Intraday Value at Risk (IVaR) Using Tick-by-Tick Data with Application to the Toronto Stock Exchange*

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Abstract

The objective of this paper is to investigate the use of tick-by-tick data for market risk measurement. We propose an Intraday Value at Risk (IVaR) at different horizons based on irregularly time-spaced high-frequency data by using an intraday Monte Carlo simulation. An UHF-GARCH model extending the framework of Engle (2000) is used to specify the joint density of the marked-point process of durations and high-frequency returns. We apply our methodology to transaction data for the Royal Bank and the Placer Dome stocks traded on the Toronto Stock Exchange. Results show that our approach constitutes reliable means of measuring intraday risk for traders who are very active on the market. The UHF-GARCH model performs well out-of-sample for almost all the time horizons and the confidence levels considered even when normality is assumed for the distribution of the error term, provided that intraday seasonality has been accounted for prior to the estimation.

JEL classification: C22, C41, C53, G15

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1 Introduction

Our objective is to propose an Intraday Value at Risk (IVaR) based on tick-by-tick data by extending the VaR techniques to intraday data. VaR refers to the maximum expected loss that will not be exceeded under normal market conditions over a predetermined period at a given confidence level (Jorion, 2001, p.xxii). In other words, VaR corresponds to the quantile of the conditional distribution of price changes over a target horizon and for a certain confidence level. Financial institutions generally produce their market VaR at the end of the business day to measure their total risk exposure over the next day. For regulated capital adequacy purposes, banks usually compute the market VaR daily and then re-scale it to a 10-day horizon. Today, VaR has been largely adopted by financial institutions as a foundation of day-to-day risk measurement.

However, the traditional way of measuring and managing risk has been challenged by the current trading environment. Over the last several years the speed of trading has been constantly increasing. Day-trading, once the exclusive territory of floor-traders is now available to all investors. "High frequency finance hedge funds" have emerged as a new and successful category of hedge funds. Consequently, risk management is now obliged to keep pace with the market. For day traders, market makers or other very active agents on the market, risk should be evaluated on shorter than daily time intervals since the horizon of their investments is generally less than a day. For example, day-traders liquidate any open positions at closing, in order to pre-empt any adverse overnight moves resulting in large gap openings. Brokers must also be able to calculate trading limits as fast as the clients place their orders. Significant intraday variations in asset prices affect the margins a client has to deposit with a clearing firm and this should be taken into account in the design of any margining engine. Moreover, as noted by Gourieroux and Jasiak (1997), banks also use intraday risk analysis for internal control of their trading desk. For example, a trader could be asked at 11 a.m. to give his IVaR for the rest of the day.

Over recent years, most exchanges have set up low-cost intraday databases, thus facilitating access to a new type of financial information where all transactions are recorded according to their time-stamp and market characteristics (price, volume, etc.). Denoted ultra-high-frequency data in the literature (Engle, 2000), these transaction or tick-by-tick data represent the limit case for recording events (transactions, quotes, etc.): one by one and as they occur. Since the time between consecutive events (defined as duration) is no longer constant, standard time series analysis techniques are inadequate when applied directly to these transaction data.

Research on high-frequency data has progressed rapidly since Engle and Russell (1998) introduced the Autoregressive Conditional Duration (ACD) model to take into account the irregular spacing of such data. Despite a greater interest in searching for appropriate econometric models, in testing market microstructure theories, and in estimating volatility more precisely,[1] very few contributions link high-frequency data to risk management. Much effort has been spent on developing more and more sophisticated Value at Risk (VaR) models for daily data and/or longer horizons but, to the best of our knowledge, the benefit of using tick-by-tick data for market risk measures has not been sufficiently explored.

With increased access to intraday financial databases and advanced computing power, an important question arises concerning the utility of high frequency data for market risk measurement: How is one to define practical IVaR measures for investors or market makers operating on an intraday basis? The irregular feature of high-frequency data also makes it hard to asses the results obtained from models based on such data. The problem is further complicated by certain microstructure effects at the intraday level, effects such as the bid-ask bounce or the discreteness of prices. Giot (2002) estimates a conditional parametric VaR using ARCH models with equidistantly time-spaced returns re-sampled from irregularly time-spaced data. He also applies high-frequency duration models (log-ACD) to price durations,[2] but the results from the models for irregularly time-spaced data are not completely satisfactory.

[1] Research initiated by Andersen *et al.* (2000, 2001a,b,c) on realized volatility measures (defined as the summation of high-frequency intraday squared returns) has shown dramatic improvements in both measuring and forecasting volatility.

[2] As first noted by Engle and Russell (1998), price durations (the minimum amount of time needed for the price of an asset to change by a certain amount) are closely linked to the instantaneous volatility of the price process.

In this paper we investigate the use of high-frequency data in risk management and we introduce an IVaR at different horizons based on tick-by-tick data. We use the Monte Carlo simulation approach for estimating and evaluating the IVaR.[3] We also propose an extension of GARCH models for tick-by-tick data (in particular, the ultra-high-frequency (UHF) GARCH model introduced by Engle, 2000) to specify the joint density of the marked-point process of durations and high-frequency returns. The advantage of this model is that it explicitly accounts for the irregular time-spacing of the data by considering durations when modelling returns. Our specification of the UHF-GARCH model is, however, more flexible than that of Engle (2000). Instead of considering returns per square root of time, we model returns divided by a nonlinear function of durations, thus endogenizing the definition of time units and letting the data speak for themselves.

A by-product of this study is represented by the out-of-sample evaluation of the predictive abilities of the UHF–GARCH model in a risk management framework. Up to now the approach taken for computing VaR with intraday data has consisted in aggregating the irregularly time-spaced data, in order to retrieve a regular sample to which traditional methods are applied.[4] This approach not only requires finding the optimal aggregating scheme, but also inevitably leads to the loss of important information contained in the intervals between transactions. We propose an alternative approach which is more closely related to the literature on market microstructure and which deals directly with irregularly time-spaced data by using models adapted to the particularities of such data. From a statistical point of view, the use of more data (all the available observations in this case) and of their specific dynamics would be expected to improve the estimation of a model.

Our results show that the UHF-GARCH model performs well out-of-sample when evaluated in a VaR framework for almost all the time horizons and the confidence levels considered, even in the case when normality is assumed for the distribution of the error term, at least in our application.

The rest of the paper is organized as follows. In Section 2, we introduce the general

[3]See Christoffersen (2003) and Burns (2002) for computing VaR using GARCH models and daily data.
[4]See for example Beltratti and Morana (1999).

econometric model used to describe the dynamics of durations and returns. In Section 3, we discuss the concept of IVaR and present the intraday Monte Carlo simulation approach to estimate it. An extension of Engle's (2000) UHF-GARCH model is proposed for modelling the tick-by-tick returns. In Section 4, we apply our methodology to transaction data on two Canadian stocks traded on the Toronto Stock Exchange (TSE): the Royal Bank of Canada (RY) stock and the Placer Dome (PDG) stock. Finally, Section 5 concludes and gives some possible research directions.

2 The General Econometric Model

Asymmetric-information models from the market microstructure literature suggest that both time between trades and price dynamics are related to the existence of new information on the market and that time is not exogenous to the price process. In Easley and O'Hara (1992), informed traders trade only when there are information events that influence the asset price. Therefore, in this model, long intertrade durations are associated with no news and, consequently, with low volatility. Opposite relations between duration and volatility follow from the Admati-and-Pfleiderer model (1988) where frequent trading is associated with liquidity traders. Hence, low trading means that liquidity traders are inactive, leaving a high proportion of informed traders on the market, which translates into higher volatility.

Predictions from theoretical models are relatively ambiguous. As pointed out by O'Hara, "the importance of time is ultimately an empirical question ..." (1995, p. 177). In the end, the common viewpoint is that durations and volatility are closely linked. Consequently, to estimate an IVaR that draws on insights from this microstructure literature, we use a joint modelling of arrival times and price changes.

To account for the irregularity of durations between consecutive trades, the data is statistically viewed as a marked-point process. The arrival times form the points, and random variables such as price, volume, and the bid-ask spread form the marks.

Consider two consecutive trades that happened at times t_{i-1} and t_i, at prices p_{i-1} and p_i, respectively. Let $x_i = t_i - t_{i-1}$ be the duration between these trades, and $r_i = \log(p_i/p_{i-1})$ be the corresponding continuously compounded return. Following Engle's framework (2000), a realization of the process of interest is fully described by the sequence:

$$\{(x_i, r_i), i = 1, ..., n\}.$$

The ith observation has joint density, conditional on the past filtration $\mathcal{F}_{i-1}$ given by

$$(x_i, r_i) | \mathcal{F}_{i-1} \sim f\left(x_i, r_i | \breve{x}_{i-1}, \breve{r}_{i-1}; \theta\right), \tag{1}$$

where θ is a d-vector of parameters ($d < n$), finite and invariant over events, and $\breve{x}_{i-1}$ and $\breve{r}_{i-1}$ denote the past of the variables X and R, respectively, up to the $(i-1)$th transaction. The information set available at time t_{i-1} is represented by $\mathcal{F}_{i-1}$.

The joint density in (1) can be written as the product of the marginal density of the durations and the conditional density of the returns given the durations, all conditioned upon the past of durations and returns:

$$f\left(x_i, r_i | \breve{x}_{i-1}, \breve{r}_{i-1}; \theta\right) = g(x_i | \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_1) q(r_i | x_i, \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_2), \tag{2}$$

where $g(x_i | \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_1)$ is the marginal density of the duration x_i with parameter θ_1, conditional on past durations and returns, and $q(r_i | x_i, \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_2)$ is the conditional density of the return r_i with parameter θ_2, and conditional on past durations and returns as well as the contemporaneous duration x_i. Using (2), the log likelihood can be written as

$$\mathcal{L}(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\log g(x_i | \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_1) + \log q(r_i | x_i, \breve{x}_{i-1}, \breve{r}_{i-1}; \theta_2)\right]. \tag{3}$$

Assuming that θ_1 and θ_2 are variation free in the sense of Engle, Hendry, and Richard (1983),[5] durations could be treated as being weakly exogenous and the two parts of the

[5] That is, if $\theta_1 \in \Theta_1$ and $\theta_2 \in \Theta_2$, then $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$.

likelihood function could be maximized separately. However, the evidence suggests that this assumption is not always valid. In fact, Dolado, Rodriguez-Poo, and Veredas (2004) reject the null hypothesis of weak exogeneity for seven out of the ten models considered when analyzing high-frequency data for five stocks traded on the NYSE. Here, we estimate the two parts of the log-likelihood function simultaneously.

We model the dynamics of trade durations using the log-ACD model proposed by Bauwens and Giot (2000) (presented in the next subsection) and the price dynamics by extending Engle's UHF-GARCH framework (2000) as shown in Section 3.

2.1 The ACD and The Log-ACD Models

Engle and Russell (1998) introduced the ACD model as a counterpart of the GARCH model to describe the duration clustering observed in high-frequency financial data. Since then, a plethora of models has emerged,[6] strongly supported theoretically by the recent market microstructure literature's emphasis on the information contained in the time between market events.[7]

Let ψ_i be the conditional duration given by

$$\psi_i = E(x_i|\mathcal{F}_{i-1}) \tag{4}$$

and supposed to be a nonnegative measurable function of $\mathcal{F}_{i-1}$.

The ACD model is based on the assumption that all the temporal dependence between durations is captured by the conditional duration mean function, such that x_i/ψ_i is independent and identically (i.i.d.) distributed,

$$x_i = \psi_i \varepsilon_i, \tag{5}$$

where the process $\varepsilon = \{\ \varepsilon_i,\ i \in \mathbb{Z}\}$ is a strong white noise, that is ε is an *i.i.d.* process.

[6] See Bauwens and Giot (2001), and Hautsch (2004) for extensive surveys of the existing models.

[7] See Diamond and Verrecchia (1987), Admati and Pfleiderer (1988), Easley, and O'Hara (1992) among others.

We assume ε_i to be non-negative with probability one and admit a density $p(\varepsilon)$ such that $E(\varepsilon_i) = 1$.

By choosing different specifications for the conditional durations in (4) and the density $p(\varepsilon)$, several forms of the ACD model can be obtained. A popular one is the ACD (m, q) which is based on a linear parameterization of (4):

$$\psi_i = \omega + \sum_{j=1}^{m} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}, \tag{6}$$

where $\omega > 0$, $\alpha_j \geq 0$, $j = 1, ..., m$ and $\beta_j \geq 0$, $j = 1, ..., q$, which are sufficient conditions to ensure the positivity of the conditional durations.

Several choices can be made for the density $p(\varepsilon)$. Engle and Russell (1998) used the standard exponential distribution (that is the shape parameter is equal to one) and the standardized Weibull distribution with shape parameter equal to γ and scale parameter equal to one. The resulting models are called the Exponential ACD (EACD) and the Weibull ACD (WACD), respectively. The choice of the distribution for the error term affects the conditional intensity function. The exponential specification implies a flat conditional hazard function which is quite restrictive and easily rejected in empirical financial applications (see e.g. Engle and Russell, 1998). The Weibull distribution (that reduces to the exponential distribution if γ equals 1) allows for a monotonic hazard function (which is either increasing if $\gamma > 1$, or decreasing if $\gamma < 1$). For greater flexibility, Grammig and Maurer (2000) advocate the use of a Burr distribution leading to the Burr ACD model, while Lunde (1999) proposes the generalized gamma distribution. Both distributions allow for hump-shaped hazard functions to describe situations where, for small durations, the hazard function is increasing and, for long durations, the hazard function is decreasing. Once the density function $p(\varepsilon)$ is specified, the parameters of the ACD model are estimated using the maximum likelihood estimation technique. In this paper we make use of the generalized gamma distribution with unit expectation which leads to the so-called GACD model.

In order to avoid the necessity of imposing non-negativity constraints on the parameters of the conditional mean function, Bauwens and Giot (2000) introduce a logarithmic version of the ACD model, called the log-ACD model, that specifies the autoregressive equation on the logarithm of ψ_i:

$$\psi_i = \exp\left(\omega + \sum_{j=1}^{m} \alpha_j \varepsilon_{i-j} + \sum_{j=1}^{q} \beta_j \ln \psi_{i-j}\right). \tag{7}$$

Equation (7) is sometimes referred to as a Nelson type ACD model because of its similarity with Nelson's EGARCH model (1991). The same hypotheses are made about ε as in the ACD model and the same probability distribution functions can be chosen. If ε_i follows a generalized gamma distribution with parameters γ_1, γ_2 ($\gamma_1 > 0$ and $\gamma_2 > 0$), its density function is given by

$$p(\varepsilon|\gamma_1, \gamma_2) = \begin{cases} \frac{\gamma_1 \varepsilon^{\gamma_1\gamma_2 - 1}}{\gamma_3^{\gamma_1\gamma_2}\Gamma(\gamma_2)} \exp\left[-\left(\frac{\varepsilon}{\gamma_3}\right)^{\gamma_1}\right], & \varepsilon > 0, \\ 0 \quad , \text{ elsewhere.} \end{cases}$$

where $\Gamma(\cdot)$ denotes the gamma function and $\gamma_3 = \Gamma(\gamma_2)/\Gamma\left(\gamma_2 + \frac{1}{\gamma_1}\right)$. The generalized gamma distribution nests the Weibull distribution when $\gamma_2 = 1$. Estimation is performed in the same way as for the ACD model by considering the new specification of ψ_i.

2.2 The UHF-GARCH model

Once we model the durations between trades conditional on past information, we need to specify a model for price changes, conditional on the current duration and past information. Since high-frequency data are irregularly time-spaced, Engle (2000) proposes a GARCH model that takes into consideration this feature of tick-by-tick data. Let the conditional variance per transaction be

$$h_i = V_{i-1}(r_i|x_i), \tag{8}$$

where the conditioning information set contains the current durations as well as the past returns and durations. However, as argued by Engle (2000) and Meddahi, Renault and Werker (2003), the variance of interest is not the "traditional" variance but the variance per unit of time, naturally defined as:

$$V_{i-1}\left(\frac{r_i}{\sqrt{x_i}}|x_i\right) = \sigma_i^2, \tag{9}$$

which naturally implies that $h_i = x_i\sigma_i^2$.

Under the assumption $E_{i-1}\left(r_i|x_i\right) = 0$, the volatility per unit of time is then modeled as a simple GARCH(1,1) process:

$$\sigma_i^2 = \widetilde{\omega} + \widetilde{\alpha}\left(\frac{r_{i-1}}{\sqrt{x_{i-1}}}\right)^2 + \widetilde{\beta}\sigma_{i-1}^2, \tag{10}$$

where $\widetilde{\omega} > 0$, $\widetilde{\alpha} \geqslant 0$ and $\widetilde{\beta} \geqslant 0$.

Although it seems natural to model the variance as a function of time when using irregularly time-spaced data, the above modelling for the unit of time appears as quite restrictive, since the conditional heteroskedasticity in the returns could depend on time in a more complicated way. In the next section, we shall propose a useful extension of the above UHF-GARCH model that is more flexible and seems to provide a better adjustment of the data, at least in our empirical applications.

3 Monte Carlo IVaR

3.1 IVaR: Definition

To define our IVaR, let $Y = \{y_k, k \in \mathbb{Z}\}$ be the process of the asset return re-sampled at regular time intervals equal to T units of time. We consider a realization of length n' of the process Y $\{y_k; k = 1, ...n'\}$ with y_k obtained at times t'_k such that $t'_k - t'_{k-1} = T$. Thus, y_k

denotes the T-period return which is simply the sum of correspondent tick-by-tick returns

$$y_k = \sum_{i=1}^{\tau(k)-1} r_i, \tag{11}$$

where $\tau(k)$ is such that the cumulative duration exceeds T for the first time. This means that

$$\sum_{i=1}^{\tau-1} x_i \leq T \text{ and } \sum_{i=1}^{\tau} x_i > T.$$

Thus, by modelling the durations specifically with the ACD model, we are able to determine every $\tau(k)$ and keep track of the time step.

The IVaR with confidence level $1-\alpha$, is formally defined as

$$\Pr(y_k < -IVaR_k(\alpha) \mid \mathcal{G}_k) = \alpha. \tag{12}$$

where $\mathcal{G}_k$ denotes the information set that includes all the tick-by-tick data (durations and returns) up to time t'_k . Common confidence levels used in the applications are $1-\alpha = 95\%, 97.5\%, 99\%$ and 99.5%; we shall consider these values in the empirical study. It follows from (12) that the IVaR is such that $-IVaR_k(\alpha) = Q_k(\alpha \mid \mathcal{G}_k)$ where $Q_k(\alpha \mid \mathcal{G}_k)$ is the quantile of the conditional distribution of y_k.

Various methods exist for computing a VaR: parametric or variance-covariance methods (e.g. RiskMetrics, GARCH), nonparametric (e.g. Historical Simulation), and semi-parametric (e.g. CAViar, extreme value theory). We propose a GARCH-type model, as presented in the next subsection, to specify the time-varying volatility of the tick-by-tick returns r_i and a Monte Carlo simulation approach to simulate the future regularly spaced returns y_k from which the IVaR is computed.

3.2 The Extended UHF-GARCH model

In this subsection, we propose a more flexible specification of the UHF-GARCH model presented in Section 2.2 in which the time weighting is determined endogenously. We model the volatility of returns as the product of a function of duration and a GARCH component. More precisely, let

$$\frac{V_{i-1}\left(r_i|x_i\right)}{x_i^{\gamma}} = \sigma_i^2, \tag{13}$$

which implies that

$$h_i = x_i^{\gamma}\sigma_i^2. \tag{14}$$

The parameter γ that specifies the duration weighting for the volatility of a particular stock has to be estimated. This formulation allows us to specify a more general form of heteroskedasticity in the conditional variance of the returns.

In Section 4, we use transaction prices instead of mid-quotes for forecasting volatility in an order-driven market. Transaction prices may be affected by various market microstructure effects, such as nonsynchronous trading and the bid-ask bounce (Tsay, 2002), that may induce serial correlation in high-frequency returns. Therefore, to remove microstructure effects, we follow Grammig and Wellner (2002) and Ghysels and Jasiak (1998) and use an ARMA(1,1) model on the tick-by-tick returns:

$$r_i = c + \varphi_1 r_{i-1} + e_i + \theta_1 e_{i-1}. \tag{15}$$

We model the GARCH component as

$$\log\sigma_i^2 = \widetilde{\omega} + \sum_{j=1}^{P}\widetilde{\beta}_j \log\sigma_{i-j}^2 + \sum_{j=1}^{Q} a_j \left\{\frac{|e_{i-j}|}{h_{i-j}^{1/2}} - E\left(\frac{|e_{i-j}|}{h_{i-j}^{1/2}}\right)\right\} + \sum_{j=1}^{Q}\widetilde{\alpha}_j\left(\frac{e_{i-j}}{h_{i-j}^{1/2}}\right), \tag{16}$$

where $e_i = z_i\sqrt{h_i}$ and $z = \{z_i, i \in \mathbb{Z}\}$ denotes a strong white noise, that is the z_i's are *i.i.d.* with a zero mean and unit variance. The unknown parameters are c, φ_1, θ_1, $\widetilde{\omega}$, $\widetilde{\beta}_j$, a_j, $\widetilde{\alpha}_j$ and γ. When $\gamma = 1$ the model is similar, though not equivalent, to Engle's UHF-GARCH model (2000) represented by equations (8), (9) and (10). We specify the mean equation (15) on the returns r_i rather than on the returns weighted by a function of durations, because we want to simulate the future distribution of returns from which the IVaR could be extracted. As shown in the empirical part of the article, the estimate of parameter γ is far below unity and is statistically significant, at least in our empirical application. When $\gamma = 0$, the model becomes a standard GARCH applied to the high-frequency data by ignoring the irregular spacing of returns when modelling the volatility. The use of an EGARCH*(p,q)* model is justified by the advantage of keeping the volatility component positive during simulations regardless of the variables added to the autoregressive equation. The absence of non-negativity constraints on the parameters also simplifies numerical optimization.

3.3 Intraday Monte Carlo Simulation

In this subsection, we describe the Monte Carlo simulation approach used for computing the IVaR based on tick-by-tick data. As the time intervals between consecutive observations are irregular, the question about how to assess the results needs to be considered. Since the model we use fully specifies the distribution of returns in event time, one-step forecasts can be computed analytically. However, we need to run simulations in order to obtain forecasts of returns in any arbitrary length of calendar time, T. More specifically, the ACD model will define the time step of our simulations and the extended UHF-GARCH model introduced in the previous subsection will generate the corresponding tick-by-tick returns. We compute the forecasted returns for regular calendar-time intervals as the sum of irregular (tick-by-tick) intraday returns. Using the simulated distribution of returns over the chosen time interval T, we calculate the IVaR by extracting the desired percentile. The IVaR obtained is thus an IVaR for regular time intervals (therefore, comfortably comparable to regular real returns), but computed using tick-by-tick data and adapted to the non-regularity of time intervals.

More precisely, we proceed as follows:

(1) The original sample is divided into two parts, one for estimation and one for forecast/validation. The estimation sub-sample serves to calibrate a log-ACD-GARCH model for durations and tick-by-tick returns as presented in the previous sections.

(2) We draw random numbers from the standard normal distribution and the standard generalized gamma distribution, respectively, to compute the innovations which will serve

to generate scenarios for future durations, returns, and volatilities.

(3) In order to obtain the simulated durations within the first fixed interval, we forecast the durations in an iterative way using equations (5) and (7) as long as the sum of forecasted tick-by-tick durations does not exceed the time interval of interest.

(4) Using equations (14), (15), and (16), we forecast, conditional on the simulated durations, the tick-by-tick returns corresponding to this first regular interval. By summing up all these returns we obtain the (regular) return for the first interval.

(5) We continue the procedure until the desired number of fixed intervals or regular returns is obtained. This corresponds to the first path.

(6) We repeat steps (1) to (5) for the desired number of paths.

(7) For each time-fixed interval, the IVaR corresponds to the quantile of the simulated distribution of returns for that interval.

(Insert Figure 1 here)

Figure 1 resumes the main steps of the simulation. As one can see, the algorithm displays similarities with the traditional Monte Carlo simulation used for computing a multi-period (generally 10-day) VaR. The main difference is at step (3) where we make use of the ACD model to keep trace of the time step. While, in the standard Monte Carlo approach the time unit is not very important because all observations are equally spaced, here we relax this constraint and are able to proceed tick-by-tick by using models adapted to the irregularity of time intervals, such as the ACD and the UHF-GARCH models. We illustrate this procedure using real data in the next section.

4 Empirical Study

4.1 Data

In this section we compute the IVaR according to the methodology previously described using high-frequency data from the "Market Data Equity Trades and Quotes Files" CD-

ROM of the Toronto Stock Exchange (TSE). To our knowledge, this is the first econometric study analyzing (irregular) high-frequency data from the Canadian stock market. Previous studies used data from the NYSE (e.g., Engle and Russell 1998; Giot 2002), or from the Paris Bourse (e.g., Jasiak 1998; Gouriéroux, Jasiak and Le Fol, 1999), the German Stock Exchange (e.g., Grammig and Wellner 2002), and the Moscow Interbank Currency Exchange (Anatolyev and Shakin 2004). Thus, this study adds another dimension to previous work. We focus on the Royal Bank of Canada stock (RY) and the Placer Dome stock (PDG) for the period from April 1st to June 30, 2001 which contains 63 trading days. The Royal Bank is Canada's largest bank as measured by market capitalization (about US$ 33.4 billion) and assets. Placer Dome is Canada's second largest gold miner with a market capitalization of US$ 8.2 billion at the end of 2004. Both stocks belong to the Toronto 35 Index. The Toronto 35 Index was developed by the TSE in 1987 and consists of the 35 largest and most liquid stocks in Canada. As of April 30, 2001 the relative weights of the RY stock and PDG stock in the Toronto 35 Index were 4.44% and 1.15%, respectively.

In 1999, the Toronto Stock Exchange (TSE) became Canada's sole exchange for the trading of senior equities. The TSE is the seventh largest equities market in the world as measured by domestic market capitalization, with US$ 1,178 billion at the end of 2004 (source: World Federation of Exchanges Annual Report 2004). The TSE currently trades equities for approximately 1,485 listed firms, with a daily trading volume averaging CAD$ 3,3 billion in 2004. The TSE operates as an automated, continuous auction market. Limit orders enter the queues of the order book and are matched according to a price-time priority rule. There is a pre-opening session from 7:00 to 9:30 am during which market participants can submit orders for possible execution at the beginning of the regular session. The pre-opening session involves the determination of a Calculated Opening Price (COP) that equals the price at which the greatest volume of trades can trade or, if it is not unique, the price at which there is the least imbalance or the price closest to the previous closing price (see Davies, 2003 for details about the pre-opening session at the TSE and the role of the registered trader at the pre-opening). Regular trading starts at 9:30 and ends at 16:00. Opening trades are at the COP and, as they have a different dynamic we eliminate them from the empirical analysis . Each stock is assigned to a Registered Trader (RT) who is required to act as a market maker and to maintain a fair, orderly, and continuous two-sided market for that stock. The RT contributes to market liquidity and depth and reduces volatility by buying or selling against the market. He must also guarantee all oddlot trades and trade all orders of a certain size, known as the Minimum Guaranteed Fill (MGF) orders, within a set price difference between buy and sell orders. The RT resembles the specialist at the NYSE but he does not act as an agent for client order flow and does not have exclusive knowledge of the limit order book. Unlike the NYSE, trading on the TSE is completely electronic, without any floor trading. An order book open to subscribers insures a highly transparent market.[8]

The TSE intraday database contains date-and-time stamped bid-and-ask quotes, transaction prices, and volume for all firms. Special codes identify special trading conditions. Prior to the analysis, a couple of operations need to be conducted on the data. Following the literature, we removed all non-valid trades and interdaily durations and kept only those transactions made during regular trading hours. As already mentioned, open trades are also deleted in order to avoid effects induced by the opening auction. For simultaneous transactions, we consider a weighted average price and remove all remaining observations with this time stamp, thus considering these observations as split transactions.

Previous studies dealing with high-frequency data commonly used mid-quotes instead of transaction prices (e.g., Engle 2000, Engle and Lange 2001, Manganelli 2002). While this may be appropriate for the NYSE which is a quote-driven market, working with transaction prices appears a better choice in our case since we are interested in VaR estimation and want to forecast returns for real transactions. To liquidate positions in an order-driven market one has to transact either on the ask or the bid and therefore using the midquote-

[8]The future of traditional floor trading at the NYSE was questioned after the merger on April 20, 2005 of the NYSE with the electronic trading network Archipelago.

change quantiles may understate the true VaR.[9]

Extremely large durations and returns between two successive trades are very unlikely, therefore we filter out all the observations with absolute returns larger than 10 standard deviations and durations larger than 25 standard deviations, resulting in 2 outliers for the RY stock and 9 outliers for the PDG stock. These leaves us with a total of 51,660 observations for the RY stock and 27,956 observations for the PDG stock.

(Insert Figure 2 here)

Figure 2 displays the histograms of the transaction returns for the two stocks considered. The price increment for the two stocks RY and PDG for the period analyzed was one penny (C\$0.01)[10] We observe a disproportionately large number of zero returns (almost 60%) which seems rather typical for high-frequency data, especially for single stocks. For the IBM dataset used by Engle and Russell (1998), Tsay (2002, p.182) reported that about two-thirds of the intraday transactions were without price change. Gorski, Drozdz and Speht (2002) report the same phenomenon for DAX[11] returns and call it the *zero return enhancement*. Bertram (2004) also finds a large number of zero price changes for equity data from the Australian Stock Exchange and argues that zero returns follow from the absence of significant new information on the market. According to the efficient market hypothesis, a price changes when new information arrives on the market and, consequently, traders simply continue to trade at the previous price when the amount of information is insufficient to move the price.

Information about the raw data is given in Table 1. Of the two stocks, RY with an average duration of almost 29 seconds is traded almost twice as frequently as PDG, while PDG is traded on average every minute. We also note overdispersion of durations, i.e., the standard deviation is higher than the mean. This is typically found in the literature

[9] We are grateful to Joachim Grammig for shedding light on this issue.

[10] Starting on January 29, 2001 the TSE introduced the penny tick size for stocks selling at over \$0.50.

[11] Deutsche Aktienindex (DAX) represents the index for the 30 largest German companies quoted on the Frankfurt Stock Exchange.

for the trade duration process and it may suggest that the exponential distribution cannot properly describe the durations. The transaction returns have a sample mean equal to zero for both stocks and a standard deviation equal to 0.001 and 0.002, respectively. RY exhibits positive sample skewness while PDG has a negative skewness. Both stocks' returns display a kurtosis higher than that of a normal distribution.

(Insert Table 1 here)

4.2 Seasonal Adjustment

As noted by Engle and Russell (1998), high-frequency data exhibits a strong intraday seasonality explained by the fact that markets tend to be more active at the beginning and towards the end of the day. While most of the studies on high-frequency data ignore interday variations in variables, Anatolyev and Shakin (2004) found that durations and return volatilities of the Russian stocks considered fluctuate throughout different trading days. To prevent the distortion of results, these interday and intraday seasonalities must be taken out prior to the estimation of any model . We inspected our data for such evidence. We noted for example that durations are higher on Mondays and Fridays than during the rest of the week. A Wald test in a regression of average RY durations on five day-of-the-week dummies rejects the null hypothesis of equality of all coefficients, thus providing evidence of interday seasonal effects. No interday effects are identified for PDG durations and returns. When found, we remove interday seasonality under a multiplicative form, following the approach taken by Anatolyev and Shakin (2004):

$$x_{t,inter} = \frac{x_t}{\overline{x}_s}, \tag{17}$$

$$r_{t,inter} = \frac{r_t}{\sqrt{\overline{r^2}_s}}, \tag{18}$$

where $\overline{x}_s$ corresponds to the average duration for day s if observation t belongs to day s, and $\overline{r^2}_s$ is the average of squared returns for day s.

To take out the time-of-day effect, Engle and Russell (1998) suggest computing "diurnally adjusted" durations by dividing the raw durations by a seasonal deterministic factor related to the time at which the duration was recorded. We obtain intraday seasonally adjusted (isa) durations and returns in the following way:

$$x_{t,intra} = \frac{x_{t,inter}}{E(x_{t,inter}|\mathcal{F}_{t-1})},$$

$$r_{t,intra} = \frac{r_{t,inter}}{\sqrt{E(r_{t,inter}^2|\mathcal{F}_{t-1})}}.$$

where the expectation is computed by averaging the variables over thirty-minute intervals for each day of the week and then using cubic splines on the thirty-minute intervals to smooth the seasonal factor. Thus, intraday patterns are different for different days of the week. Figures 3 and 4 show, respectively, the estimated intraday factor for durations and squared returns for the RY stock. Similar seasonal-factors patterns are found for the PDG stock and therefore they are not reported. The patterns are analogous to what has been found in previous studies. Durations are shorter at the beginning and at the end of the day and longer in the middle. The return volatility is lower in the middle of the day than at the beginning and at the end. This reflects the behavior of traders who are very active at the beginning of the trading session and adjust their positions to incorporate the overnight change in information. Towards the end of the day, traders are changing their positions in anticipation of the close and to pre-empt the risk posed by any information that could arrive during the night. We also notice a difference between the patterns for different days.

(Insert Figures 3 and 4 here)

Descriptive statistics for deseasonalised data are given in Table 2. We notice the salient features of high-frequency data, such as overdispersion in trade durations and high autocorrelations. The Ljung-Box statistics for fifteen lags Q(15) tests the null hypothesis that the first 15 autocorrelations are zero. The large values of these statistics greatly exceed the 5% critical value of 25, indicating strong autocorrelation of both durations and returns. The skewness is close to zero for both stocks. There is still excess kurtosis for both stocks even if at a lesser degree compared to the raw data. We chose the normal distribution when estimating the GARCH model and we obtained satisfactory simulation results. It is well known that the conditional normality assumption in ARCH models generates some degree of unconditional excess kurtosis (see, e.g., Bollerslev et al., 1992).

(Insert Table 2 here)

4.3 Estimation Results

In this section we apply the model presented in Sections 2 and 3 to the deseasonalised data for the two stocks. Observations of the first month are used for estimation and those of the last two months serve for forecast and validation. The likelihood function is maximized using Matlab v. 7 with the Optimization toolbox v. 3.0 and numerical derivatives are used for computing the standard errors of the estimates.

We first tested our durations for the clustering phenomenon using the test of Ljung-Box with 15 lags, Q(15). The high coefficients (reported in Table 2) suggest the presence of ACD effects in our durations data at any reasonable significance level. The high positive serial dependence of the squared returns greatly exceeding the critical values, as illustrated by the Ljung-Box test statistics, noted $Q_2(15)$, represents evidence of volatility clustering and justifies the application of a GARCH-type model.

Estimation results are presented in Table 3 together with the p-values for the adequacy tests applied to the standardized residuals. We tried to find the best fit for our data and reestimated the model chosen without the variables that were statistically insignificant. We tested the adequacy of the log-GACD model using the Ljung-Box test applied to the standardized residuals.[12] We judged the quality of the GARCH fit using the Ljung-Box

[12] We have also applied the test statistics for ACD adequacy introduced by Duchesne and Pacurar (2005) using the Bartlett kernel. The results are similar and therefore not reported.

tests for standardized residuals and their squares.

(Insert Table 3 here)

We retain a log-GACD(2,2)-ARMA(1,1)-UHF-EGARCH(1,1) model for our data. We find that a log-GACD(2,2) specification is successful in removing the autocorrelation in durations. For both stocks the p-values of the Ljung-Box test with 15 lags are superior to 0.05. The parameters of the generalized gamma distribution are all significant. For each stock, the sum of the autoregressive parameters $\beta_1 + \beta_2$ is close to one, revealing high persistence in durations.

With regard to the high-frequency returns, the ARMA(1,1)-UHF-EGARCH(1,1) specification accounts satisfactorily for the dependence of both the returns and squared returns of the PDG stock, as evidenced by the p-values of the order-15 Ljung-Box test statistics that are all greater than 0.05. For the RY stock, the Ljung-Box test statistics indicate that some serial dependence is still present in the data, which is inconsistent with the model's adequacy. However, the dependence is dramatically reduced compared to the original data. The Ljung-Box statistic with 15 lags $Q(15)$ for autocorrelation of returns was reduced from 8195 to 62 (the associated 95% critical value being 24.99). Similarly, the Ljung-Box statistic $Q_2(15)$ for autocorrelation of squared returns was reduced from 15417 to 36. Similar results have been found by Engle (2000) using Ljung-Box test statistics. The problem of passing tests of model adequacy seems to remain an issue when using irregular high-frequency data. We have tried higher-order models both for the mean and the variance equations but we have not been able to gain any considerable improvement. We keep the ARMA(1,1) - UHF- EGARCH(1,1) specification of the model for its parsimony and because we are rather interested in assessing its forecasting ability under a risk management framework. As in Engle (2000), the MA(1) coefficient represented by θ_1 is negative and highly significant for both stocks. The AR(1) term represented by φ_1 is positive. This can be explained by the fact that traders split large orders into smaller orders to obtain a better price overall and therefore make prices move in the same direction and thus induce a positive autocorrelation of returns (Engle and Russell, 2005).

The positive autocorrelation can also be related to negative feedback trading (Sentana and Wadhwani, 1992). Engle (2000) has also found evidence of positive autocorrelation in high-frequency returns. The autoregressive parameter $\widetilde{\beta_1}$ for the EGARCH(1,1) model is close to one for the RY stock, indicating a higher persistence in volatility than for the PDG stock. The parameter $\widetilde{\alpha_1}$ is statistically insignificant (not reported in the table) for both stocks, so no leverage effect is supported. The parameter γ is approximately equal to 0.05 for both stocks and it is statistically different from zero.

Now that we have calibrated the model to our data, we may proceed to the simulation of future durations and returns.

4.4 IVaR Backtesting

In this section we simulate tick-by-tick durations and returns using the estimated coefficients of our model and the observations of the last day as starting values. We sum up the irregular simulated returns over a fixed-time interval. Five different interval lengths are used: 15, 25, 35, 45 and 90. Since all models are applied on deseasonalised data, the time unit of the interval does not represent a calendar unit (e.g. seconds, minutes). A correspondence can nevertheless be established for each stock given the number of resampled intervals generated, depending on the trading intensity of that stock for the 44 days of the validation period. For example, a length = 15 for the RY stock results in 2470 intervals for 44 trading days, which is equivalent to an average interval of 7 minutes. Assessing the results in a regularly spaced framework allows us to evaluate the performance of the model in a traditional way, thus circumventing the problem of finding an appropriate benchmark. We have generated 5000 independent paths and we have extracted the IVaR as a percentile from the simulated distribution of returns.

We first analyze the performance of the model by using Kupiec's test (1995) for the percentage of failures, which is also embedded in the regulatory requirements on the backtesting of VaR models. Using a standard procedure in the literature, we compute the empirical failure rate ($\widehat{\alpha}$) as the percentage of times actual returns (y_k) are greater than the estimated IVaR. If the IVaR estimates are accurate, the failure rate should equal α. Kupiec's test checks whether the observed failure rate is consistent with the frequency of exceptions predicted by the IVaR model. Under the null hypothesis that the model is correct we have $\widehat{\alpha} = \alpha$ and Kupiec's likelihood ratio statistic takes the form:

$$LR = 2\left[\ln(\widehat{\alpha}^m(1-\widehat{\alpha})^{n-m}) - \ln(\alpha^m(1-\alpha)^{n-m})\right]$$

where m is the number of exceptions and n is the sample size. This likelihood ratio is asymptotically distributed as a $\chi^2(1)$ under the null hypothesis. The left panels of Tables 4 and 5 show the p-values for the Kupiec test for the two stocks with a 5%, 2.5%, 1% and 0.5% IVaR level. Bold entries denote a failure of the model at the 95% confidence level, since the p-value is inferior to 0.05. The results show that the model performs well for both stocks. For almost all the intervals and the tails considered the p-values are superior to 0.05. The only exception is for smallest interval ($T = 15$) and higher VaR levels ($\alpha = 5\%, 2.5\%$) which could be explained by the fact that the interval length is too small for obtaining non-zero returns when resampling the tick-by-tick real returns and, therefore, the theoretical number of IVaR violations cannot be achieved. For an interval equal to 25 and a 5% IVaR level, the model is rejected at a 95% confidence level but not at a 97.5% confidence level.

We also apply the dynamic quantile (DQ) test of Engle and Manganelli (2004) to check for another property a VaR measure should display: the hits (IVaR violations) should not be serially correlated. According to Engle and Manganelli (2004), this can be tested by defining a sequence:

$$Hit_k \equiv I(y_k < -IVaR_k) - \alpha.$$

The expected value of Hit_k is 0 and the DQ test is computed using the regression of the variable Hit_k on its past, on current IVaR, and any other variables:

$$Hit_k = XB + \epsilon_k$$

Then, $DQ = \widehat{B}'X'X\widehat{B}/(\alpha(1-\alpha)) \sim \chi^2(l)$, where l is the number of explanatory variables and $\widehat{B}$ is the OLS estimate of B. We perform the test using 5 lags of the hits and the current IVaR as explanatory variables. Results are given in the right panels of Tables 4 and 5 which report the p-values of the DQ test. The results are satisfactory for the RY stock and coherent with the results from Kupiec's test. In most cases the p-values are larger than 0.05. For the PDG stock, some estimates still show some predictability, especially for the smallest interval. Overall, it seems that the model performs best for the two stocks when a 1% IVaR level is considered.

(Insert Tables 4 and 5 here)

Figure 5 illustrates the typical IVaR profile obtained from the model. One might argue that the Monte Carlo simulation is very time-consuming. While this may be true for backtesting purposes that generally require a sufficiently large number of validation intervals, computing the next forecasted IVaR (which is normally required for practical purposes) is reasonably fast. For example, producing the next IVaR with a Pentium 4, one CPU 3.06 GHz and 5000 simulated paths for an interval equal to 90, takes us approximately 3.5 minutes for each of the two stocks considered. For our samples, an interval equal to 90 corresponds on average to 41 minutes for RY and 79 minutes for PDG. The computing time can easily be reduced to less than one minute by using more than one CPU.

(Insert Figure 5 here)

5 Conclusion

In this paper, we have proposed a way of computing intraday Value at Risk using tick-by-tick data within the UHF-GARCH framework introduced by Engle (2000). Our specification of the UHF-GARCH model is more flexible than that of Engle (2000) since it endogenizes the definition of the time unit and lets the data speak for themselves. We applied our methodology to two actively traded stocks from the TSE (RY and PDG). While the literature is full of efforts to develop sophisticated VaR models for daily data, here we investigate the use of irregularly time-spaced intraday data for risk management. This is particularly useful for defining an IVaR appropriate for agents who are very active in the market. As a by-product of our study, we provide an out-of-sample evaluation of the predictive abilities of an UHF-GARCH model in a risk management framework, a question that has not yet been addressed in the literature.

We developed an intraday Monte Carlo simulation approach which enabled us to forecast high-frequency returns for any arbitrary interval length, thus avoiding complicated time manipulations in order to return to a convenient regularly spaced framework. In our setup the ACD model yields the consecutive steps in time while the UHF-GARCH model allows us to simulate the corresponding conditional tick-by-tick returns. Regularly spaced intraday returns are simply the sum of tick-by-tick returns simulated conditional on the forecasted duration. We considered using a normal distribution to estimate the UHF-GARCH model but, instead of using the predicted volatility given by that model and computing the VaR in a parametric way, we preferred to simulate the distribution of returns and extract the IVaR from it. Therefore, the approach is not totally model dependent.

Our results for the RY and PDG stocks indicate that the UHF-GARCH model performs well out-of-sample even when normality is assumed for the distribution of the error term, provided that the intraday seasonality has been accounted for prior to the estimation. This may be explained by the fact that we use a semiparametric method for computing the VaR as an empirical quantile from the simulated distribution of returns. In this way, it becomes possible to define an IVaR for any horizon of interest based on tick-by-tick data. Thus, our methodology for computing VaR with tick-by-tick data may constitute a reliable approach for measuring intraday risk.

Potential users of our approach would be traders who need intraday measures of risk; brokers and clearing firms looking for more accurate computations of margins; or any other entity interested in computing the VaR during a trading day in order to improve risk control. To compute the next IVaR using the method we propose, one has to monitor the time using the starting time (i.e. 11 a.m.) and the simulated durations for each path and, consequently, to apply the appropriate seasonal factors to re-introduce both interday and intraday seasonality in returns. The IVaR extracted from the simulated raw returns then has the usual interpretation.

Several extensions follow naturally from this study. First, a comparison of the VaR based on tick-by-tick data with predictions obtained from volatility models of the ARCH type (on regularly spaced observations) and realized volatility models in the spirit of Giot and Laurent (2004) would help clarifying the advantage of each approach. Second, one may wonder whether banks could benefit from incorporating tick-by-tick information into their VaR models. Financial institutions and particularly banks currently compute the 1-day VaR based on end-of-day positions, taking into account only daily closing prices and thus ignoring possibly wide intraday fluctuations and, consequently, the risks associated with them. Therefore, one could use our approach to see whether better results can be obtained by using all the trade information available in intraday databases rather than extracting a single observation to characterize the activity of a whole day. Third, the impact of using more sophisticated UHF-GARCH models together with distributions that account for fat tails and the excessive number of zero returns could also be investigated. Fourth, given that several methods for dealing with the intraday seasonality have been proposed in recent literature, one may study the impact of different deseasonalization procedures on the IVaR estimates. Finally, a challenging but rewarding extension would be the development of a portfolio IVaR based on irregularly time-spaced high-frequency data.

One may argue that the true VaR is underestimated by using transaction data since transactions are observed only if the spread is favorable to the trader. In this respect, since transaction events are certainly timed liquidity risk is not taken into account [13]. A VaR based on knowledge of the order book such as that proposed by Giot and Grammig

[13] We thank Joachim Grammig for pointing it out.

(2005) could provide an upper bound of the estimate.

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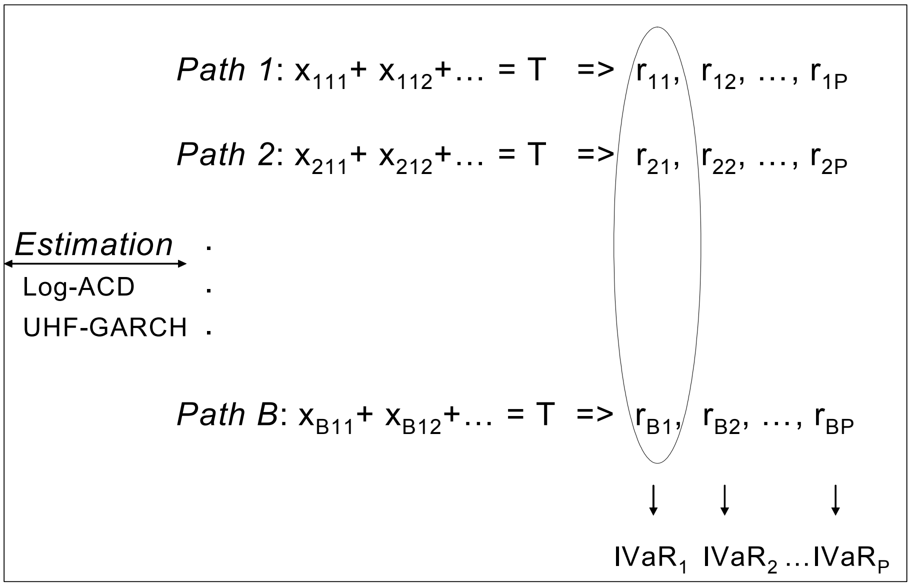
Figure 1: Illustration of the Intraday Monte Carlo Simulation Approach :



B is the number of independently simulated paths. T is the length of the chosen time interval. P is the number of regular intervals used for validation. x_{ijk} is the k trade duration corresponding to the path i for the interval j. r_{ij} is the regular return corresponding to the interval j for the path i. i = *1,...,B*. j = *1,...,P*. k = *1, ...,* t such that $\sum_{k=1}^{t} x_{ijk} \leqslant T$ and $\sum_{k=1}^{t+1} x_{ijk} > T$.

Figure 2: Histograms of Intraday Returns for Royal Bank (RY) and Placer Dome (PDG).

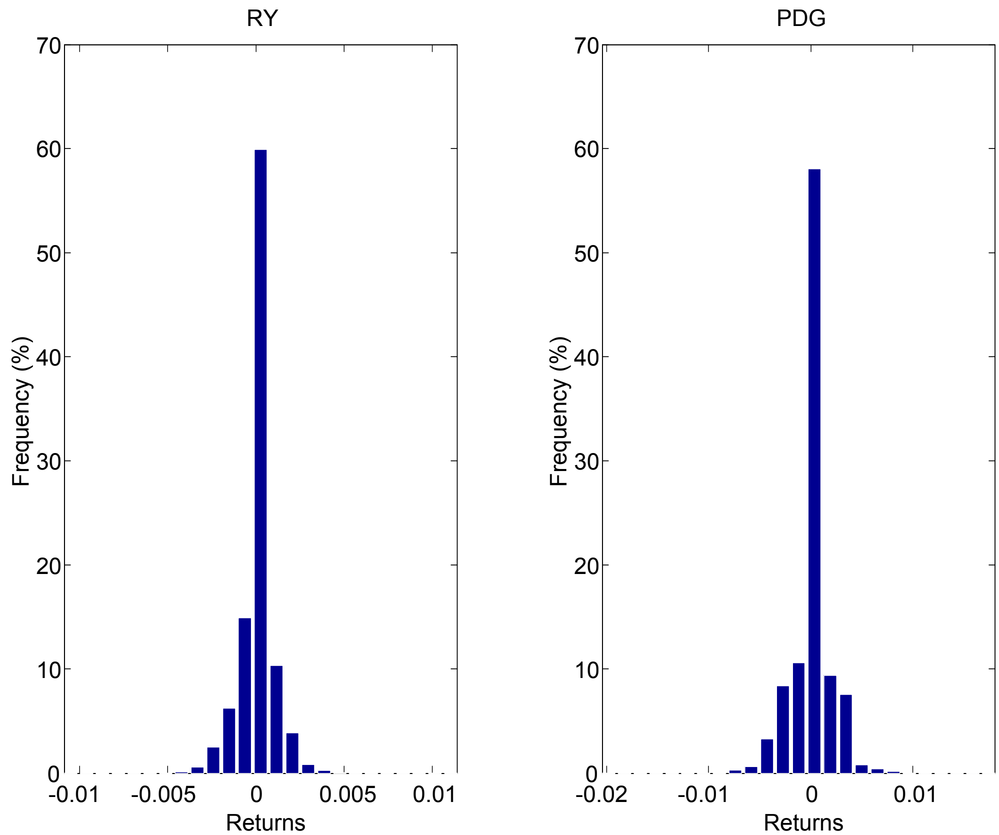


Figure 3: Estimated Intraday Factors for RY Durations

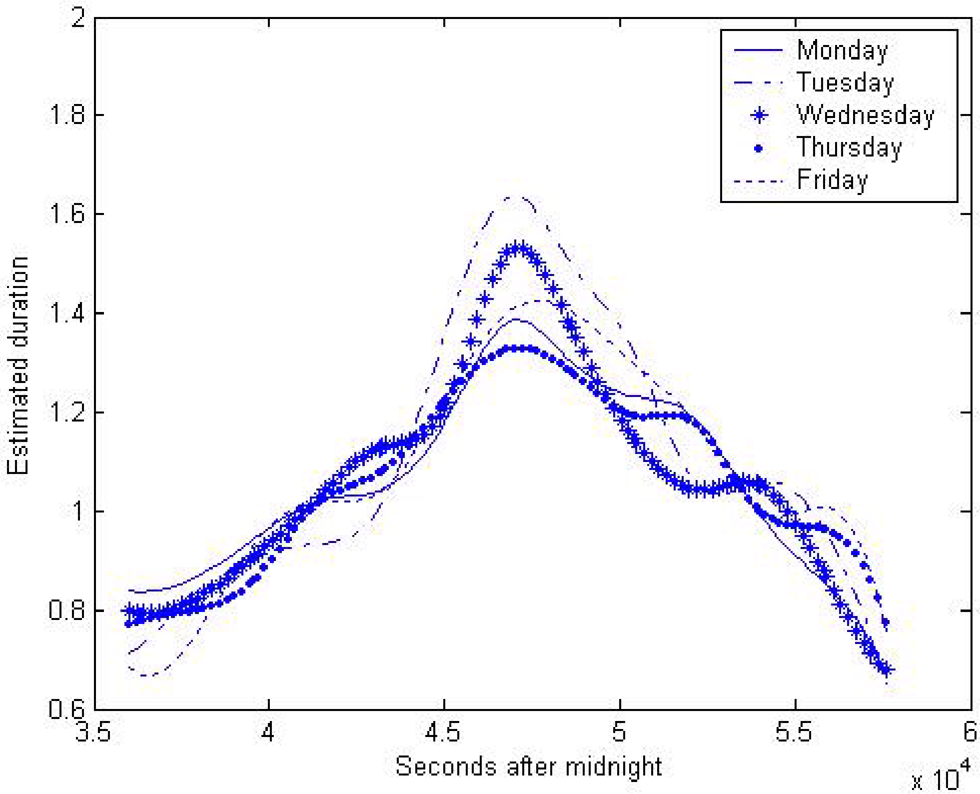


Figure 4: Estimated Intraday Factors for RY Squared Returns

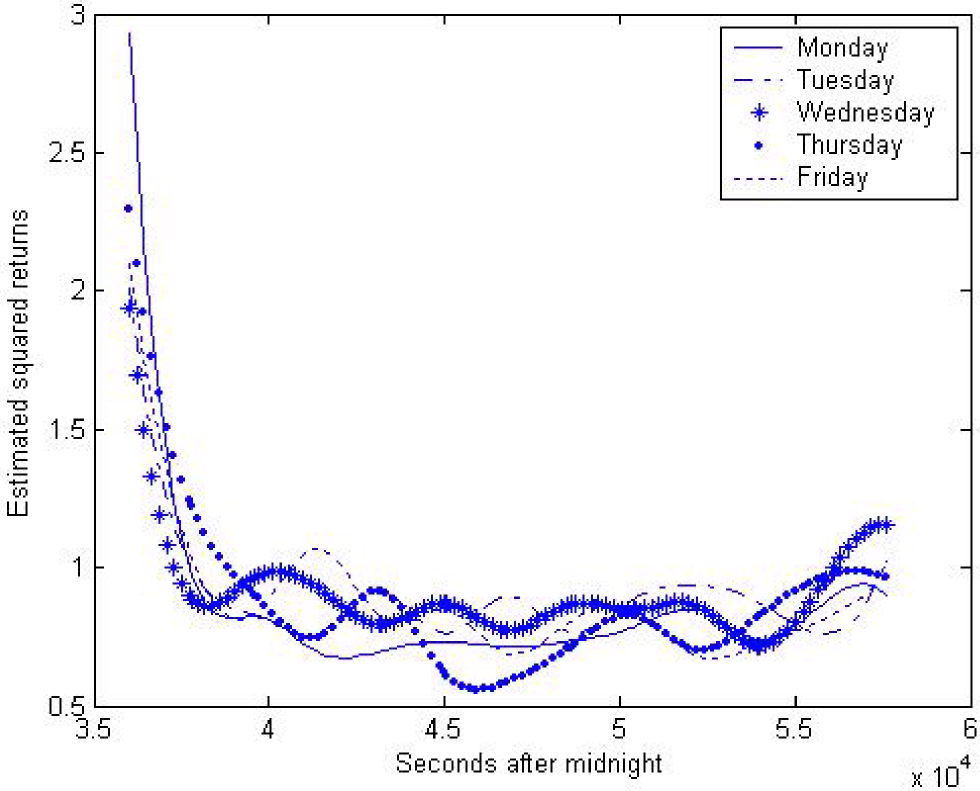


Figure 5: Intraday Returns vs IVaR for RY (interval = 45 or 22 minutes)

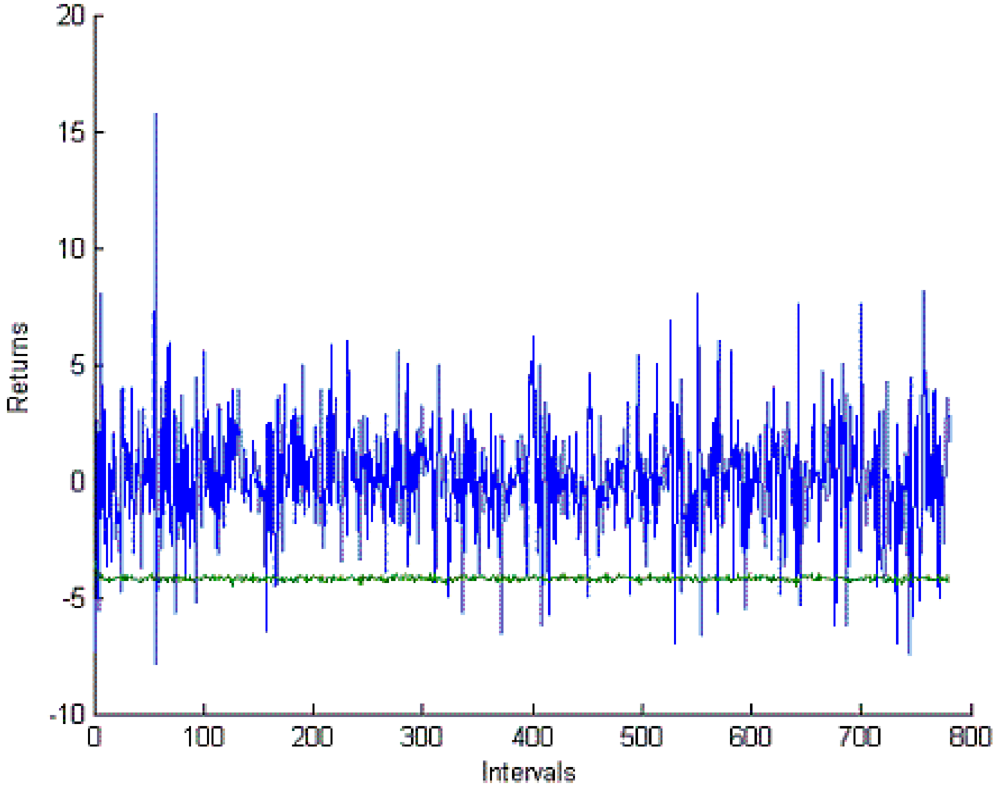


Table 1: Descriptive Statistics of Raw Data for Royal Bank (RY) and Placer Dome (PDG) Stocks

	Mean	Std. dev	Skew	Kurt	Max	Min
Durations						
RY	28.44	35.19	2.98	17.54	526	1
PDG	52.59	82.11	5.04	59.87	2248	1
Returns						
RY	0.000	0.001	0.024	9.890	0.011	-0.011
PDG	0.000	0.002	-0.169	11.195	0.018	-0.020

The sample period runs from April 1st to June 30, 2001. It consists of 51,660 observations for the RY stock and 27,956 observations for the PDG stock. *Mean* is the sample mean, *Std. dev* is the sample standard deviation, *Skew* is the sample skewness coefficient, *Kurt* is the sample kurtosis, *Max* is the sample maximum, *Min* is the sample minimum.

Table 2: Descriptive Statistics of Deseasonalised Data for Royal Bank (RY) and Placer Dome (PDG) Stocks

	Mean	Std. dev	Skew	Kurt	Max	Min	Q(15)	$Q_2(15)$
Durations								
RY	0.99	1.14	2.50	12.36	16.68	0.01	1325	387
PDG	0.93	1.50	7.68	170.59	63.33	0.01	5015	185
Returns								
RY	0.000	0.815	0.005	7.560	5.960	-6.422	8196	15417
PDG	0.001	0.890	-0.009	6.735	6.404	-6.421	2355	3118

The sample period runs from April 1st to June 30, 2001. It consists of 51,660 observations for the RY stock and 27,956 observations for the PDG stock. *Mean* is the sample mean, *Std. dev* is the sample standard deviation, *Skew* is the sample skewness coefficient, *Kurt* is the sample kurtosis, *Max* is the sample maximum, *Min* is the sample minimum. $Q(15)$ is the Ljung-Box test statistics with 15 lags, $Q_2(15)$ is the Ljung-Box test statistics applied to squared returns using 15 lags. The associated 95% critical value is 24.996.

Table 3: Estimates of ACD-GARCH Models

	RY		PDG	
Parameter	Estimate	t-Student	Estimate	t-Student
β_0	-0.017	-30.46	-0.030	-19.26
α_1	0.075	129.13	0.137	86.58
α_2	-0.057	-99.32	-0.105	-67.24
β_1	1.391	265.93	1.421	313.17
β_2	-0.408	-77.41	-0.434	-95.49
γ_1	0.425	95.04	0.391	55.36
γ_2	4.444	50.26	3.772	29.42
ϕ_1	0.069	4.11	0.078	2.58
θ_1	-0.605	-48.50	-0.369	-13.34
ω	-0.032	-9.27		
$\underset{\smile}{a_1}$	0.242	22.58	0.221	8.25
$\widetilde{\beta_1}$	0.921	232.00	0.778	19.59
γ	0.055	3.15	0.057	2.33
p-value $Q_d(15)$	0.182		0.611	
p-value $Q(15)$	0.000		0.208	
p-value $Q_2(15)$	0.002		0.833	

This table contains the parameters estimates of the ACD-GARCH models for RY and PDG. *t-Student* are the t-statistics associated with the parameters estimates. *p-value* $Q_d(15)$ represents the p-value associated with the Ljung-Box test statistic computed with 15 lags applied to the ACD standardized residuals. *p-value* $Q(15)$ and *p-value* $Q_2(15)$ represent, respectively, the p-values of the Ljung-Box test statistic computed with 15 lags for serial correlation in the standardized EGARCH residuals and in their squares.

Table 4: IVaR Results for RY

	Kupiec test				DQ test				
VaR level T	5%	2.5%	1%	0.5%	5%	2.5%	1%	0.5%	N
15	**0.000**	**0.000**	0.154	0.694	**0.000**	**0.021**	0.879	0.672	2470
25	**0.036**	0.231	0.928	0.317	0.365	0.344	0.360	0.003	1434
35	0.153	0.792	0.969	0.624	0.208	0.444	0.999	0.329	1012
45	0.862	0.914	0.676	0.632	0.974	0.881	0.999	0.982	781
90	0.592	0.903	0.651	0.957	0.220	0.847	0.992	0.999	385

This table contains the p-values for Kupiec's test and the DQ test of Engle and Manganelli (2004) using 5 lags and the current VaR as explicative variable. T is the length of the interval used for simulation. N represents the number of intervals used for validation. Bold entries denote a failure of the model at the 95% confidence level since the p-values are inferior to 0.05.

Table 5: IVaR Results for PDG

	Kupiec test				DQ test				
VaR level T	5%	2.5%	1%	0.5%	5%	2.5%	1%	0.5%	N
15	**0.000**	**0.018**	0.577	0.560	**0.009**	0.304	**0.041**	**0.000**	1294
25	0.642	0.837	0.837	0.678	0.387	0.251	0.063	**0.000**	755
35	0.762	0.944	0.274	0.675	0.136	**0.001**	0.991	0.992	530
45	0.121	0.692	0.662	0.531	**0.038**	0.670	0.568	0.711	409
90	0.762	0.215	0.994	0.381	0.544	0.590	0.934	0.645	201

This table contains the p-values for Kupiec's test and the DQ test of Engle and Manganelli (2004) using 5 lags and the current VaR as explicative variable. T is the length of the interval used for simulation. N represents the number of intervals used for validation. Bold entries denote a failure of the model at the 95% confidence level since the p-values are inferior to 0.05.