Open issues in equity derivatives modelling

Lorenzo Bergomi

Equity Derivatives Quantitative Research
Société Générale
lorenzo.bergomi@sgcib.com

Talk Outline

- **Equity derivatives at SG**
- **A brief history of equity derivative products**

  Prehistory – 1997

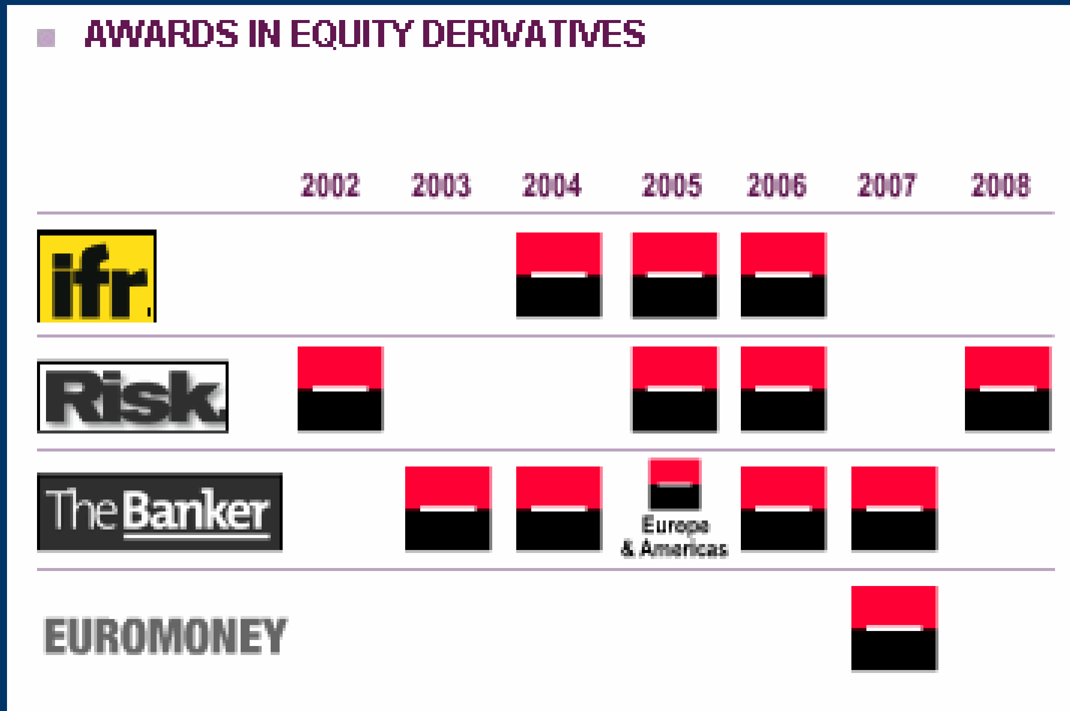
  History 1997 – 2003

  Modern times 2003 –

- **Modelling issues, algorithmic issues**
- **Risk measurement and management**
- **Conclusion**

Equity derivatives at SG

- SG regarded by industry participants as No 1 in equity derivatives



A brief history of equity derivative products

Prehistory – 1997

Products		Risks
Barrier options / Digitals		Skew: level / dynamics (little)
Max / Min options	$\left(\max_t (S_t) - K\right)^+$	same
Asian options	$\left(\frac{1}{N}\sum S_{t_i} - K\right)^+$	Smile
Basket options	$\left(\frac{1}{N}\sum S_T^i - K\right)^+$	Correlation (level)
Volatility swaps	$\sqrt{\frac{1}{T}\sum_k \ln\left(\frac{S_k}{S_{k-1}}\right)^2} - \hat{\sigma}_K$	Smile, VolOfVol
Simple cliquets	$\left(\frac{S_{T_2}}{S_{T_1}} - K\right)^+$	Forward smile

- Models / algos
  - Black Scholes / local vol
  - PDE / straight Monte Carlo

A brief history of equity derivative products

Capital-guaranteed products distributed by retail networks

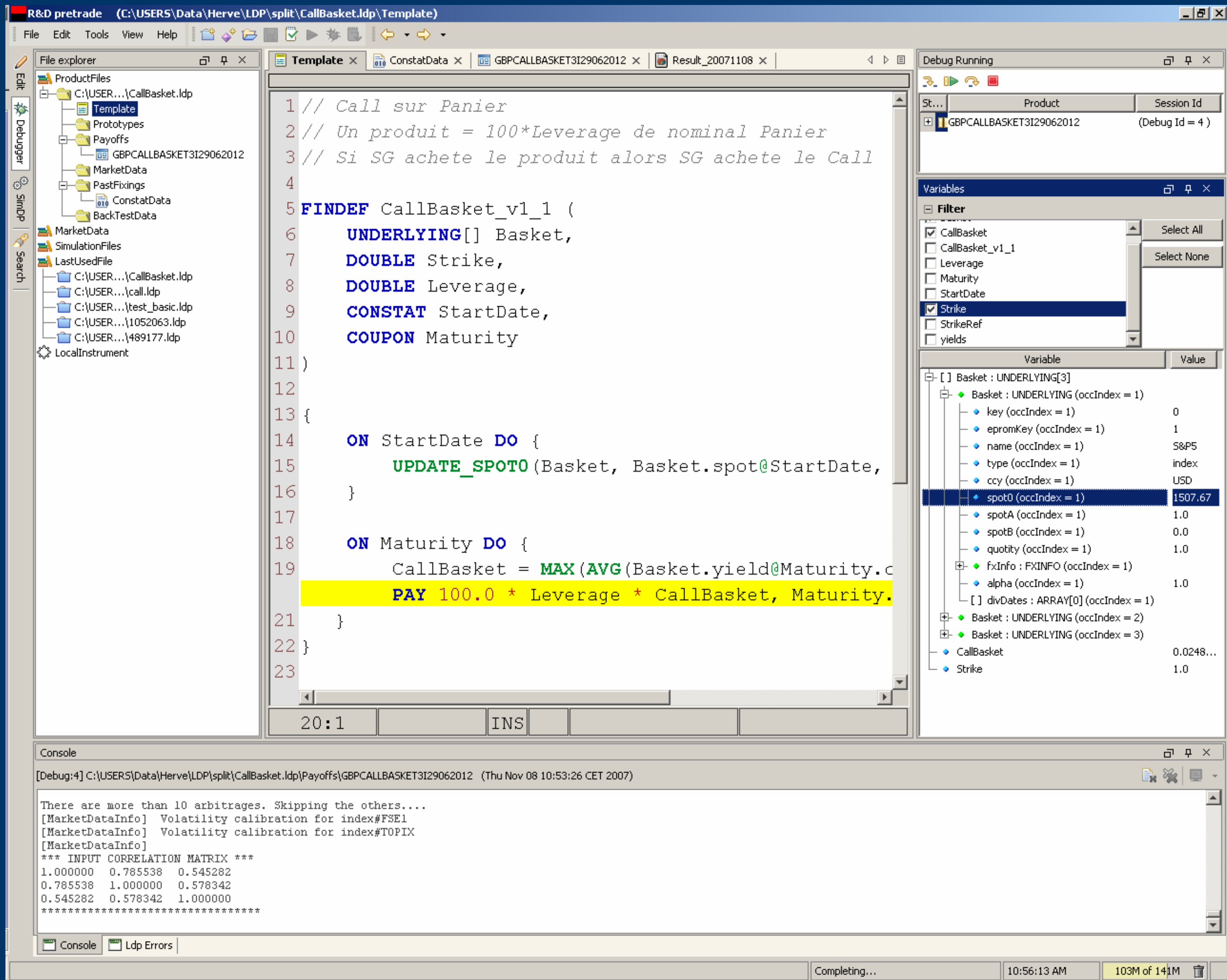
- Everest 1997 — 5 years / 12 stocks — ⇨ $100\% + \min\left(\frac{S_T^j}{S_0^j}\right)$

- Emerald 2004 — 10 years / 20 stocks — Every year, the stock whose perfomance since $t = 0$ is the largest gets frozen and removed from the basket, and its level is floored at 200% ot its initial value.

  ⇨ 100% + maximum perormance of yearly basket values since $t = 0$, floored at 0.

… and many, many, many, other variations

⇨ **trying to find closed-form formulas for specific exotic payoffs now irrelevant and useless**


R&D pretrade (C:\USERS\Data\Herve\LDP\split\CallBasket.ldp\Template)
File Edit Tools View Help
File explorer
ProductFiles
C:\USER...\CallBasket.ldp
Template
Prototypes
Payoffs
GBPCALLBASKET3I29062012
MarketData
PastFixings
ConstatData
BackTestData
MarketData
SimulationFiles
LastUsedFile
C:\USER...\CallBasket.ldp
C:\USER...\call.ldp
C:\USER...\test_basic.ldp
C:\USER...\1052063.ldp
C:\USER...\489177.ldp
LocalInstrument
Template
ConstatData
GBPCALLBASKET3I29062012
Result_20071108
// Call sur Panier
// Un produit = 100*Leverage de nominal Panier
// Si SG achete le produit alors SG achete le Call
FINDEF CallBasket_v1_1 (
UNDERLYING[] Basket,
DOUBLE Strike,
DOUBLE Leverage,
CONSTAT StartDate,
COUPON Maturity
)
{
ON StartDate DO {
UPDATE_SPOT0(Basket, Basket.spot@StartDate,
}
ON Maturity DO {
CallBasket = MAX(AVG(Basket.yield@Maturity.c
PAY 100.0 * Leverage * CallBasket, Maturity.
}
}
20:1
INS
Debug Running
St...
Product
Session Id
GBPCALLBASKET3I29062012
(Debug Id = 4)
Variables
Filter
CallBasket
CallBasket_v1_1
Leverage
Maturity
StartDate
Strike
StrikeRef
yields
Select All
Select None
Variable
Value
[ ] Basket : UNDERLYING[3]
Basket : UNDERLYING (occIndex = 1)
key (occIndex = 1) 0
epromKey (occIndex = 1) 1
name (occIndex = 1) S&P5
type (occIndex = 1) index
ccy (occIndex = 1) USD
spot0 (occIndex = 1) 1507.67
spotA (occIndex = 1) 1.0
spotB (occIndex = 1) 0.0
quotity (occIndex = 1) 1.0
fxInfo : FXINFO (occIndex = 1)
alpha (occIndex = 1) 1.0
[ ] divDates : ARRAY[0] (occIndex = 1)
Basket : UNDERLYING (occIndex = 2)
Basket : UNDERLYING (occIndex = 3)
CallBasket 0.0248...
Strike 1.0
Console
[Debug:4] C:\USERS\Data\Herve\LDP\split\CallBasket.ldp\Payoffs\GBPCALLBASKET3I29062012 (Thu Nov 08 10:53:26 CET 2007)
There are more than 10 arbitrages. Skipping the others....
[MarketDataInfo] Volatility calibration for index#FSE1
[MarketDataInfo] Volatility calibration for index#TOPIX
[MarketDataInfo]
*** INPUT CORRELATION MATRIX ***
1.000000 0.785538 0.545282
0.785538 1.000000 0.578342
0.545282 0.578342 1.000000
****
Console
Ldp Errors
Completing...
10:56:13 AM
103M of 141M

A brief history of equity derivative products

- **Variance Swaps** — 3 months ⇨ 5 years — stocks / indices

  Pays realized variance – usually measured using daily returns

$$\sum_k \ln\left(\frac{S_k}{S_{k-1}}\right)^2 - \hat{\sigma}^2 T$$

- **Napoleon** — 5 years / 1 index

  Every year, pays coupon reduced by worst of 12 monthly performances of the index.

$$\left(C + \min_k\left(\frac{S_k}{S_{k-1}}\right)\right)^+$$

- **Accumulator** — 3 years / 1 index

  At maturity pays the sum – if it is positive – of the monthly performances, capped and and floored.

$$\left(\sum_k \max\left(\min\left(\frac{S_k}{S_{k-1}} - 1{,}1\%\right), -1\%\right)\right)^+$$

Modern times

- **Corridor variance swaps**

  Daily variance only counted when underlying is inside given interval

  $$\sum_k 1_{S_k \in [L,H]} \left(\ln\left(\frac{S_k}{S_{k-1}}\right)^2 - \hat{\sigma}^2 \Delta t \right) \qquad [L,H],\ [L,+\infty],\ [0,H]$$

- **Correlation swaps**

  Pays realized correlation over 3 years by stocks of an index

- **Gap notes**

  Maturity = 1 year, a series of daily puts on daily returns of an index

  with strikes 85%, 90%

- **Options on realized variance**

  On indices, maturities: 3 months to 2 years

  $$\left(\frac{1}{T} \sum_\tau \ln\left(\frac{S_\tau}{S_{\tau-1}}\right)^2 - \hat{\sigma}_K{}^2 \right)^+$$

- **Timer options**

  Vanilla payoff, paid when realized variance Q_t reaches set level:

  $$Q_\tau = \sum_{i=1}^{\tau} \ln\left(\frac{S_{i+1}}{S_i}\right)^2$$

- **Hybrids**

  Equities / Rates / Forex / Commodities   Arbitrary payoffs

Modelling issues – 1

- Why not just delta-hedge ?
  - Variance of residual P&L too large ⇨ use other options

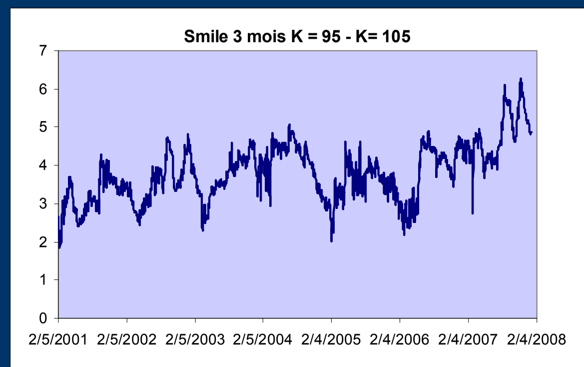
⇨ **Options are hedged with options**

- Once we start using options as hedging instruments
  - Less sensitivity to historical parameters, more sensistivity to implied parameters

⇨ **Model the dynamics of implied parameters**

- Example of simple cliquet $\left(\frac{S_{T_2}}{S_{T_1}} - 1\right)^+$

$$t \quad\quad T_1 \underbrace{\quad\quad}_{\hat{\sigma}_{12}} T_2 \qquad P(\hat{\sigma}_{12}, r, \cdots)$$



Modelling issues – 2

- **How should calibration be done ? Do we really need to calibrate ?**
  - Not compulsory: charge a hedging cost. We hedge parameter p by trading instrument O so that sensitivity to p vanishes:

$$\frac{dP}{dp} = \lambda \frac{dO}{dp}$$

  - Model price P is adjusted so as to include hedging cost:

$$\begin{aligned} \text{Price} &= P(\hat{p}) + \lambda\left(O(p_{\text{Market}}) - O(\hat{p})\right) \\ &\approx P(p = p_{\text{Market}}) \end{aligned}$$

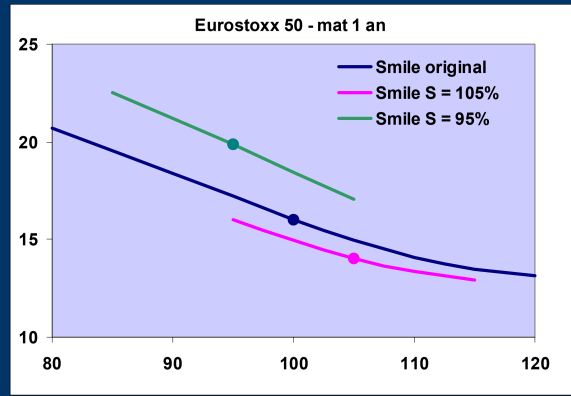
- **Then what is the point in calibrating ?**
  - Ensures price factors in hedging costs incurred at $t = 0$ – **not future costs !**

⇨ **Necessary to calibrate model on relevant set of hedging instruments**

⇨ **Useless if one is unable to specify how to hedge the exotic with the hedge instruments**

Modelling issues – 3

- **Volatility risk – models**



- **« Old models »**
  - Local volatility
  - Heston
  - SABR
  - Models based on process of instantaneous variance:
  - Jump / Lévy

$$\begin{aligned} dS &= \ldots dt + \sqrt{V}\, S\, dW_S \\ dV &= \ldots dt + (\quad)\, dW_V \end{aligned}$$

- **Challenge: Build models that give control on joint dynamics of implied volatilities and spot:**
  - **First step: model dynamics of curve of forward variances**
  - **Next step: model dynamics of the implied volatility surface**
    - Direct modelling of dynamics of implied volatilities is a dead end
    - Low-dimensional Markov representation desirable
    - How much freedom are we allowed ?

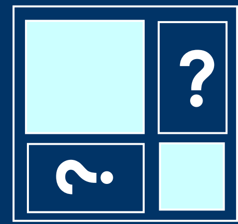
Modelling issues – 4

- **Hybrids**
  - Equities
  - Interest rates
  - Forex
  - Commodities

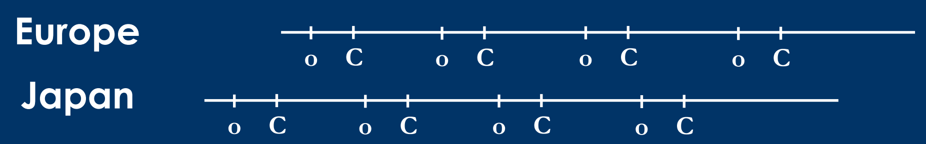
- **Hybrid models are not built by simply glueing together models for each asset class**
  - **Passive hybrids: payoff involves one asset class only**
    - Long-dated equity, Forex options
    - Credit / Equity: convertible bonds
  - **Active hybrids : payoff involves all asset classes**
    - Require state-of-the art models for each asset class
    - Even local vol calibration for equity smiles not easy when interest rates are stochastic

Modelling issues – 5

- **Correlation – how de we put together correlation matrices ?**
  - How do we build the large correlation matrices needed in hybrid modelling ?
  - Simpler question: imagine a 1-factor stoch. vol model and a payoff involving 2 securities
    - How do we set the cross-correlations ?



  - Even simpler question – how do we measure correlations ?
  - Example of European / Japanese stocks – no overlap



- **Correlation – how de we measure correlation risk ?**

- **Corrrelation – how to model correlation smile ?**

Algorithmic issues

- Monte Carlo
  - How can we speed up pricing ?
  - Quasi-random numbers
  - Discretization of SDEs ?

  - Callable / putable options

  - Computing sensitivies to
    - Initial conditions
    - Parameters of dynamics (volatilities / correlations, etc..)

Conclusion

- **These are exciting times for doing quantitative finance**
  - Lots of new instruments / product / algorithmic issues
  - Rich mathematical toolbox from which to pick