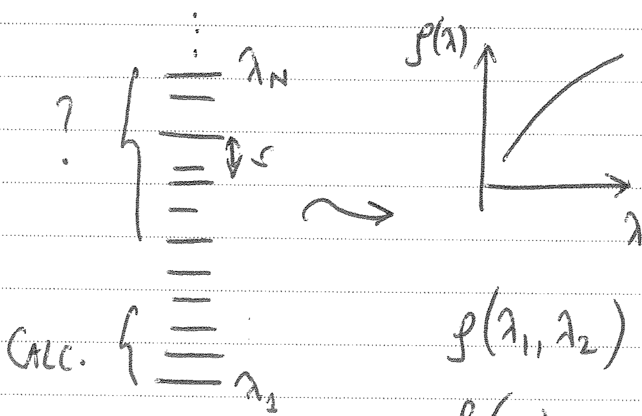
0. INTRODUCTION

0.A Wigner + von Neumann

50's — 40's

Complex Nuclear spectra



$\rho(\lambda_1, \lambda_2)$

$\ell(s)$

cf Prime Numbers

Numerical analysis

$$H\vec{x} = \vec{y}$$

$\sigma = \dfrac{\lambda_{max}}{\lambda_{min}}$: Conditioning factor

Random tests : $H_{ij} = H_{ji} = \pm 1$

↳ pdf of σ ?

What if H is a random matrix?

Hope : <u>universality</u>

<u>RMT</u> : Statistics of $\begin{cases} \text{eigenvalues} \\ \text{eigenvectors} \end{cases}$ of Random Matrices

60's Dyson, Mehta [Brownian motion model; interacting charges model]

90's Voiculescu [free random matrices]

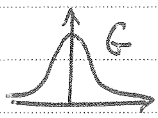
0.B Micro vs. macro universality – Bulk vs. Edge

H: IID RV with $P(H) \underset{|H|\to\infty}{\sim} \dfrac{A_\pm}{|H|^{1+\mu}}$

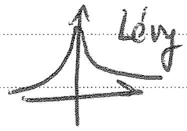
* Macro-universality

IID sums, $N \to \infty$

- $\mu > 2$: $\dfrac{1}{\sqrt{N}} \sum_i H_i \to$



- $\mu < 2$: $\dfrac{1}{N^{1/\mu}} \sum_i H_i \to$



IID extremes

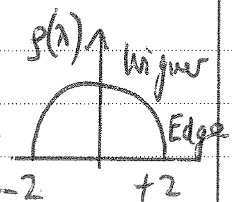
- $\mu = +\infty$ Gumbel
- $\mu < +\infty$ Fréchet

IID sym. mat.

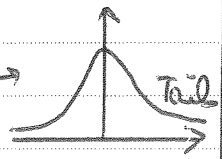
$H_{ij} = H_{ji}$

$N \times N \to \infty$

- $\mu > 2$: $\dfrac{1}{\sqrt{N}} H\vec{v} = \lambda \vec{v} \to$



- $\mu < 2$: $\dfrac{1}{N^{1/\mu}} H\vec{v} = \lambda \vec{v} \to$



Lévy matrices

Extreme eigenvalue

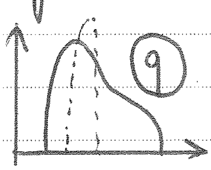
- $\mu > 4$: $\lambda_{max} - 2 \sim \dfrac{1}{N^{2/3}}$ Tracy-Widom
- $\mu < 4$: Fréchet

IID emp. correlation

H_{it} $N \times T \to \infty$

$\dfrac{1}{T} \sum_t H_{it} H_{jt} = \dfrac{1}{T} H H^t$

$Q = \dfrac{T}{N}$ $q = \dfrac{1}{Q}$



- $\mu > 2$: Marcenko-Pastur [MP]
- $\mu < 2$: Lévy MP

Extreme eigenvalue

- $\mu > 4$ TW
- $\mu < 4$ Fréchet

H sym, $\mu > 2$

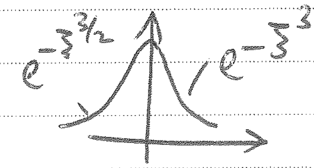
* Micro-universality

– Level spacing distribution $P(s) \sim s e^{-s^2}$

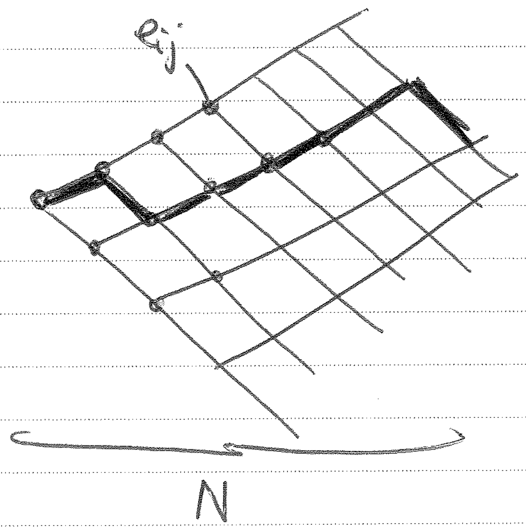
– Density fluctuations: $\langle N^2 \rangle_\Delta - \langle N \rangle_\Delta^2 \sim \ln \Delta$

- – Strong correlations may change $\rho(\lambda)$ but NOT $P(s)$
- – $P(s)$ sensitive to
  - symmetries e.g. $H_{ij} = H_{ji}^*$
  - Lévy ??

0.C Tracy-Widom & Directed Polymers

TW : $\lambda_{max} - 2 = \frac{\xi}{N^{2/3}}$ $F_{TW}(\xi)$ 

DP :



$$E_{\mathbb{P}} = \sum_{\text{path } \mathbb{P}} e_{ij}$$

$$E_0 = \min_{\text{paths}} E_{\mathbb{P}}$$

$$E_0 \underset{N\to\infty}{\sim} -\bar{e}N + \xi N^{1/3} \qquad \xi : F_{TW}(\xi)$$

0.D Applications of RMT

Nuclear spectra | Data Analysis
Quantum Transport | Finance
Wireless communication | ⋮
Riemann zeros
Random landscapes