

RANDOM MATRIX THEORY AND FINANCIAL APPLICATIONS : OUTLINE

0. INTRODUCTION

0.A History: Wigner and Von Neumann.

0.B *Bulk* spectral statistics: macro-universality and micro-universality. Central limit theorem, Wigner semi-circle and Marčenko-Pastur distribution. Level repulsion.

0.C *Edge* statistics and Tracy-Widom distributions. Directed polymers in random media and other related problems.

0.D Applications: from quantum chaos to finance – Riemann zeroes and multi-antennas.

1. RETURN STATISTICS, CORRELATION MATRICES & PORTFOLIO THEORY

1.A Returns multivariate statistics on different time scales. Gaussian & Student ensembles, correlation matrices. Epps effect. Empirical estimates and the large N,T problem.

1.B Risk and optimal portfolios. In sample vs. out of sample risk. Quality factor. Matrix cleaning: shrinkage and minimal diversification, RMT cleaning.

1.C Other risks: options and volatility correlations, tail correlations

1.D Non-equal time correlations, general rectangular correlation matrices. Singular Value Decomposition and the large N,M,T problem.

1.E Microscopic models for correlation matrices and connectivity graphs.

2. CLASSICAL RMT RESULTS

2.A Spectral transforms: Stieltjes transform, Green and Blue functions, R-transform and S-transform

2.B Four-ways to the semi-circle: a) joint level distributions and Dyson charge analogy; b) recursion equations and CLT; c) the replica method; d) free convolution.

2.C Other RMT results: Porter-Thomas, participation ratios, correlations, Lévy matrices, finite-N corrections, Tracy-Widom.

3. FREE MATRICES AND APPLICATIONS

3.A Free matrices and independent eigenbases; spectrum of sums and products of random matrices.

3.B Applications: Wigner, Marčenko-Pastur (MP), EMA MP

3.C Other applications: MP for arbitrary correlations, Student MP, maximum likelihood Student; Random SVD

4. EXTREME EIGENVALUES

4.A Tracy-Widom and Rank 1 perturbations; condensation transition. The MP case.

4.B Power-law tails: From Tracy-Widom to Fréchet and the fourth moment condition

4.C Dynamics of the top eigenvector. Spherical Ornstein-Uhlenbeck process.

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