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BOOK OF ABSTRACTS

An Estimator of the Quadratic Variation of a Process with Finite Energy

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Quadratic variation determines the price of options in the classical B & S market model. It is also well-known that to certain extent the hedging price is robust with respect to quadratic variation. The hedging price is the same for a big class of models with the same quadratic variation ([2]). Here we propose an estimator of the quadratic variation for a class of processes with finite energy. A typical example is the case, where the stock price is driven by X, and X is a sum of standard Brownian motion W and fractional Brownian motion B^H with Hurst index $H > \frac{1}{2}$, independent of W. Our results are extensions to [3]. The talk is based on [1].

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Tempered stable and tempered infinitely divisible models with volatility clustering

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Most of the important models in finance rest on the assumption that randomness is explained through a normal random variable. However there is ample empirical evidence against the normality assumption, since stock returns are heavy-tailed, leptokurtic and skewed, see [15],[5] and [14] for example. Returns from financial assets show well-defined patterns of leptokurtosis and skewness which cannot be captured by the normality assumption. Furthermore a conclusion of the literature is that although the empirical evidence does not support the normal distribution, it is not always consistent with an α-stable distribution. The distribution of returns for assets has heavier tails relative to the normal distribution and thinner tails than the α-stable distribution. Partly in response to those empirical inconsistencies, there is a search for suitable alternatives to the α-stable distribution. One such alternative is the family of tempered stable (TS) and tempered infinitely divisible (TID) distributions.

In the first part of the presentation, we will review the definition of the class of TS distributions. The formal definition of this family of distributions has been proposed by Rosiński [16] where a completely monotone function is chosen to transform the Lévy measure of a stable distribution. The KoBol [3], the CGMY [4], the Inverse Gaussian (IG), the tempered stable of Tweedie [18] and the KR ([10],[9]) are only some parametric examples in this class, that have an infinite dimensional parametrization by a family of measures [19]. Further extensions or limiting cases are also given by the fractional tempered stable framework [7], the bilateral gamma [12] and the generalized tempered stable distribution ([5],[13]).

In the second part of the presentation, we will introduce the TID class. Tempered stable distributions may have all moments finite and exponential moments of some order. The idea of selecting a different class of tempering function has been already considered in the literature, see [8]. By following the approach of Rosiński [16] and considering a particular family of tempering functions, a new class of distributions is introduced with same suitable properties of the tempered stable class, but may admit exponential moments of any order [1]. By multiplying the Lévy measure of a stable distribution with a positive definite radial function, see [17], instead of with a completely monotone function as in [16], we obtain the class of tempered infinitely divisible (TID) distributions. In some cases the characteristic function of a TID random variable is extendible to an entire function on $\mathbb{C}$, (i.e., it admits any exponential moment). A parametric example of a TID distribution is given by the MTS distribution [11]. It is not a tempered stable of the Rosiński type even though its properties are very close to that class.

In general, the use of infinitely divisible distributions is obstructed by the difficulty of calibrating and simulating them. In this presentation, we address some numerical issues resulting from tempered stable and tempered infinitely divisible modelling, with a view toward the density approximation and simulation [2].

By considering the Duan's GARCH model [6], we will present an infinitely divisible GARCH framework [9] in the last part of the presentation. We then construct a new GARCH model with the infinitely divisible distributed innovation and different subclasses of that GARCH model that incorporates three observed properties of asset returns: volatility clustering, fat tails, and skewness. We will present the algorithm to find the risk-neutral return processes for those GARCH models using the change of measure for the tempered stable and tempered infinitely divisible distributions. To compare the performance of these GARCH models, we report the results of the parameters estimated for the S&P 500 index and investigate the in-sample and out-of-sample performance for the S&P 500 option prices.

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An Alternative Characterization of Time-Consistent Sets of Measures

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Since Epstein an Schneider introduced their multiple priors model in [2] and with it the concept of rectangularity which has been much discussed in the literature. The search for an equivalent definition was among others solved by Delbaen in [1] with his concept of time-consistency. This feature is needed to be able to make dynamic consistent decisions when looking at choices that involve the factor time.

In [3] Riedel constructs time-consistent sets of measures with the help of predictable processes in a discrete setting. Naturally the question arose if this construction exhausts all time-consistent sets of measures in this setting. For that reason we start with a discrete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbb{P}_0)$ for which we only assume that the filtration has a constant and finite splitting function. We also first restrict ourselves to a finite time horizon.

We then take a set of measures $\mathcal{P}$ for which we assume

- $\mathbb{P}_0 \in \mathcal{P}$ and $\forall\, \mathbb{P} \in \mathcal{P} : \mathbb{P} \sim \mathbb{P}_0$
- $\mathcal{D}_t = \left\{ \frac{d\mathbb{P}}{d\mathbb{P}_0}\big|_{\mathcal{F}_t} \mid \mathbb{P} \in \mathcal{P} \right\}$ is weakly compact in $L^1(\Omega, \mathcal{F}, \mathbb{P}_0)$ for all fixed $t \in [0, T]$.
- $\mathcal{P}$ is time-consistent.

These assumptions and the assumption on the filtration give us a martingale basis $\{\omega_{1,t}, ..., \omega_{\nu-1,t}\}_{t\in[0,T]}$ which allows us to uniquely construct a set of predictable processes $\mathcal{A}^{\mathbb{P}} := \left\{(\alpha^{\mathbb{P}}_{1,t}), ..., (\alpha^{\mathbb{P}}_{\nu-1,t})\right\}_{t\in[0,T]}$ belonging to each measure $\mathbb{P} \in \mathcal{P}$. That is for every set $\mathcal{P}$ we get a set $\mathcal{A} := \{\mathcal{A}^{\mathbb{P}} \mid \mathbb{P} \in \mathcal{P}\}$.

This leads us to the main result of the paper which is summarized in the following:

Theorem: *For every set of measures $\mathcal{P}$ satisfying the above assumptions there is a set of predictable processes $\mathcal{A}$ such that*

$$\mathcal{P} = \left\{ \mathbb{P} \,\Big|\, \left(\frac{d\mathbb{P}}{d\mathbb{P}_0}\right)_t = \tilde{\mathcal{E}}_t(\alpha)\ ,\ \alpha \in \mathcal{A} \right\} \qquad \textit{where}$$

$$\tilde{\mathcal{E}}_t(\alpha) = \exp\left(\sum_{s=1}^{t}\sum_{h=1}^{\nu-1} \alpha_{hs}\Delta\omega_{hs} - \sum_{s=1}^{t} \ln \mathbb{E}\left[\exp\left(\sum_{h=1}^{\nu-1} \alpha_{hs}\Delta\omega_{hs} \right) \right] \right)$$

Additionally $\mathcal{A}$ inhabits the following features:

- $0 \in \mathcal{A}$
- *$\mathcal{A}$ is compact.*
- *$\mathcal{A}$ is stable under pasting, i.e. for $\alpha, \beta \in \mathcal{A}$ and every stopping time τ the process γ defined by* $\gamma_t = \begin{cases} \alpha_t & \textit{if } t \leq \tau \\ \beta_t & \textit{else} \end{cases}$ *is also in $\mathcal{A}$.*

What we can also show is that the conversion of this theorem is true. This means if a set of predictable processes has the features mentioned in the theorem above, it will define a time-consistent set of measures via the same construction used to prove the theorem. Here it can be shown that the resulting set $\mathcal{P}$ again satisfies the assumptions asked for in the beginning.

So all in all we have in this special setting found an alternative characterization for time-consistent sets of measures which might be helpful in solving optimal stopping problems in the future.

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On the Convergence of Higher Order Hedging Schemes

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The aim of this paper is to investigate the convergence of the hedging mean squared error of a discretely re-balanced hedge portfolio containing two hedge instruments with respect to the number of re-balancings. We consider a complete diffusion market setting containing a number of instruments with smooth enough payoff functions (i.e. European call options) where the hedge portfolio is rebalanced on an equidistant time grid.

In Zhang (1999) the order of convergence of the mean squared hedging error using the underlying as hedge instrument was found to be n^{-1} in the case of European options. Gobet-Temam (2001) showed that when the derivative has a more irregular payoff the order of convergence may decrease. For the digital option the order of convergence is found to be $n^{-\frac{1}{2}}$. By using a nonequidistant time grid an order of convergence of n^{-1} in the case of the digital option can be reached. This is shown in Geiss (2002). This paper is mainly an extension of the paper by Zhang (1999) however the techniques used in the proof are more related to those used in Gobet-Temam (2001).

In this paper we extend these previous results to cover the case of contracts that is hedged using two hedge instruments (the underlying and some other derivative) on an equidistant time grid. We find that the mean squared hedging error decreases as $n^{-\frac{3}{2}}$ when letting n approach infinity. Thus the order of convergence increases substantially when adding one more hedge instrument to the hedge portfolio. We also derive an expression of this leading $n^{-\frac{3}{2}}$-order term.

In the case of a constant coefficient market (i.e. the Black and Scholes market) we derive an explicit expression of the leading $n^{-\frac{3}{2}}$ order term. Further more we investigate by simulation how well the asymptotic expression approximates the true mean squared error for finite n.

The next section introduces some notation, basic facts and previous results together with our results. In section 3 we perform a simulation study investigating how the results from section 2 relates to a simulated example. We conclude our findings in section 4.

1 Setting and results

We will consider a market model where the risky asset under the risk neutral probability measure $\mathbb{Q}$ has the dynamics

$$dX(t) = rX(t)dt + \sigma(X(t))X_t dW(t)\,, \tag{1}$$

with $X(0) = x_0$ and where $\{W(t)\}_{t\in[0,T]}$ is a Wiener process. The risk free asset has the dynamics

$$dB(t) = rB(t)dt\,.$$

The above market model may be recognized as a local volatility model with a time invariant diffusion coefficient. In this setting it is often convenient to work with the transformed process $Y(t) = \log(X(t))$. One typically wants the processes to have smooth enough transition densities, which translates to smoothness properties of the pricing functions of derivatives in this market. We will let the coefficients of (1) fullfill the following conditions:

A1. $\sigma(x)$, $x\sigma_x(x)$ and $x^2\sigma_{xx}(x)$ should be bounded and continuous and there should exist constants $H > 0$ and $\delta \in (0,1)$ such that $|x^2\sigma_{xx}(x) - \bar{x}^2\sigma_{xx}(\bar{x})|/|x-\bar{x}|^\delta \leq H$, for $(x,\bar{x}) \in \mathbb{R}^2$.

A2. There should exist a constant $h > 0$ such that $\sigma(x) \geq h$ for all $x > 0$.

Now consider a T_1 claim denoted F_1 with payoff function $\Phi_1(x) = (x - K_1)^+$. We would like to replicate this contract using the underlying and some other derivative F_2 with payoff function $\Phi_2(x) = (x - K_2)^+$ and which expires at $T_2 > T_1$. The price of the contracts at time t will be denoted $F_1(t, X(t))$ and $F_2(t, X(t))$ respectively. We will let $F_{1,x}(t, X(t))$ and $F_{2,x}(t, X(t))$ denote their respective derivatives with respect to the underlying and equivalently for higher order derivatives. Also introduce the process $\tilde{X}(t) = e^{-rt}X(t)$.

Since the market considered is complete the derivatives can be perfectly replicated using the underlying as a hedge instrument. In this case the amount to be held in the underlying in order to replicate the contract equals $F_{i,x}(t, X(t))$. This is what is usually called delta hedging.

We now turn our attention to the case where we use both the underlying and F_2 to hedge F_1. We will let $h^B(t)$, $h^X(t)$ and $h^{F_2}(t)$ denote the amount held in the bank account, the number of shares of the underlying and the number of shares of the hedge derivative respectively. We will let $V(t)$ denote the value of the hedge portfolio at time t

$$V(t) = h^X(t)X(t) + h^{F_2}(t)F_2(t, X(t)) + h^B(t) .$$

Clearly we want the value of this portfolio to equal the value of F_1. Furthermore in order to replicate the issued contract one way is to make the hedge portfolio both delta and gamma neutral, i.e. neutral with respect to the first and second derivative with respect to the underlying, thus

$$\begin{aligned} F_1(t) &= h^X(t)X(t) + h^{F_2}(t)F_2(t, X(t)) + h^B(t) , \\ F_{1,x}(t) &= h^X(t) + h^{F_2}(t)F_{2,x}(t, X(t)) , \\ F_{1,xx}(t) &= h^{F_2}(t)F_{2,xx}(t, X(t)) . \end{aligned}$$

By solving this we get the portfolio

$$\begin{aligned} h^X(t) &= F_{1,x}(t, X(t)) - \frac{F_{2,x}(t, X(t))F_{1,xx}(t, X(t))}{F_{2,xx}(t, X(t))} , \\ h^{F_2}(t) &= \frac{F_{1,xx}(t, X(t))}{F_{2,xx}(t, X(t))} , \\ h^B(t) &= F_1(t, X(t)) - h^X(t)X(t) - h^{F_2}(t)F_2(t, X(t)) . \end{aligned}$$

We will refer to this type of hedging as gamma hedging.

In our setting we will re-balance the hedge portfolio at a prespecified time grid t_i^n, where n denotes the number of re-balancing points. In the equidistant case we have that $t_i^n = iT_1/n$ where $i = \{0, \ldots, n-1\}$. The difference between the hedge portfolio and the derivative at time $t = T_1$ with n number of re-balancing will be denoted by $\mathcal{R}_\Delta(n)$ in the delta hedging case and $\mathcal{R}_\Gamma(n)$ in the gamma hedging case. In the delta hedging case $\mathcal{R}_\Delta(n)$ can be written as

$$\mathcal{R}_\Delta(n) = \int_0^T \left(F_{1,x}(t, X(t)) - F_{1,x}(\varphi_n(t), X(\varphi_n(t)))\right) d\tilde{X}(t) ,$$

where $\varphi_n(t) = \sup\{t_i^n \mid t_i^n < t\}$, and in the gamma hedging case we have that

$$\begin{aligned} \mathcal{R}_\Gamma(n) = &\int_0^T \left(h^X(t) - h^X(\varphi_n(t))\right) d\tilde{X}(t) \\ &+ \int_0^T \left(h^{F_2}(t) - h^{F_2}(\varphi_n(t))\right) d\tilde{F}_2(t) . \end{aligned}$$

In Zhang (1999) the discretely rebalanced delta hedge case is investigated. The mean squared hedging error is found to be of order n^{-1}:

$$\mathbb{E}[\mathcal{R}_{\Delta}^2(n)] = \frac{T}{2n}\mathbb{E}\left[\int_0^T e^{-2rt}\sigma^4(X(t))X^4(t)\left(F_{1,xx}(t,X(t))\right)^2 dt\right] + o\left(\frac{1}{n}\right).$$

In this paper we investigate the hedging error induced by discretely rebalancing the gamma hedge portfolio. We find that the mean squared hedging error is of order $n^{-\frac{3}{2}}$:

Proposition 1.1 *Under A1 and A2 the following holds*

$$\mathbb{E}[\mathcal{R}_{\Gamma}^2(n)] = \left(\frac{T_1}{n}\right)^{\frac{3}{2}} C_{\frac{3}{2}} \lim_{t\to T_1} g(t) + o\left(\frac{1}{n^{\frac{3}{2}}}\right),$$

where

$$g(t) = (T_1 - t)^{3/2}\mathbb{E}\left[e^{-2rt}F_{1,xxx}^2(t)X^6(t)\sigma^6(X(t))\right], \tag{2}$$

and

$$C_a = \sum_{k=1}^{\infty}\int_0^1\int_0^x\int_0^w \frac{1}{(k-v)^a}dv\,dw\,dx.$$

Remark For the case when $\alpha = 3/2$ we have that $C_{3/2} \approx 0.65$.

2 Simulation study

In this section we will consider a market model with constant diffusion coefficient $\sigma(x) = \sigma$, i.e. the Black and Scholes market, thus

$$dX(t) = rX(t)dt + \sigma X(t)dW(t).$$

In this case (2) can be calculated explicitly. Our aim in this section is to see how close this expression is to an estimate of the hedging error that is obtained by Monte Carlo simulation.

We find in the case of a Black and Scholes market (after rather tedious calculations) that

$$\mathbb{E}[\mathcal{R}_{\Gamma}^2(n)] = \left(\frac{T_1}{n}\right)^{\frac{3}{2}} \frac{\sigma^2K^2}{4\pi\sqrt{2T_1}}e^{-2rT_1}e^{-\frac{(\ln(x_0/K)+(r-\sigma^2/2)T_1)^2}{2\sigma^2T_1}}C_{\frac{3}{2}} + o\left(\frac{1}{n^{\frac{3}{2}}}\right). \tag{3}$$

We now want to see how well this expression approximates the true mean squared hedging error. In Figure **??** the above expression is depicted for a number of different re-balancings together with estimates of the mean squared hedging error from a Monte Carlo simulation for some different choices of K_2 and T_2. The number of trajectories, N_{MC}, in the Monte Carlo simulation was set to $N_{MC} = 1000$.

As can be seen all Monte Carlo estimates are relatively close to the value of (3). As expected the hedge derivative most similar to F_1, that is when $T_2 = 0.6$ and $K_2 = 100$, is the one giving the lowest squared error. Further more it can be seen that for higher T_2 the squared error over all choices of K_2 seem to be quite low.

3 Conclusions

We have shown that the mean squared hedging error using two hedge instruments converges to zero with order $n^{-\frac{3}{2}}$ as the number of re-balancings n goes to infinity. An expression of the leading $n^{-\frac{3}{2}}$-order term has also been derived.

By simulation it is shown that the derived expression of the leading term approximates the true mean squared hedging error quite well, see Figure **??**. It can also be seen that, as expected, using hedge instruments that are similar to the instrument to be hedged gives lower mean squared hedging error.

Further research could be directed to the investigation of hedging schemes using an arbitrary number of hedge instruments. Is it possible give a general statement of the order of convergence as a function of the number of hedge instruments? Another direction would be to investigate the higher order terms in the expansion of the hedging mean squared error, in order to find an optimal choice of hedge instrument in a collection of possible hedge instruments.

Optimal consumption policies in illiquid markets

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We study a mixed discrete/continuous time stochastic control problem arising from a portfolio/consumption choice problem in a market model with random trading times introduced in [1] and also studied in [2]. In this market model under liquidity risk, stock prices can be observed and traded only at random times of a Poisson process corresponding to quotes in the market. The investor is also allowed to consume continuously from the bank account and her/his objective is to maximize the expected discounted utility from consumption. The resulting optimization problem is a nonstandard mixed discrete/continuous time stochastic control problem, which leads via the dynamic programming principle to a coupled system of nonlinear integro-partial differential equations (in short IPDE).

In [2], the authors proved that the value functions to this stochastic control problem are characterized as the unique viscosity solutions to the corresponding coupled IPDE. This characterization makes the computation of value functions possible (see [1]), but it does not yield the optimal consumption policies in explicit form.

The main contribution of this paper is to derive smoothness C^1 results for the value functions, going beyond the viscosity property. Actually, by using arguments of (semi)concavity and convex Hamiltonian for the IPDE in connection with viscosity solutions, we prove the continuous differentiability of the value functions. Such regularity result allows then to get the existence of an optimal control through a verification theorem and to characterize the optimal portfolio/consumption strategy both in feedback form in terms of the classical derivatives of the value functions and as the solution of a second-order ODE.

Finally, numerical illustrations of the behavior of optimal consumption strategies between two trading dates are given.

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Hedging of American Options under Transaction Costs

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A classical result of the theory of frictionless market asserts that the set of initial capitals needed to hedge a European option ξ with the maturity(=exercise) date T is a semi-infinite closed interval $[x_*, \infty[$ whose left extremity $x_* = \sup_\rho E\rho_T\xi$ where $\rho = (\rho_t)$ runs through the set of martingale densities for the price process S. By "to hedge", we means to dominate the random variable ξ by the terminal value of a self-financing portfolio. For the case of American-type option which pay-off is an adapted càdlàg stochastic process $f = (f_t)_{t\leq T}$, the assertion is basically the same : $x_* = \sup_{\rho,\tau} E\rho_\tau f_\tau$ where τ (an exercise date) runs through the set of stopping times dominated by T. Here, "to hedge" means to dominate, on the whole time interval, the pay-off process by a portfolio process. In both cases, as was shown by Dmitri Kramkov [12], the results can be deduced from the optional decomposition theorem applied to a corresponding Snell envelope.

In the theory of markets with transaction costs hedging theorems for European options are already available for discrete-time as well as for continuous-time models. The model is given by an adapted cone-valued process $G = (G_t)_{t=0,1,\dots,T}$ in $\mathbf{R}^d$, where the G_t's model the solvency (random) cones : the positions where the agent is solvent. The hedging problem is to describe the set Γ of initial values x for which one can find a self-financing portfolio X such that $x + X_T$ dominates ξ in the sense of the partial ordering induced by the cone G_T. It happens that, under appropriate assumptions,

$$\Gamma = \{x \in \mathbf{R}^d : \ Z_0 x \geq EZ_T\xi \ \ \forall Z \in \mathcal{M}_0^T(G^*)\}$$

where $\mathcal{M}_0^T(G^*)$ is the set of martingales evolving in the (positive) duals G_t^* of the cones G_t. The elements of $\mathcal{M}_0^T(\widehat{K}^*)$ are called consistent price systems. For the continuous-time model the description remains the same but the theorem becomes rather delicate because of some modeling issue (see [11] and [3]).

The hedging problem for the vector-valued American option $U = (U_t)$ in the discrete-time framework with transaction costs was investigated in the paper [2] by Bruno Bouchard and Emmanuel Temam (see also the earlier article [4]). It happens that one cannot follow the same idea as in the frictionless market (using stopping times): we need a richer set of "dual variables" to describe the set Γ. Bouchard and Temam proved the identity

$$\Gamma = \left\{ x \in \mathbf{R}^d : \ \bar{Z}_0 x \geq E\sum_{t=0}^{N} Z_t U_t \ \ \forall Z \in \mathcal{Z}_d(G^*, P) \right\}$$

where $\mathcal{Z}_d(G^*, P)$ is the set of discrete-time adapted process $Z = (Z_t)$ such that the random variables $Z_t, \bar{Z}_t \in L^1(G_t^*)$ for all $t \leq T$ with the notation $\bar{Z}_t := \sum_{s=t}^T E(Z_s|\mathcal{F}_t)$.

In our paper, we prove the following identity in a continuous time setting and for a cadlag pay-off process U_t :

$$\Gamma = \left\{ x \in x \in \mathbf{R}^d : x\overline{Z_0}^\nu \geq E^\nu ZU, \forall \nu \in \mathcal{N}, \forall Z \in \mathcal{Z}(G^*, P, \nu) \right\}$$

Where $\mathcal{N}$ is the set of every (deterministic) finite positive measure on $[0, T]$, and $\mathcal{Z}(G^*, P, \nu)$ denote the set of adapted càdlàg processes $Z \in L^1(P^\nu)$ such that $Z_t, \bar{Z}_t^\nu \in L^0(G_t^*, \mathcal{F}_t)$ for all $t \leq T$ with $\bar{Z}_t^\nu = E\left(\int_{[t,T]} Z_s \nu(ds) \middle| \mathcal{F}_t\right)$. We call the elements of this set *coherent price systems*: the hedging endowments are those whose "values" are larger than the expected weighted "values" of the pay-off process for every coherent price system used for the "evaluation" of the assets.

To prove this identity, we have to use a rather involved model, basically the one used in [3]. We first cover the pay-off process on the dyadics, then on the whole interval. The used tools are basically the same as in [11] : we want to apply a bipolar theorem, hence we have to check that our sets have the good properties. Note that we needed to ask the portfolio processes to be ladlag and predictable, following [3], to prove that the set of the processes which are replicable on the n-dyadic is Fatou-closed.

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Own-Company Stockholding and Work Effort Preferences of an Unconstrained Executive

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Stemming from the agency theory fundamentals of Ross (1973), Jensen and Meckling (1976), Holmstrom (1979) and others, there has been much concern for the ‘incentivization’ link from equity-based executive compensation to corporate financial performance. The associated academic literature is extensive. Counterpoint to past research, we consider the motivation for an executive with unconstrained (unincentivized) compensation to voluntarily performance-link his personal wealth. We develop a model framework that identifies the joint own-company stockholding and work effort strategy of a utility-maximizing executive. The executive's compensation is assumed to be incorporated into his up-front total personal wealth, which he invests variously in a risk-free money market account, a diversified market portfolio, or his own company's stock. The financial market is defined on a filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t\geq 0})$ satisfying the usual hypothesis and large enough to support two independent standard Brownian motions, $W^P = (W^P_t)_{t\geq 0}$ and $W = (W_t)_{t\geq 0}$. The investment opportunities available to our executive are a risk-free money market account, a diversified market portfolio and his own company's stocks. The risk-free money market account has the price process $B = (B_t)_{t\geq 0}$, with dynamics

$$\mathrm{d}B_t = r\, B_t\, \mathrm{d}t\,, \quad B_0 = 1\,, \tag{4}$$

where r is the instantaneous risk-free rate of return, hence $B_t = e^{r\,t}$. The price process of the market portfolio, $P = (P_t)_{t\geq 0}$, follows the stochastic differential equation (SDE)

$$\mathrm{d}P_t = P_t\,(\mu^P\,\mathrm{d}t + \sigma^P\,\mathrm{d}W^P_t)\,, \quad P_0 \in \mathbb{R}^+\,, \tag{5}$$

where μ^P is the expected return rate of the market portfolio, σ^P is the market portfolio volatility and $W^P = (W^P_t)_{t\geq 0}$ denotes a standard Brownian motion. The company's non-systematic stock price process, $S^{\mu,\sigma} = (S^{\mu,\sigma}_t)_{t\geq 0}$, is a controlled diffusion with SDE

$$\mathrm{d}S^{\mu,\sigma}_t = S^{\mu,\sigma}_t\,(\mu_t\,\mathrm{d}t + \sigma_t\,\mathrm{d}W_t)\,, \quad S_0 \in \mathbb{R}^+\,, \tag{6}$$

where μ is the company's expected return rate in excess of the beta-adjusted market portfolio's expected excess return rate (i.e. the expected return compensation for non-systematic risk), and σ is the company's non-systematic volatility, both controlled by the executive. The 'full' stock price process is simply a portfolio combination of P and S dependent on the company's beta.

The executive is able to beneficially influence the value of his company via work effort; he gains utility from the increased value of his direct stockholding (within his overall personal portfolio), but loses utility for his work effort. The executive influences the company's stock price dynamics by choice of the control strategy (μ, σ), which is specified to be associated with work effort. Value is added if μ is greater than r, indicating excess return compensation for non-systematic risk. The executive's instantaneous disutility of work effort is represented by $c_t(\mu_t, \sigma_t)$ for control strategy (μ_t, σ_t) at time t. We assume a Markovian disutility rate, i.e., $c_t(\mu_t, \sigma_t) = c(t, v, \mu_t, \sigma_t)$ where $c : [0, T] \times \mathbb{R}^+ \times [r, \infty) \times \mathbb{R}^+ \to \mathbb{R}^+_0$ is a continuous and suitably differential function.

A feature of our framework is that the executive's work effort, specified in terms of two control variables, non-systematic expected return and volatility (μ and σ), can be restated in terms of a single control variable, the non-systematic Sharpe ratio ($\lambda = (\mu - r)/\sigma$, where r is the risk-free rate of return). This reduces the dimension of the problem and introduces a parameterization based on the well-known Sharpe ratio performance measure. The disutility of work is then represented by the Markovian disutility rate $c^\star_t(\lambda_t) = c^\star(t, v, \lambda_t)$ for control strategy λ_t at time t.

We then consider the *optimal investment and control decision problem*

$$\Phi(t, v) = \sup_{(\pi,\lambda)\in A'_\gamma(t,v)} \mathbb{E}^{t,v}\left[U(V^\pi_T) - \int_t^T c^\star(u, V^\pi_u, \lambda_u)\,\mathrm{d}u\right]\,, \quad \text{for } (t, v) \in [0, T] \times \mathbb{R}^+\,, \tag{7}$$

where $A'_\gamma(t, v)$ is a suitable set of admissible strategies.

The executive's optimal personal investment and work effort strategy is then derived in closed-form using stochastic control theory and the corresponding Hamilton-Jacobi-Bellman equations. Other technical papers similarly concerned with dynamic principal-agent models include Cadenillas, Cvitanic and Zapatero (2004) and Ou-Yang (2003), for example. In particlar, the closed-form solutions are derived

for the following forms of the utility and disutility function:
The utility function U satisfies

$$U(v) = \begin{cases} \dfrac{v^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \\ \log(v), & \text{for } \gamma = 1, \end{cases} \tag{8}$$

and the cost of effort (or disutility) $c^\star$ is assumed to satisfy:

$$c^\star(t, v, \lambda) = \kappa\, v^{1-\gamma} \frac{\lambda^\alpha}{\alpha}, \quad \gamma > 0, \tag{9}$$

where $\kappa > 0$ is the inverse work productivity, $\alpha > 2$ the disutility stress, and the scaling factor $v^{1-\gamma}$ is based on a similar formulation for the intertemporal utility from consumption in a constant relative risk aversion setting. By this setting, the executive is characterized by a risk aversion parameter (γ), and two work effectiveness parameters (κ, representing inverse work productivity, and α, representing disutility stress).

Our closed-form results demonstrate that an executive with superior work effectiveness (i.e. higher quality) will undertake more work effort for his company. Furthermore, depending on any change in the company's non-systematic volatility associated with the executive's work effort (i.e. control strategy), due to risk aversion a higher quality executive will not necessarily undertake a higher own-company stockholding. For application to empirical data, our framework allows an executive quality measure to be backed-out from the observed own-company stockholdings of unconstrained executives (assuming knowledge of non-systematic company volatility). Alternatively, with assumption of executive quality and risk aversion, our framework allows identification of the deviation in own-company stockholding that results from constraining an executive with performance contracting.

Having these closed-from solutions, we demonstrate an indifference utility rationale to determine the required compensation of the executive. We show that the executive's indifference utility compensation increases with his work productivity and decreases with his disutility stress.

A future extension for our framework is to specify a constrained executive subject to an imposed own-company stockholding representative of performance contracting, and to contrast his work effort strategy with that of our unconstrained executive.

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Binomial approximations for barrier options of Israeli style

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We show that prices and shortfall risks of game (Israeli) barrier options in a sequence of binomial approximations of the Black–Scholes (BS) market converge to the corresponding quantities for similar game barrier options in the BS market with path dependent payoffs and the speed of convergence is estimated, as well. The results are new also for usual American style options and they are interesting from the computational point of view, as well, since in binomial markets these quantities can be obtained via dynamical programming algorithms. The paper continues the study of [1] and [2] but requires substantial additional arguments in view of pecularities of barrier options which, in particular, destroy the regularity of payoffs needed in the above papers.

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Asymptotics for multifractal modelization of financial data

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Multifractal random processes are known to accurrately reproduce a large number of stylized facts on financial data: long memory of the volatiliy, heavy-tailed distributions of the log-returns, scale-invariance, etc. We briefly present the construction of continuous-time multifractal random walks that can be found in [1]. We further discuss some of their statistical properties with application to financial data.

We show in particular the importance of the asymptotic framework when one deals with these processes: whether the sampling frequency is high or the observation length is large. As of today, multifractal processes all make use of a parameter called integral scale which is a decorrelation time. Thus, if one has a huge quantity of data, it may be the case of the observation of a large number integral scales, which is mathematically similar to the familiar and easy framework of independant data. However, it may also be the case that only a few integral scales are observed with a high sampling frequency. Then the data are strongly dependant, which makes statistical estimation much more difficult.

In the case of financial data, the integral scale is about one year, so that we mainly observe strongly dependant data. Then, it has been shown that standard estimators of key quantities such as tail exponents may greatly underestimate the true exponent, eventhough the number of available data is huge. The most general framework for estimation issues would be a "mixed asymptotic" where both the observation length $2^{n\chi}$ and the sampling frequency 2^n grow at different rates which depend on a parameter $\chi \in [0, \infty]$. We provide further arguments in favor of this setting: in particular, the multifractal regularity and scale invariance of the observed process both depend on the value of χ.

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Inflation Linked Derivatives: Pricing Model for a Multi-Country Setting

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In this study we propose an arbitrage pricing model for inflation linked derivative instruments. We consider a multi-country setting where domestic and foreign nominal and real bonds are traded. Imposing no-arbitrage assumption immediately yields the usual definition of real exchange rate (RER). Moreover we get drift conditions, implied by the no-arbitrage, for real and nominal term structures of the domestic and foreign economies. Assuming martingale property for the the real exchange rate we find a relation between the real interest rates of the two economies. Introducing a forward contract into our model results with the forward real exchange rate which can be written in terms of the price of the domestic and foreign inflation indexed bonds. We calibrate our model to UK and US data showing that international factors are important when it comes to pricing and hedging derivative instruments.

Keyword: Inflation, nominal rates, real rates, inflation linked derivatives, real exchange rates.

A method of moments approach to pricing double barrier options with the underlying modelled by a general class of jump diffusions.

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A common empirical observation is that financial returns data possesses heavy tails and excess kurtosis, features that cannot be captured by the geometric Brownian motion model. Successful modifications have been proposed to improve this standard model include stochastic volatility models and Jump models (and combinations of both). In principle the method presented should be able to handle both cases, however our practical examples have focused on jump diffusion processes.

This method can handle a large class of options but our numerical examples are focused on barrier options. Barrier options are one of the most widely traded exotic option classes. So much so that they are liquidly traded, because of this there is a need for direct calibration which require fast pricing methods. Barrier options are options that are (de-)activated when the underlying price process up-crosses and/or down-crosses certain pre-specified levels.

The knock-out barrier is deactivated if the underlying cross the barrier. The knock-in barrier on the other hand is not activated until the underlying cross the barrier. Looking at a knock-in and a knock-out barrier with the same barrier and other parameters, one has a value when the other doesn't, this is called the in-out parity that states that the sum of the value of a knock-in and a knock-out barrier option is worth the same as an option of the same type without the barrier. This allows us to focus on one type of barrier option.

To price double barrier options under the geometric Brownian motion [2] and [5] developed a Laplace transform approach. Davydov and V. Linetsky [1] used eigenfunction expansions to price double barrier options in a wider diffusion setting. Both of the mentioned approaches draw on specific properties of the underlying driving process and cannot be readily extended to different settings.

The method of moments for exit time distributions in a diffusion setting using linear programming was developed by Helmes et al. [3]. The algorithm was extended by Lasserre et al. [4] to be able to price a class of exotic options using semidefinite programming. They also provide convergence results.

The first step of the methods of moments approach is to describe the price of the contract as a linear combination of the moments of certain measures. The measures used are the exit location measure and the expected occupation measure. The exit location measure describes how the process ends, in the case of a Barrier option this would contain two parts. The distribution of the underlying at the exercise date given that the process has not crossed the barrier and secondly the distribution of crossing the barrier before the time of exercise. The expected occupation measure describes the behaviour of the process until it hits the domain of the exit location measure. The method allows for the measures to be partitioned and we can use linear combination of the moments of these partitioned measures to describe the payoff. This implies that we can price any contract that is either a piecewise polynomial of the underlying at exit time, or the integral until exit time of a piecewise polynomial of the process. In short if we can price contracts valued by

$$v = \mathbb{E}\left[e^{-\alpha_\tau} h(S_\tau) + \int_0^\tau e^{-\alpha_s} g(S_s) ds\right]$$

where h and g are piecewise polynomials. We allow for discounting of the type $\alpha_t = \int_0^t r(s)ds$ where $r(s)$ is a polynomial.

We calculate the needed moments using an linear system of equations. This system is built using Dynkin's formula applied to polynomials, the requirement for this to become a linear system of equations is that the infinitesimal generator of the stochastic process maps polynomials to polynomials. This can be seen to be the case for example for Lévy processes, but also for additive processes when the time dependent parameters are polynomials and a lot of general jump diffusions.

In order to get a solvable problem we need to add moment conditions, these are conditions that a series of numbers corresponds to moments of some measure. These come in two types either a system of linear inequalities or the requirement that certain matrices are positive semidefinite.

Adding these parts together we get a linear or semi-definite programming problem. Each of which have there own dedicated solvers to be used.

We will illustrate the method with some numerical examples.

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Modeling and Estimation of Dependent Credit Rating Transitions

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Simultaneous defaults in large portfolios of credit derivatives can induce huge losses. To take the dependence of defaults into account, the literature applies interacting particle systems to model the rating transitions of firms. The advantage of these models is, that we can describe the direct interaction between firms in an easy way. An example for such direct interaction is the subprime mortgage crisis in summer 2007. Since many lenders defaulted simultaneously, a contagion effect took place and showed that the downgrading of a single bank directly caused the downgrading of other banks in the same sector.

The previous literature describes several ways to consider dependent credit rating transitions of firms. Giesecke and Weber (2003) apply the voter model to construct the dependence structure of the defaults. The voter model is a spin system that means there are only two possible states for a single firm, default or not. If a firm defaults, then the neighbours of the firm default with a certain probability as well. A more general approach is used by Bielecki and Vidozzi (2001). They assume that there are no simultaneous jumps of the firms. The intensity of a rating transition is modeled by the intensities of a rating transition of the single firms, which depend on the current ratings of the other firms. The rating change of one firm increases the intensity of rating changes of the other firms. In the paper of Frey and Backhaus (2007) the particle system is a mean-field interaction model. That means the firms are divided into several groups. Only the number of defaulted firms in the different groups influence the default intensity of the other firms.

In contrast to the previous papers, this talk presents a model, where the firms are allowed to change the rating class simultaneously. For example, if the oil price increases and induces higher costs for the whole aviation industry, it is reasonable, that several airlines are downgraded at the same time. Furthermore, the state space S of a single firm are $K \in \mathbb{N}$ credit rating classes, not just default or not.

For this we model the rating transitions with the so-called coupled random walk process, introduced by Spitzer (1981). In this process we have independent Poisson processes with parameter $\lambda_x \geq 0$ for the rating classes. When the Poisson process of rating class x jumps, we choose another rating class y according to a stochastic transition function. Now every firm with rating x changes the rating to y, independently of the other firms, with probability p_x, depending on rating class x. Economically, if a single firm is downgraded, then the other firms in this class are directly linked and with a certain probability they are downgraded as well.

The outline of the talk is the following. We introduce the model and define the Q-matrix of the homogeneous Markov jump process, which describes the ratings of the firms over time. As parameters we assume the generator μ of the movement of a single firm and the dependence vector $p = (p_x)_{x \in S}$, consisting of the probabilities, that a firm changes the rating class when the corresponding Poisson process jumps. To fit the model, using historical rating transitions, we compute the maximum likelihood estimators of the parameters. Finally, we illustrate how the dependence parameter p influences the loss of a large portfolio. To this end, we simulate the Markov jump process for different p and compute the profit and loss of a portfolio of defaultable zero-coupon bonds issued by firms, whose credit ratings move according to our model. If p equals one, that means there is strong dependence between the firms, the probability of high losses and profits increases, compared to independent moving firms. Therefore, different shapes of profit and loss distribution functions are feasible, and our model allows a better fit of the distribution than a model with independent rating changes.

Pricing and Hedging of CDO-squared tranches by using a one factor Lévy model

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Copula models have become the market standard for the pricing of CDO tranches. These models present two main advantages: the dependence structure between default time can be specified independently of the marginal credit curves and the pricing rests on a semi-analytical method. These two advantages still hold in the case of CDO-squared tranche pricing. The aim of the first part of this talk consists of an extension of the commonly used Gaussian copula model to the class of Lévy copula models which describe better the empirical return distributions given the fatter tail of the Lévy distributions. More particularly, it provides a comparison of the exponential copula Lévy model with the classical Gaussian copula model for the pricing of CDO-squared tranches. Several approximations of the recursive approach are considered: a full Monte Carlo approximation, a multivariate Normal approximation of the joint inner CDO loss distribution firstly proposed by Shelton and a multivariate Poisson approximation of the joint number of defaults affecting the inner CDOs. More particularly, a sensitivity analysis is carried out for three particular days characterised by a low, medium and high value of the quoted iTraxx and CDX index spreads. This analysis points out the performance of the Normal and Poisson approximation methods for both the Gaussian and exponential Lévy models. Indeed, these two methods allow a significant decrease of the computational time which can turn out to be of a crucial importance, especially for calibration aims. Moreover, simulations show that the multivariate Poisson approximation method outperforms the multivariate Normal approximation, especially under the exponential copula model.

The second main part of this talk features a comparison of the Gamma and Gaussian Deltas under the multivariate Normal approximation for a period extended from the 20^{th} of September 2007 until the 13^{rd} of February 2008. The Deltas are computed with respect to weighted and unweighted versions of the CDS pool as well as with respect to another CDO-squared tranche. Simulation studies show that the Gamma Deltas vary in line with the Gaussian Deltas whose variance explains more than 97 % of the variability in the Gamma Deltas and that the tranche hedging strategy seems to be really promising for CDO-squared tranches, as it is already the case for CDO products, given the small volatility of the equity-junior mezzanine hedge ratio.

Stochastic Jump Intensity, Stochastic Volatility and Stochastic Higher Moments in Asset Returns: An Empirical Investigation

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Stochastic volatility and jumps have been becoming the fundamental factors in studying continuous-time asset pricing models. In the latest decade, the investigations have nearly exclusively focused on modeling stochastic volatility using the square-root process (Cox, Ingersoll and Ross, 1985; Heston, 1993) and modeling return jumps using compound Poisson process. With different datasets in asset returns and/or options, these studies (Bakshi, Cao and Chen, 1997; Bates, 2000; Pan, 2002; Andersen, Benzoni and Lund, 2002; Eraker Johannes and Polson, 2003; Eraker, 2004; Broadie, Chernov and Johannes, 2007) have reached almost the same results: stochastic volatility alone can not capture the distributional characteristics of asset returns and explain implied volatility skew/smile of options and a (Poisson) jump component in the return process is indispensable. The further empirical work, from both the statistical and economic criteria, indicates the incapability of the above stochastic volatility jump-diffusion models and points to the double-jump stochastic volatility models (Eraker et al., 2003; Broadie et al., 2007). These studies argue that the diffusive stochastic volatility process can not drive the volatility to move abruptly and another jump component is needed in the volatility process.

In this paper, I adopt a different approach to study the continuous-time asset pricing models: firstly, having noticed that to assume return jump a rare event is implausible and also inconsistent with the discretely observed sample data (Madan, 2001; Carr et al., 2002; Geman, 2002), I thus rely on the infinite activity Lévy process. A fundamental difference between the infinite activity Lévy process and the finite activity compound Poisson process is that the former can produce infinite number of jumps at any finite time interval and regards return jump as a common event. The superiority of the infinite activity Lévy models have already been reported in Huang and Wu (2004), Li, Wells and Liu (2007), Favero, Li and Ortu (2007) and others.

Secondly, I model asset price dynamics with the exponential Brownian motion and infinite activity Lévy process time-changed by the different independent activity rates. By time-changing both Brownian motion and infinite activity Lévy process, we introduce not only the stochastic diffusion volatility but also the stochastic jump arrival rate. Brownian motion and Lévy process can be interpreted as systematic (market) risk and idiosyncratic (credit) risk respectively and stochastic diffusion volatility and stochastic jump arrival rate indicate that the information flow of these two risks is time varying with different speed and may cause rapid price fluctuations. This is in contrast to the jump-diffusion stochastic volatility model, where the jump intensity is usually assumed to be constant. Now the stochastic return volatility has two sources: one is from diffusion part and the other from jump part. They both can contribute to the abrupt move of the return volatility without depending on jump component in diffusion activity rate. In fact, the empirical study in Section **??** shows that the jump arrival rate mean-reverts very fast and has a high volatility of volatility parameter. They both indicates that the return volatility could be pushed to a high level in sudden.

Lastly, the time-change approach adopted also introduces stochastic higher moments (skewness and kurtosis) in asset returns. Recent study in derivatives markets has already suggested that the higher moments such as skewness vary significantly over time. This can be seen from the different implied volatility structures of out-of-the-money options and in-the-money options along time. The phenomenon is more striking in the exchange rate and currency option markets (Carr and Wu, 2007 and Bakshi, Carr and Wu, 2007).

Many Lévy processes have already been investigated and applied in financial modeling. In this paper, I mainly focus on two very popular infinite activity Lévy processes: the Variance Gamma process (VG; Madan et al., 1998) and the Normal Inverse Gaussian process (NIG; Barndoff-Nielsen, 1998). Both processes have analytical characteristic functions, probability densities and Lévy densities and thus are more mathematically tractable. Indeed, they are two of the most popular infinite activity Lévy processes in finance and can suffice for many financial modeling purposes. Even though they are both infinitely active, they represent two types of infinite activity Lévy processes. The Variance Gamma process is of finite variation whereas the Normal Inverse Gaussian process takes on infinite variation.

Despite the fact that the infinite activity Lévy models become popular both in academics and practice, the inferences for these models are still on developing. Recently, a couple of estimation methods have been developed in empirical study. Bakshi et al. (2007) develop a maximum likelihood estimation method; Favero et al. (2007) propose an iterative method combined with the characteristic function based continuous GMM; Li et al. (2007) apply MCMC methods and Li (2008) studies the time-changed infinite activity Lévy models with sequential Monte Carlo methods. This paper implements model estimation with Bayesian methods.

Contrary to the classical methods, Bayesian estimation regards all the parameters Θ and the states H in a model as random variables and tries to find their posterior distribution $p(\Theta, H|Y)$ conditional on the observations Y. Markov Chain Monte Carlo (MCMC) methods are usually applied to sample from this posterior distribution for parameter estimation, state estimation and model comparison. Bayesian estimation with MCMC is particularly suitable to the continuous-time financial models (Johannes and Polson, 2003). It simultaneously estimates parameters and latent states through computing their posterior distributions and delivers exact finite sample inferences. And at the same time, it saves the computational time dramatically.

The representation of Brownian subordination of the Variance Gamma process and the Normal Inverse Gaussian process provides us convenience to implement Bayesian estimation since we could completely forget their complicated probability density functions and instead focus on the densities of subordinators. The Variance Gamma process can be constructed by time-changing a Brownian motion with drift using an independent Gamma process and the Normal Inverse Gaussian process can be obtained via subordinating a Brownian motion with drift using the inverse Gaussian process. Both the Gamma distribution and the inverse Gaussian distribution have well-known analytically tractable densities.

Ideally, the joint estimation with both return data and option data is desirable (Pan, 2002; Eraker, 2004; Favero et al., 2007). But this method is usually computationally intensive and is only feasible for the small dataset. I thus estimate the models with return data only which are long enough to contain

typical market behaviors: market crash, volatile market and tranquil market. With S&P 500 index data ranging from January 1986 to December 2000, we find that the infinite activity infinite variation NIG models perform better than the infinite activity finite variation VG models. We also find that NIG models, especially the stochastic jump arrival rate NIG model, have large capacity to generate implied volatility curves which can capture the hook effect found by Duffie, Pan and Singleton (2000) and Pan (2002). The study indicates that we do not need jump component in the volatility process, which is against the previous study in the finite activity finite variation models. The stochastic jump arrival rate can contribute a lot to the abrupt move of the return volatility.

A Modified Structural Model for Credit Risk—Utility Indifference Valuation

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This paper modifies the classical structural models by embedding them into the optimal portfolio problems. The price of the corporate bonds is derived based on the indifference between the investor's two utility maximization problems. There are two new parameters introduced into the models, namely the investor's risk aversion and the correlation between the firm's assets and the stocks it issues, which result in the different behaviors of the credit spread and the nonlinearity of the corporate bond's price. Under the Markovian framework, the default happening at the maturity, the first passage time model and the optimal bankruptcy time are considered in this paper, and the corresponding closed formulae are derived under the CARA utility by solving the HJB equations with the Cauchy problem, the boundary-value problem and the free-boundary problem respectively. Furthermore, the equivalent martingale measure for pricing is proved as the minimum relative entropy measure by the duality argument. The limiting cases are at last considered based on the viscosity solution argument and the large deviation method, which recover the results in the classical structural models.

Keyword: Credit risk, Structural models, Credit spread, Utility indifference, HJB equations.

On Using Shadow Prices in Portfolio Optimization with Transaction Costs

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In frictionless markets, utility maximization problems are typically solved either by stochastic control or by martingale methods. Beginning with the seminal paper of Davis and Norman [10], stochastic control theory has been used to solve various problems of this type in the presence of proportional

transaction costs. Martingale methods, on the other hand, have so far only been used to derive general structural results. These apply the duality theory for frictionless markets typically to a fictious shadow price process lying within the bid-ask bounds of the real price process. In this paper we show that this dual approach can actually be used for both deriving a candidate solution and verification in Merton's problem with logarithmic utility and proportional transaction costs. In particular, the shadow price process is determined explicitly.

Modelling credit dynamics – a tractable first-passage time model with jumps

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The payoff of many credit derivatives is subject to spread risk, i.e., it depends on the evolution of credit spreads through time. Moreover, credit derivatives with a leverage component are subject to *gap risk*, that is the risk of insufficient funding when jumps in credit spreads occur. To motivate our analysis we consider the leveraged credit-linked note, whose payoff is particularly sensitive to jumps in credit spreads.

We extend the class of existing models for credit spread dynamics by a model that is mathematically tractable and at the same time allows for meaningful dynamics of credit spreads. In particular, the model includes jumps in the evolution of credit spreads, which allows for calibration to a wide range of term structures and for valuation of spread products whose payoff is sensitive to jumps. Although the spread dynamics exhibit jumps, we are able to formulate our model in a way that allows to draw on results from the theory of continuous stochastic processes.

Our basic idea follows [1] who propose a structural model for valuing credit derivatives whose payoff is sensitive to default risk. Let W be a Brownian motion on a filtered probability space $(\Omega, \mathbb{F}, (\mathbb{F}_t)_{t\geq 0}, \mathbf{P})$ with $\mathbf{P}$ a risk-neutral martingale measure. A *credit quality process* is a continuous stochastic process with deterministic time-varying volatility,

$$X_t = \int_0^t \sigma_s \,\mathrm{d}W_s.$$

Default takes place when X hits a deterministic, constant barrier from above.

To allow for meaningful dynamics we extend the Overbeck-Schmidt model by specifying the volatility σ to be an adapted, càdlàg stochastic process independent of W. Let b be the barrier such that default occurs at the first time that X hits b and denote the time of the default event by τ. By a representation of X as a time-changed Brownian motion, the (risk-neutral) probability of default until time t at time s, on $\{\tau > s\}$, is

$$\mathbf{P}(\tau \leq t|\mathbb{F}_s) = \mathbb{E}\left(2\mathrm{N}\left(\frac{b - X_s}{\sqrt{\int_s^t \sigma_u^2 \,\mathrm{d}u}}\right)\Big|\mathbb{F}_s\right) \quad \mathbf{P}\text{–a.s.,} \tag{10}$$

where N denotes the Normal distribution function. Credit spreads are then derived from conditional default probabilities by risk-neutral valuation. We choose the variance process σ^2 to be a Lévy-driven Ornstein-Uhlenbeck process, i.e., σ^2 is the solution to the SDE

$$\mathrm{d}\sigma_t^2 = a(\theta(t) - \sigma_{t-}^2)\,\mathrm{d}t + \mathrm{d}Z_t, \quad t \geq 0,$$

where θ is a strictly positive, bounded, càdlàg function and Z is a compound Poisson process.

The economic rationale is as follows: the sudden arrival of bad news induces credit spreads to jump; in our model, a positive jump in the variance of the credit quality process induces a sudden increase in "default speed" (we show that whenever σ jumps, then so does the conditional default probability process $(\mathbf{P}(\tau \leq t|\mathbb{F}_s)_{s\leq t})$ and so do credit spreads). On the other hand, good news tend to propagate gradually, i.e., credit spreads tend to decrease gradually; this is reflected by the mean-reversion component of the Ornstein-Uhlenbeck process.

Computing default probabilities numerically involves evaluating a conditional expectation with regard to the quadratic variation process of the credit quality process, cf. Equation (10). Making use of the fact that (X, σ) is a Markov process and that the quadratic variation is itself driven by a compound Poisson process we efficiently compute the conditional expectation by Panjer recursion [2]. The resulting valuation algorithm is a combination of Monte Carlo simulation and numerical evaluation of default probabilities: we simulate the driving variables X_s, σ_s^2 and conditional on (X_s, σ_s^2), we compute the term structures of default probabilities and credit spreads at s.

Calibration of the model involves determining the jump frequency and jump size distribution. In our example, we obtain a good fit to a given term structure of credit spreads by choosing a two-point jump size distribution, such that small jumps occur frequently and large jumps occur rarely. The former captures the spread jump risk whereas large jumps resemble jump-to-default risk (by the continuous nature of the credit quality process and the deterministic barrier, the model excludes jump-to-default risk, as the default event is predictable; however, a very large jump in the volatility of the credit quality process may lead to default in a very short time).

Finally, we determine the dynamics of default probabilities and credit spreads and we present some valuation examples.

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Strong Taylor approximation of SDEs and application to the Lévy LIBOR model

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The aim of this work is to provide a fast and accurate approximation scheme for the Monte-Carlo pricing of derivatives in the Lévy LIBOR model. The scheme is based on the strong Taylor approximation of the random terms entering the drift of the successive LIBOR rates. It offers a tractable alternative to "freezing the drift" at an accuracy similar to the full numerical solution. Numerical illustrations will also be presented.

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Liquidity Risk and Price Impacts

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We use liquidity constraints to model the impact of a trade on prices and obtain a new characterization of self-financing strategies. We give a sufficient condition for no arbitrage. We show that with a proper choice of derivatives, contingent claims can be approximately replicated by the use of backward stochastic differential equations. The replicating cost of such a contingent claim can then be described as the viscosity solution of a partial differential equation of the Black-Scholes type.

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Non-liquid Assets and Error Theory

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Our primary objective is to study liquidity risk as a by-product of market uncertainties, in particular the so-called "limit order books". "Limit order books" describe the existence of different sell and buy prices, which we explain by using different risk aversions of the agents. The risky asset follows a local volatility diffusion governed by a Brownian motion which is uncertain. We use the error theory with Dirichlet forms to formalise the notion of uncertainty on the Brownian motion. This uncertainty generates a noise on the trajectories of the underlying asset and we use this noise to expound the presence of a bid-ask spread. In addition, we prove that this noise also has a direct impact on mid-price of risky asset. We enrich our analysis with a numerical simulation when the volatility is a power function of the asset price.

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Term structure models driven by Poisson measures as solutions of SPDEs

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In the spirit of Björk et al. [1], we investigate term structure models driven by compensated Poisson random measures of the type

$$df(t,T) = \alpha(t,T)dt + \sum_{j=1}^{d} \sigma_j(t,T)dW_t^j + \int_E \gamma(t,x,T)\tilde{\mu}(dt,dx), \quad t \in [0,T]. \tag{11}$$

Using the alternative parametrization

$$r_t(x) := f(t,t+x), \quad x \geq 0$$

which is due to Musiela [3], we may regard $(r_t)_{t\geq 0}$ as *one* stochastic process with values in H, that is

$$r : \Omega \times \mathbb{R}_+ \to H,$$

where H denotes the space of forward curves. Denoting by $(S_t)_{t\geq 0}$ the shift semigroup on H, that is $S_t h = h(t + \cdot)$, equation (11) becomes in integrated form

$$\begin{aligned} r_t(\xi) = S_t h_0(\xi) &+ \int_0^t S_{t-s}\alpha(s,s+\xi)ds + \sum_{j=1}^{d} \int_0^t S_{t-s}\sigma_j(s,s+\xi)dW_s^j \\ &+ \int_0^t \int_E S_{t-s}\gamma(s,x,s+\xi)\tilde{\mu}(ds,dx), \quad t \geq 0. \end{aligned} \tag{12}$$

From a financial modeling point of view, one would rather consider drift and volatilities to be functions of the prevailing forward curve, that is

$$
\begin{aligned}
\alpha &: H \to H, \\
\sigma_j &: H \to H, \quad j = 1, \ldots, d \\
\gamma &: H \times E \to H.
\end{aligned}
$$

As we shall see, the resulting forward curve evolution $(r_t)_{t \geq 0}$ satisfying (12) then becomes, in the sense of Da Prato, Zabczyk [3] and Peszat, Zabczyk [4]Lasserre, a so-called *mild solution* of the stochastic partial differential equation

$$
\begin{cases} dr_t &= \left(\frac{d}{dx} r_t + \alpha_{\mathrm{HJM}}(r_t)\right)dt + \sum_{j=1}^d \sigma_j(r_t) dW_t^j + \int_E \gamma(r_{t-}, x)\tilde{\mu}(dt, dx) \\ r_0 &= h_0, \end{cases} \tag{13}
$$

where $\frac{d}{dx}$ denotes the infinitesimal generator of the strongly continuous semigroup of shifts $(S_t)_{t \geq 0}$, and where α_{HJM} is chosen such that the implies bond market is free of arbitrage.

In this talk, we will establish existence and uniqueness for the HJMM (Heath–Jarrow–Morton–Musiela) term structure equation (13).

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Fast Numerical Method for Computation of Variance-Optimal Hedging Error

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The aim of this talk is to provide a fast numerical method for the computation of the variance-optimal hedging error for exponential Lévy models using the method of Schwab et al. (2005).

That means, in this talk the problem of computing the hedging error of a European option is considered. If the underlying is a diffusion, this leads to a complete market, where every such claim can be replicated. But the shortcomings of diffusion models in representing the risk related to large market movements have led to models of the underlying which allow for jumps. Those lead to incomplete markets, where the replication of a European option claim is typically impossible. In this setting the classical approach is to minimize the *variance-optimal hedging error*

$$\tilde{h} := E((V_T - v + \phi S_T)^2)$$

over all reasonable hedging strategies ϕ and possibly all endowments v. Here, V represents the option price, S the discounted price process of the underlying, the dot refers to stochastic integration, and T is the time horizon. In this case the computation of the hedging error $\tilde{h}$ amounts to computing the projection of V onto a space of stochastic integrals.

The hedging problem has already been extensively studied. An overview over the literature is given in [10] and [11]. In due course several more or less explicit representations of the hedging strategy and the error have been developed. [1] have provided an expression using the carré-du-champ operator [1], the Malliavin derivative is used in [2] or various Laplace transforms in [5]. In [3] several representations in the general semimartingale setting are given, where S does not have to be a martingale.

Explicit computation of the hedging error was done for instance in [4] and for stochastic volatility models in [9]. The first uses an expensive Monte-Carlo simulation to get the results. The second reference uses an integral transformation method, thus developing an expression for $\tilde{h}$, which can be solved by computing a complex double integral. However, this method seems to be restricted to this specific type of hedging problem.

In this talk a new method shall be presented that allows for an efficient numerical treatment and is open to other hedging situations. However, the talk will be restricted to European options having as underlying an exponential Lévy process and it will be studied under the martingale measure in one dimension.

To this end the hedging error is expressed in a new way, based upon the results of [5]. This is done in terms of a parabolic integro-differential equation, which uses the option price V as data. Given certain smoothness and integrability conditions of V, the hedging error is given by $\tilde{h} = h(T, S_0)$, where $h(t, x)$ solves the following initial value problem

$$\begin{aligned} \frac{\partial}{\partial t} h(t,x) - Ah(t,x) &= \psi^V(t,x), \quad \forall (t,x) \\ h(0,x) &= 0, \quad \forall x \end{aligned}$$

Here, A denotes the generator of S and

$$\psi^V = \tilde{c}^V - (\tilde{c}^{SV})^2 (\tilde{c}^S)^{-1},$$

where $\tilde{c}$ is the modified differential semimartingale characteristic of (S, V).

Due to the strong resemblance to the well-known Kolmogorov backward equation used to obtain the option price V, the efficient numerical treatment developed in [11] is adapted. This is done in such a way, that the implementation can be realised as add-on to the option price implementation. That means, only objects, which had to be assembled to solve the option price backward equation via [11], are used to assemble the new equation.

Along the lines of [11] the equation is first localized and then cast into a variational setting with the finite element space V_h of order p and of dimension N. That means on each interval of the discretization acts a polynomial of degree p. However, the usual finite element approach in space discretization results in equation systems with densely-populated matrices in space. A wavelet compression technique deals with this problem and reduces the number of non-trivial entries of the matrix to $O(N \log N)$. The assembly of the right hand side, i.e. the computation of $(\psi^V, v), v \in V_h$, is realised as possible add-on to the implementation of the option price computation. The overall assembly and solution of the semi-discrete problem (i.e. only discretized in space) via GMRES amounts to a complexity of $O(N(\log N)^8)$.

Time discretization is done via the discontinuous Galerkin scheme, resulting in an overall complexity of $O(N(\log N)^8)$. The talk concludes with the corresponding error bounds and numerical examples.

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Optimal consumption and portfolio for an insider in a market with jumps

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We consider a stochastic optimal control problem in a financial market model with asymmetric information. In the market, we assume that there are two kinds of investors with different levels of information: a *uninformed agent* whose information coincides with the natural filtration of the price processes and an *insider* who has more information than the uninformed agent. Using forward integral techniques, we solve the optimal consumption and investment problem for the insider. We conclude by giving some examples.

The Marginal Price under Proportional Transaction Costs for Exponential Utility

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In markets with transaction costs, the classic Black-Scholes perfect replicating strategy for option pricing is no longer possible and preferences might be introduced in order to evaluate options. Utility indifference approach, based on portfolio selection problems of choosing optimal trading policies to maximize the investorŠs utility, provides an alternative for pricing options in the presence of transaction costs. In this article, we give a formal derivation for the formula of the marginal price which was introduced by Davis (1997). We find that, for exponential utility, the marginal price of an option is constant with respect to the investor's initial number of shares held in the underlying when his position is in the Buy or the Sell region of the portfolio selection problem.