proof.

Thank you!

- M. Lelarge. Diffusion and Cascading Behavior in Random Networks. Games Econ. Behav., 75(2):752-775, 2012.
- E. Coupechoux, M. Lelarge. How Clustering Affects Epidemics in Random Networks, arXiv:1202.4974.
- E. Coupechoux, M. Lelarge. Diffusion of innovations in random clustered networks with overlapping communities.
- E. Coupechoux, Analysis of Large Random Graphs, PhD thesis 2012.

Available at http://www.di.ens.fr/~lelarge

(4) Locally tree-like



Independent computations on trees

(4) Branching Process Approximation

- Local structure of G = random tree
- Recursive Distributional Equation (RDE) or:

$$Y_i = \begin{cases} 1 & \text{if infected from 'below'} \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_i = 1 - (1 - \sigma_i)\mathbb{1}\left(\sum_{\ell \to i} Y_\ell \leq q d_i\right)$$

(4) Solving the RDE

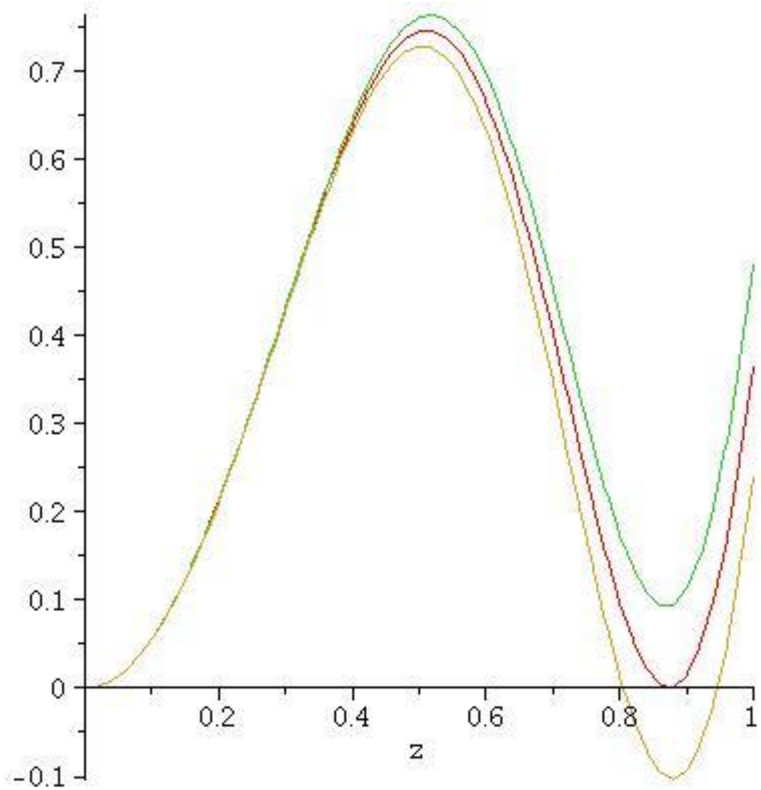
$$Y \stackrel{d}{=} 1 - (1-\sigma)\mathbb{1}\left(\sum_{\ell=1}^{\widehat{D}-1} Y_\ell \leq q\widehat{D}\right)$$

$$z = \mathbb{P}(Y = 0)$$

$$\lambda z^2 = (1-\alpha)h(z)$$

$$h(z) = \sum_{s,r \geq s - \lfloor qs \rfloor} r p_s \binom{s}{r} z^r (1-z)^{s-r}$$

(4) Phase transition in one picture



$$z^* = \max\{z \in [0,1],\ \lambda z^2 - (1-\alpha)h(z) = 0\}$$