Managing the Newest Derivatives Risks

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Agenda

Some Practical Aspects of Option Modelling:

I. Introduction

Introduction

▪ Many different markets and products…

- Foreign exchange (FX) markets with options on FX rates (USD/JPY, CNY/KRW, EUR/PLN, …), and basket options.
- Equity derivatives : from standard exotics (digital, barrier) and American options to multi-underlying (baskets, best-of/worst-of) and path-dependent instruments (Asian).
- Interest-rate derivatives : from swaps, caps and swaptions to very complex exotic swaps (ex : Euribor vs number of days where 2Y/10Y CMS spread remains in some corridor) with callable features.
- Inflation products written on the consumption price index (CPI) or on the year-on-year inflation rate (YOY).
- Credit structured products from single-name credit default swaps (CDS) to correlation products (CDO tranches) and credit index options.
- Commodity options on oil, gas, metals, agricultural goods…
- And all the hybrids you can ever imagine…

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…But common approaches.

- Market is split between liquid (‘vanilla’) options daily quoted and structured (‘exotic’) instruments specially designed for clients and for which almost no market price exist.
- Quotation of vanilla options is done in terms of implied volatilities where the Black-Scholes framework is merely a communication device for traders.
- Use of proprietary models for pricing the exotic products. Even if the underlying assets to be modelled (FX rates, stocks, commodity prices…) are different, models tend to be quite the same. All the models used in practice can be classified in some well-known families (local volatility, stochastic volatility, jumps…).
- The proprietary models need be calibrated on the prices of the vanilla instruments.
- Importance of numerical methods, computer speed and booking systems.
- Exotic traders are not supposed to make bets on the evolution of the underlying assets and vanilla options but use them to hedge the (aggregated) risks of the structured products sold to clients.

Introduction

- ***There are two approaches to dealing with pricing models for derivatives:***
  - ***<u>"Fundamental approach"</u> : assumes ex-ante some specification for the dynamics of the underlying instruments (diffusion, jump-diffusion, local-volatility diffusion model,…) that***
    - ***fits the historical dynamics of the tradable assets***
    - ***best recovers the market prices of the plain-vanilla options actively traded in the market.***
  - ***<u>"Instrumental, or the trader's approach"</u> : market quotes and model prices are compared using implied volatility. Traders are not interested in the true process for the underlying but are concerned by the "smile, the spot and forward term structures of volatility and how they evolve through time.***

Introduction

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- ***Now, could the process for the underlying be chosen with total disregard for the true process as long as it reproduces the correct behavior of the implied volatilities?***

The answer is NO for the reason that

The trader will have to, at the very least, delta hedge the positions and *clearly, the effectiveness of the hedging program depends on the specification of the dynamics of the underlying*

Introduction

- ***In practice we judge the quality of a model from two different angles:***
  1. ***Does the model produce prices within the market consensus?***
  2. ***How effective is hedging?***

A model is considered attractive not only if it prices correctly, but also if the parameters of the model remain stable when the model is recalibrated every day, the hedge ratios in terms of the hedging instruments also remain stable, and hedging is effective.

Introduction

- ***Different models for different products?***
  - There is no universal model able to price all products.
  - Traders/quants must select the relevant risks/market information on the liquid hedging instruments for hedging an exotic derivative. Any model for pricing this exotic product should provide a satisfactory description of the risks involved in the hedging instruments.
  - This hedging portfolio has to be used for calibrating the model.
- ***Why not using all available data?***
  - Available market information is large (forward values, volatilities, correlations…) and is expanding (ex : FX barrier option used to be considered as exotic and are now viewed as vanilla).
  - Calibrating on all data would have a prohibitive cost.
  - Since all markets are not arbitraged, some market data can be not consistent (ex : IR cap and swaption volatility smiles).

II. Quick Typology of Models

Few Models in Practice

- **Market split between vanilla options/exotic products implies a split between vanilla models and exotic models.**

- **Actually only a few families of models are used in practice:**

  - Black-like models for volatility-dependent products (vanilla options only) : Log-normal model, shifted log-normal model, normal model for options, Gaussian copula model for credit tranches.

  - Local volatility models for smile-dependent products (digitals) : from the Dupire original approach for FX/equity derivatives to the Hunt-Kennedy (or Markov functional) framework for IR derivatives and the local correlation model for credit.

  - Stochastic volatility models for products sensitive to the forward volatility/smile (barrier options cliquets/ratchets, options on variance swaps).

  - Jumps for extra effects (short-term smile for FX options, default times for credit).

  - Multi-asset models : from a static view of the correlation (copulas) to a dynamic one (assets described with a system of coupled SDEs).

Vanilla Models

- **Vanilla Models are used for quoting options:**
  - Options traders quote standard instruments.
  - Exotic traders use these models as extrapolation tools to produce smooth volatility surfaces on which they will calibrate their exotic models.
- **For a long time vanilla models were Black-like models.**
  - Assume a log-normal, normal, CEV or shifted-diffusion of the underlying.
  - Let one free parameter (volatility) used to quote the option price for each strike.
  - Interpolate/extrapolate this parameter to quote other options.
  - Arbitrages are possible among strikes (Gaussian copula for credit).
- **New models try to avoid arbitrages in strike/time.**
  - Simplest models are mixtures of Black-like models.
  - Most powerful models are complex diffusion models for which approximations are available for implied volatilities.
  - Best-known example is SABR for IR caps/swaptions with Hagan’s formula.

Exotic Models

- **Exotic models are used to price new derivatives for which no price can be found on the market.**
- **Exotic models provide an acceptable description of some risks that are not priced in the market.**
  - Time-evolution of market quantities (volatilities, smiles).
  - Correlation between different assets.
- **Exotic models need be calibrated on the prices of the vanilla instruments (volatilities).**
  - Some models can be auto-calibrated (Dupire, Hunt-Kennedy) once the price of vanilla options for all strikes/maturities is known thanks to the vanilla model.
  - But most models involve a heavy multi-dimensional minimization procedure.
- **Traders use exotic models to get their hedging strategy.**

Local or Stochastic Volatility?

- ***Local volatility models*** **aim at a full replication of the market smile seen from today, using a local variance dependent on the spot level.**
  - **No genuine financial interpretation.**
  - **Most famous examples are Dupire local volatility model and Derman- Kani (discrete time binomial tree version).**

- **The rationale for *stochastic volatility models* is to introduce a process on the local variance in order to control the smile dynamics.**
  - **Best-known example is the Heston model**
  - **Empirical studies support the idea that volatlity is stochastic ('volatility clustering').**

Local or Stochastic Volatility?

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- **Local volatility**
  - **Pros:**
    - Good market replication
    - Auto-calibration possible (Dupire formula, HK model).
    - Consistent modelling at the book level
  - **Cons:**
    - Poor smile dynamics
    - Delta and gamma get mixed up
- **Stochastic volatility**
  - **Pros:**
    - Finer modelling through decorrelation
    - Allows some control over the smile dynamics
    - Separate between the risk factors
  - **Cons:**
    - Many, many choices…
    - Calibration may be difficult
    - Dynamic vega hedge required to achieve replication

Stochastic Volatility models

- ***Market Standard for Stochastic Volatility Models***

  - No model is really the market standard – some are more popular than others.

  - Several features to take into account:
    - Calibration
    - Numerical tractability
    - Induced smile dynamics
    - Hedge ratios

Stochastic volatility models

- ***Affine models*** **are often used to build stochastic volatility models (Heston, Hull-White with stoch. vol.)**

  - Quasi-analytical formulae for plain-vanilla options can be obtained using Fourier-Laplace transforms.

  - Parsimonious models but calibration does not produce stable parameters, e.g. correlation between the spot and volatility very unstable.

  - However, these models are useful to produce smooth volatility surfaces.

III. The Case of FX, Equity Derivatives, Commodities and Fixed Income

Which model for which product?

<u>Issue:</u> incorporate all the available market information on the liquid hedging instruments when calibrating a model.

FX Derivatives

- **Highly liquid market for plain-vanilla and barrier options.**
- **As a consequence prices cannot be replicated by simple local volatility models (not enough degrees of freedom):**

  LSV (local stochastic volatility) models plus jump (short-term smile)

Equity Derivatives

- Highly liquid market for plain-vanilla options (200 – 300 calibration points) and, more recently, liquid market for variance swaps (15 – 20 calibration points).

- No good estimation of the correlation between stocks. Traders hedge their correlation risk by diversifying their portfolio.

- Standard model is the local volatility model. An issue is to build a satisfactory multi-asset extension.

Equity Derivatives

- **Options on single stocks: jump to default models that incorporate the information on the CDS market (asymptotic smile for low strikes)**

Equity Derivatives

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- ***Local Stochastic Volatility (LSV):***

  - The best of both worlds: a self-calibrated model with flexible smile dynamics

$$\frac{dS_t}{S_t} = a(S_t, t)b(Y_t, t)dW_t^1 + \mu_t dt$$

$$dY_t = \alpha(Y_t, t)dW_t^2 + \xi_t dt$$

  - LSV for products that depends on the forward smile: cliquet options, options on volatility and variance, options with payoff conditional on realized volatility,…

  - LSV + Jump when steep short-term smile

  - Dynamics of the volatility is controlled throught the “stochastic” terms

Equity Derivatives

Implementation issues:

- PDE-based calibration via forward induction
- Pricing based on Monte-Carlo (generally):
  - Server farm with 6,000 processors used to conduct parallel computing.
  - Variance reduction techniques:

    - Antithetic method;

    - Control variate technique;

    - Importance sampling: difficult to implement in practice as distribution shift is payoff specific.

Equity Derivatives

***Today's challenges*:**

- **Correlation smile:**

  Basket of indexes: Euro Stoxx, S&P, Nikkei

  Arbitrage: index vs. individual stock components

- **Dynamic management of the hedge:**

  How to rebalance the hedge portfolio provided we cannot trade in continuous time but only once every Δt (one day, 15 mns,…)?

- **Credit-linked modelling via CDS market for single stocks**

  A large amount of information is contained in the CDS market. When this market is liquid enough, it can be used to hedge out the credit risk inherent to any derivatives

- **Liquidity constraint management**

  Single stocks, mutual and hedge fund derivatives, non exchangeable currencies,…

Today's challenges (Cont.):

- **Dividend Modelling:**

  Basket of indexes: Euro Stoxx, S&P, Nikkei

  Arbitrage: index vs. individual stock components

- **Stochastic volatility Models with many factors:**
  - **Models to calibrate**
  - **Variance Swaps**
  - **VIX**
  - **Forward skew**
  - **Term skew (smile)**

Commodities Derivatives

Products:

The same diversity as in Equity Derivatives

Model Features:

- **Mean reversion**
- **Term structure modeling of forward prices and volatilities**
- **Smile modeling**
- **Seasonality**

Fixed Income Derivatives

Products:

Reverse Floater Target Redemption Notes (TARN)

Callable Snowballs

CMS spread options

Models:

“Hull & White” is the model that traders like very much. HW can fit the:

- **zero-coupon yield curve**
- **term structure of implied ATM volatilities for caps or swaptions**
- **usually 1 (reverse floater) or 2 (spreads) Gaussian factors**
- **PDE allows to price easily Bermudan options**

Shortcomings:

Does not capture:

- **the smile (at-the-money calibration: for a given maturity all the caplets have the same volatility)**
- **the volatility dynamics.**

Practical solutions:

- H&W with stochastic volatility (1 or 2 factors depending on the products: easy to price but difficult to calibrate).
- Quadratic Gaussian approaches are quite similar to H&W with stochastic volatility.
- Another approach involves “Smiled BGM”: hard to calibrate and to price.

It is often a local volatility extension of BGM model that allows almost arbitrary terminal distributions for Libor rates, while keeping pricing by simulation feasible. There is also a shifted log-normal version of BGM with a stochastic volatility.

Callable products need an American Monte-Carlo (usually the Longstaff & Schwartz algorithm). Slow convergence and gives only a lower bound for the call option price.

Practical solutions (Cont.):

- HK (Hunt Kennedy): a Markovian arbitrage-free, one factor model that allows exact numerical calibration of market caplet smiles. (Analogy with Dupire's model for equity derivatives.) Traders don't like HK as it generates unstable hedge ratios.

- SABR: static model but flexible to control the smile. SABR is used (Bi-SABR) to price CMS spread options.

IV. The Case of Credit Derivatives

IV.1 Standard model for pricing CDO tranches is the single-factor Gaussian Copula model

Pros: **Price quotes with one parameter: base correlation**

Cons: **Copula models have many shortcomings:**

- **They are unable to reproduce implied correlations for quoted tranches in a simple manner**
- **They are static models which are only good for single-period instruments, such as CDO tranches, whose prices depend only on marginal distributions for a series of dates,**
- **There is no dynamics for spreads and therefore cannot price forward starting tranches, options on tranches and leveraged super-senior tranches**
- **Even if you don't need a dynamic model of the evolution of spreads to price a CDO tranche, deltas with respect to CDS computed under Copula models are *inconsistent since they do not contain spread risk***

Base Correlation Problems with the Gaussian Copula Model:

- **Base Correlation skew: Gauss correlations have a strong slope**
- **Base Correlation skew leads to interpolation “noise”: thin-tranche arb, bespoke noise**
- **Credit crisis → 100% correlations**
- **Wide portfolio dispersion exacerbates problems**
- **Cannot calibrate super-senior tranches**

Some Extensions of the Copula Model:

1. Lévy processes

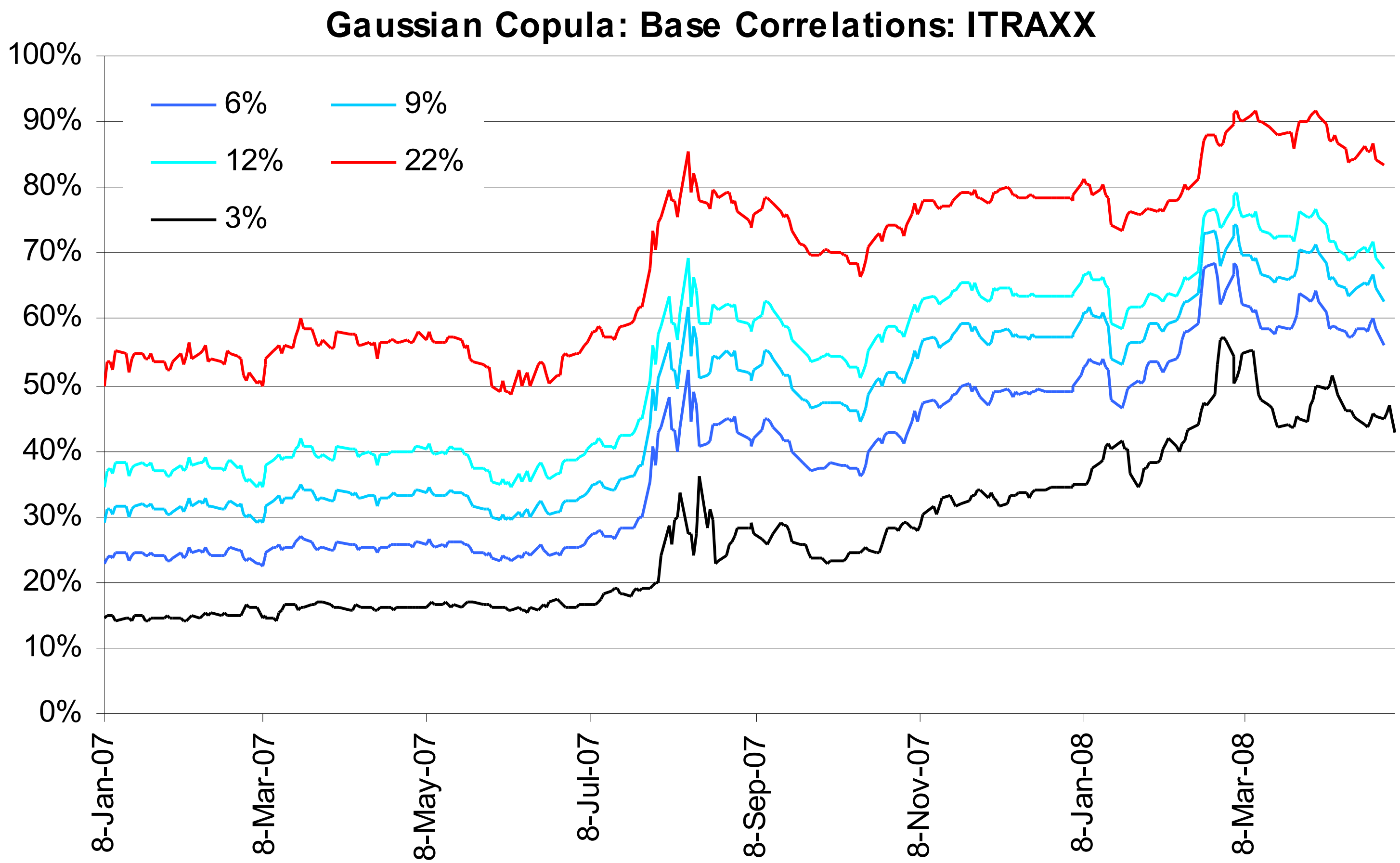
Rational: Credit market events are very shock driven – no smooth behavior of credit spreads. A CDS, a credit index can jump 20% in a day. It is then important to model jumps and extreme events.

Gamma model: Main features:

- Downside tail unbounded / upside bounded
- Greater weight in downside tail

Base Correlation Results

ITRAXX 5Y: Gaussian 2007-2008



Base Correlation Results

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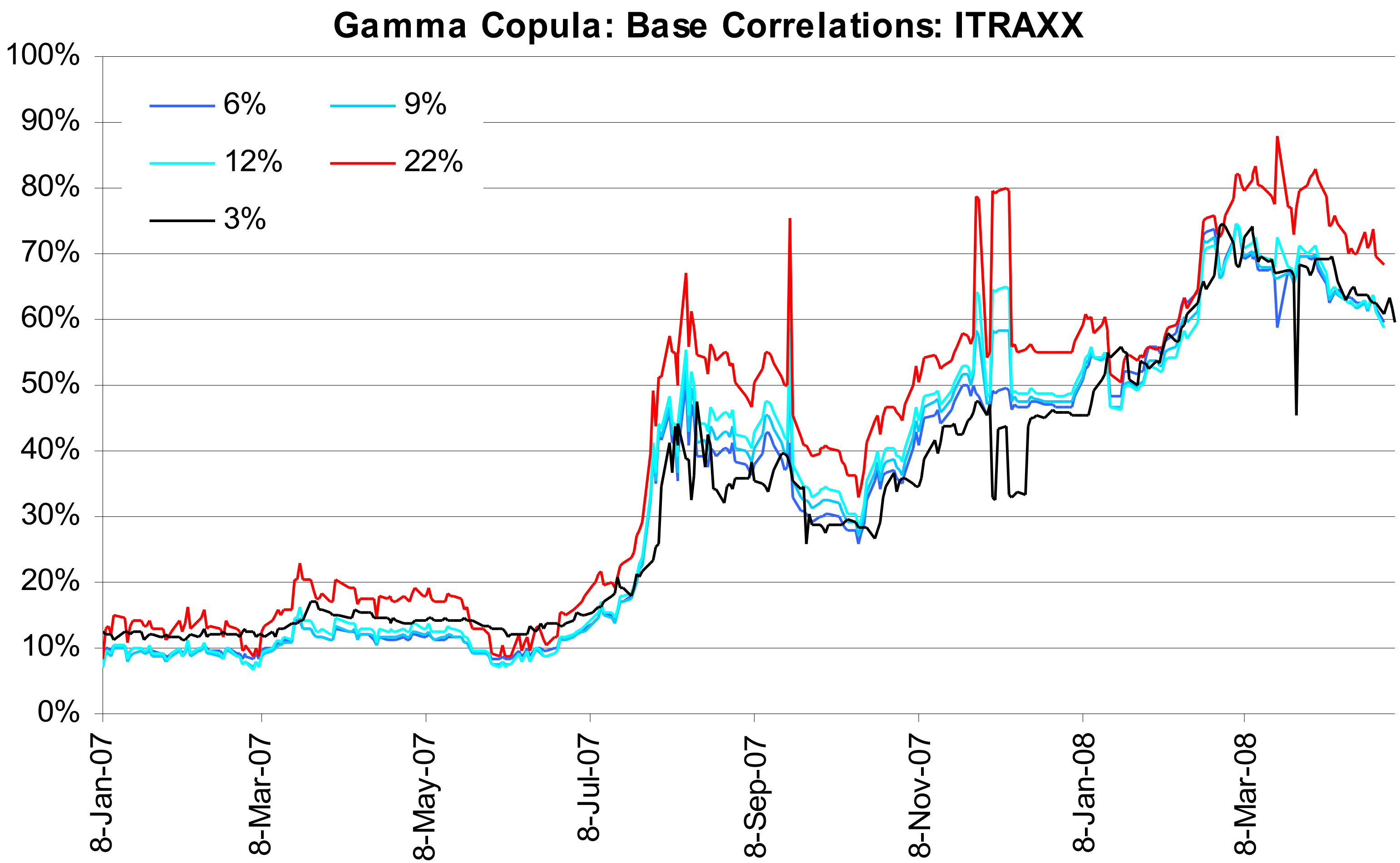
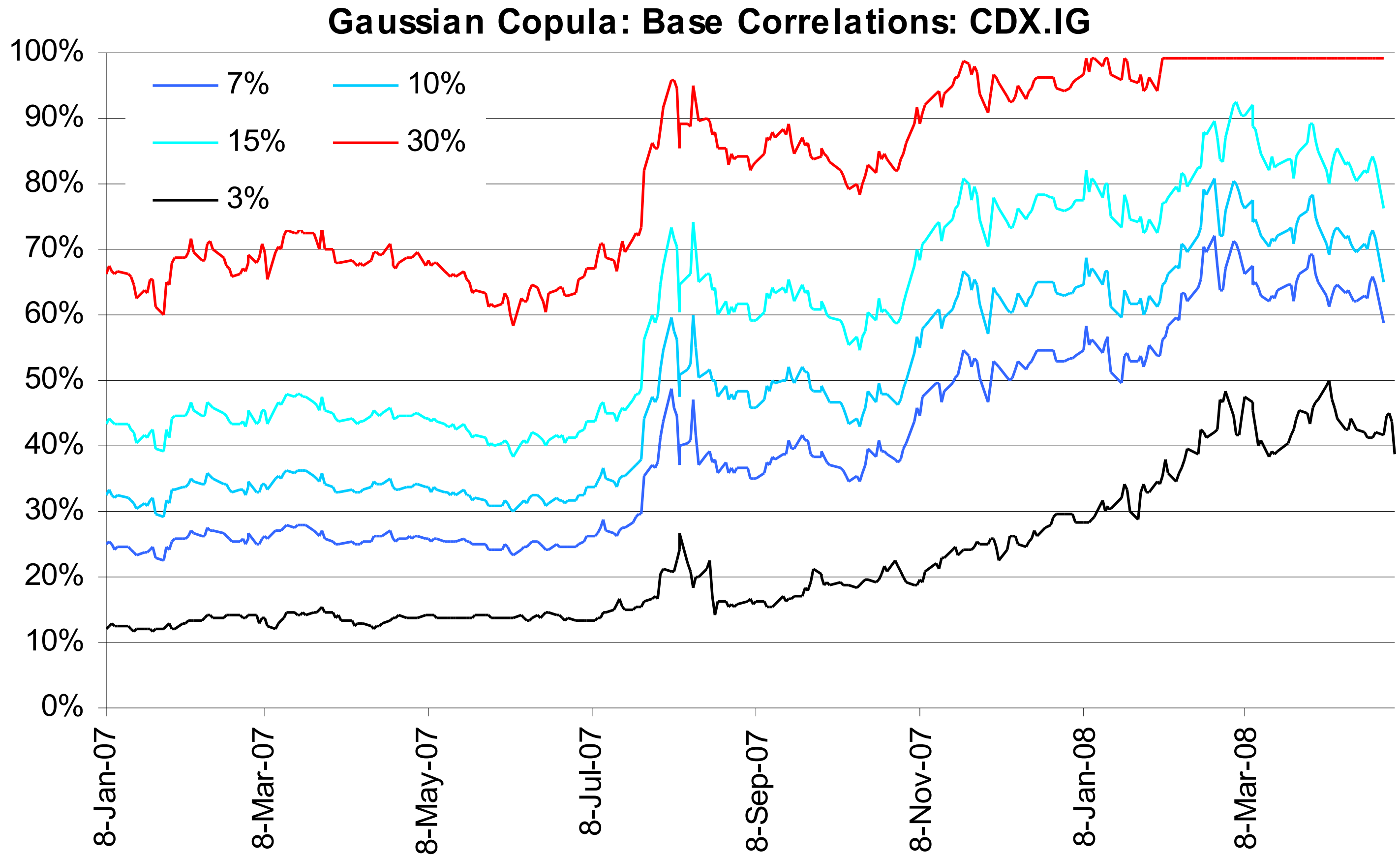
ITRAXX 5Y: Gamma



Base Correlations 2007-2008

CDX 5Y: Gaussian



Base Correlations 2007-2008

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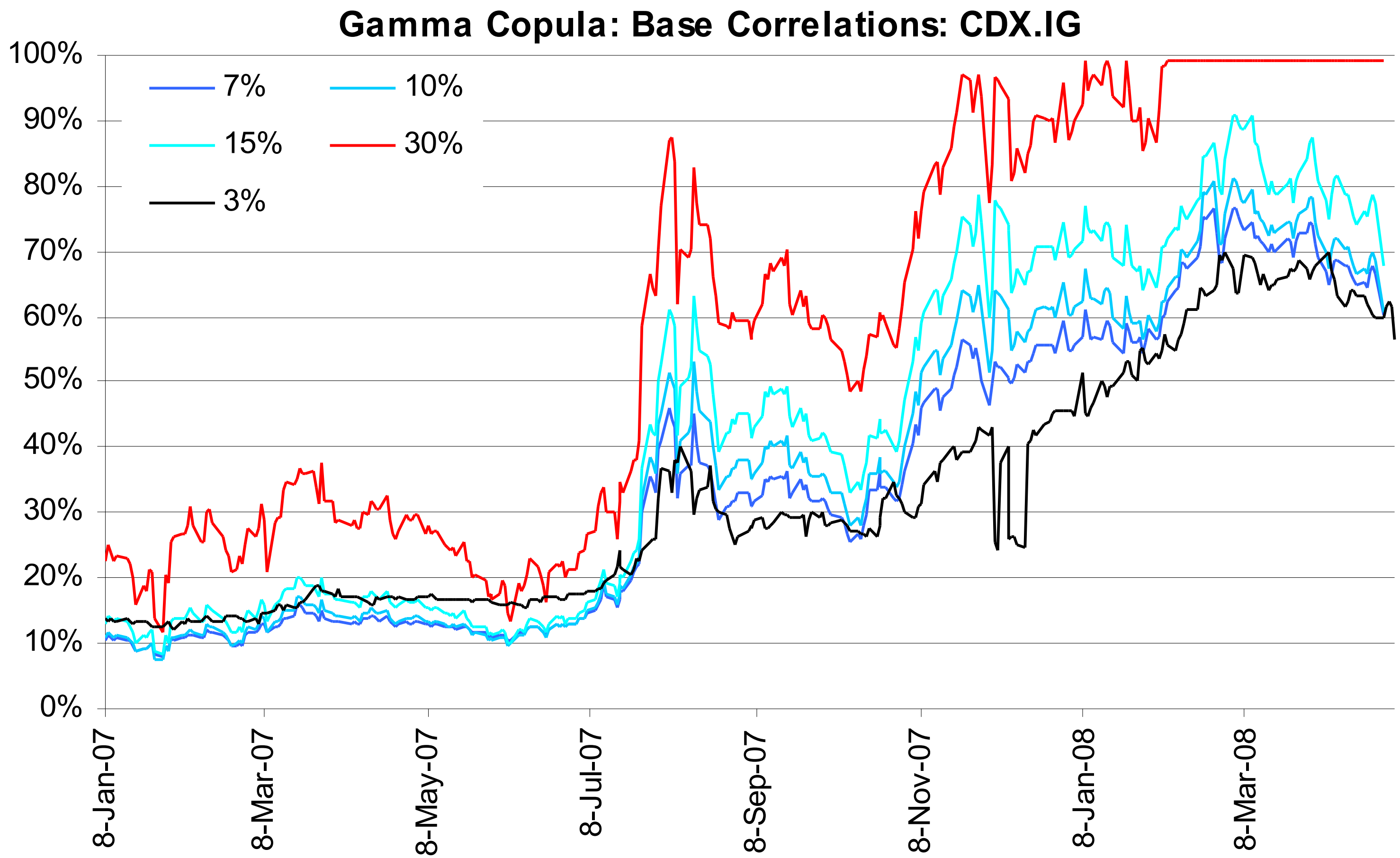
CDX 5Y: Gamma



Base Correlations 2007-2008

Good news:

- GAMMA base correlation skew is consistently flatter
- Itraxx gamma skew is extremely flat
- Qualitative features of results hold for 5Y, 7Y, 10Y

Bad news:

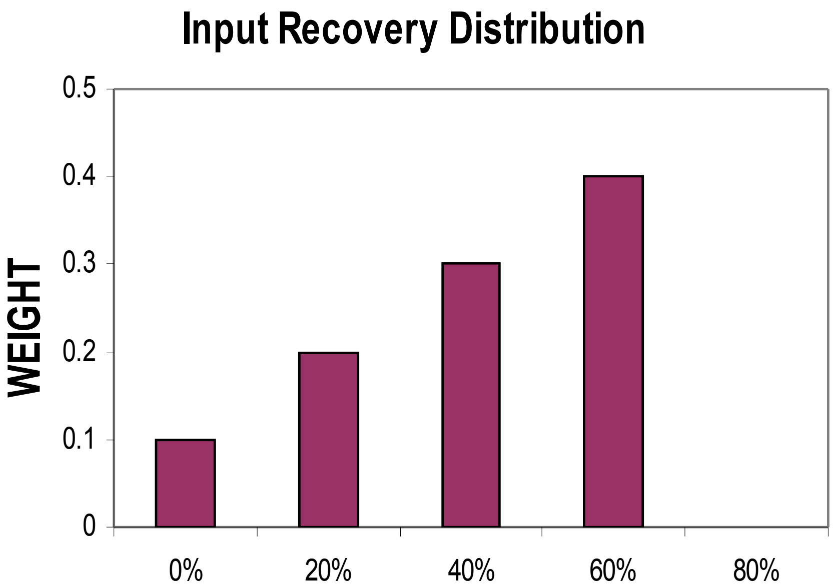
- CDX – senior correlation still > 99%

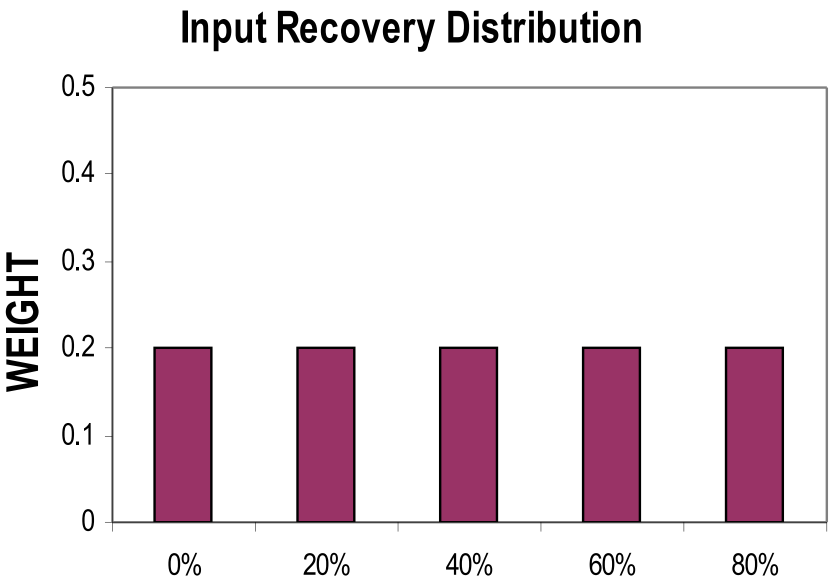
Other news:

- Gamma correlation changes more rapidly from day to day

- **Some Extensions of the Copula Model** (Cont.)**:**
  - **Stochastic Recovery (Krekel, 2008)**
    - **Choose a recovery distribution unconditional for each issuer**
    - **The variable which drives issuer default also drives the value of loss given default LGD=1-R**
    - **Conditional default rates are positively linked to LGD**

Choose a Recovery Distribution



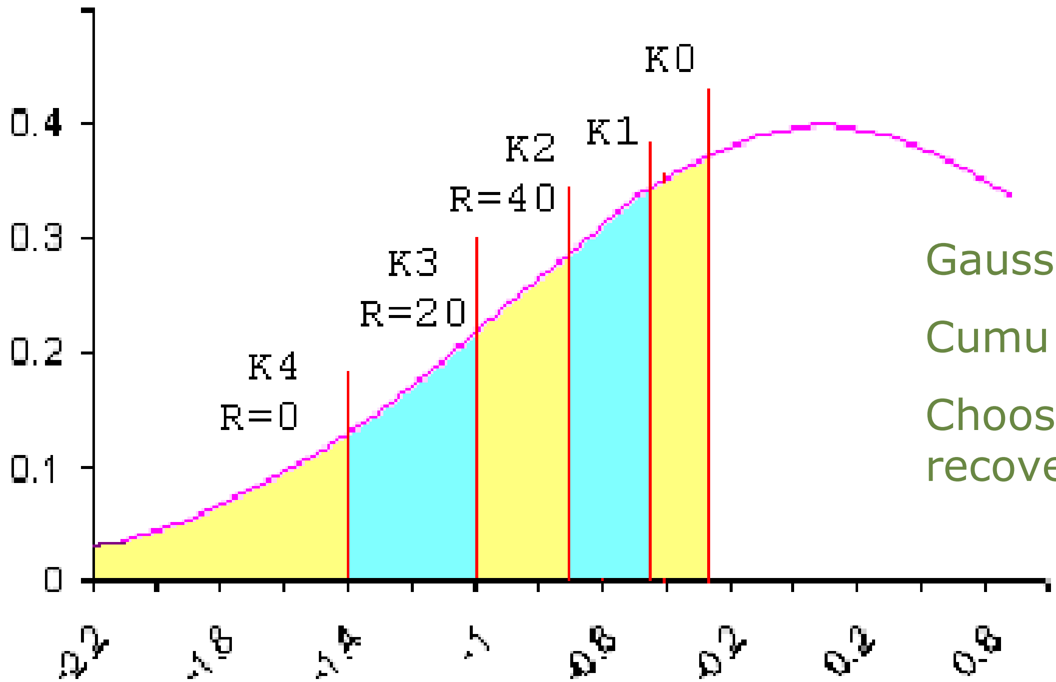


SR1: Mean 40%, skewed

SR2: Mean 40%, greater deviation

Keep in mind: with all weight at 40%, becomes static recovery

How the Model Works: Default-Recovery Triggers



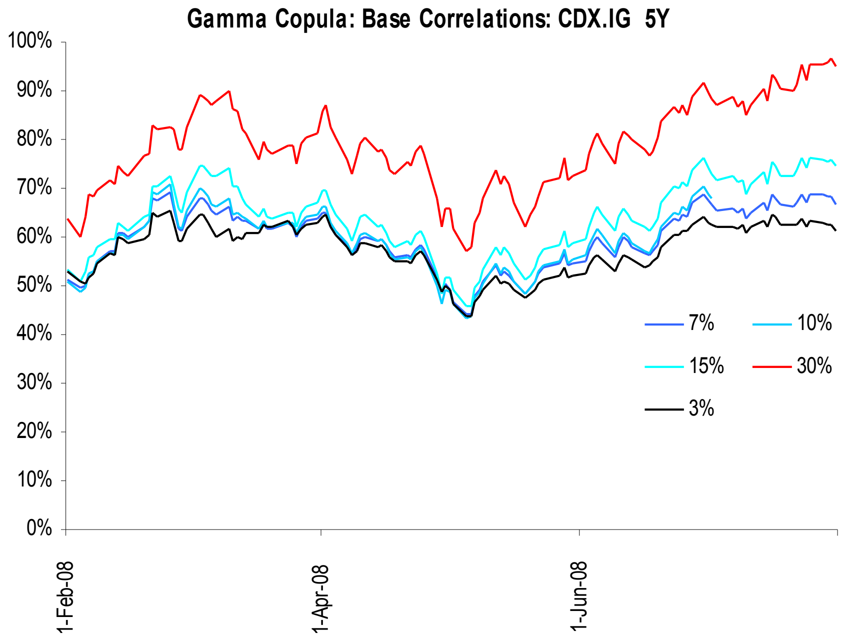
Gaussian model

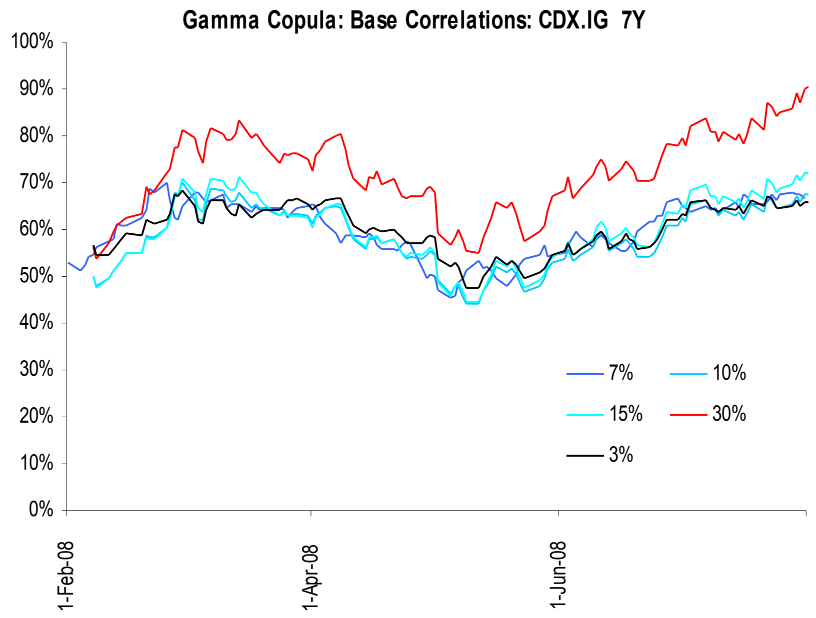
Cumu default prob : 40%

Choose SR2 (uniform recovery weights)

- Issuer m defaults before time T if the variable A(m,T) is below a default threshold K0(m,T)
- The realized recovery of issuer m depends on a set of sub-thresholds K1, K2, …

SREC Correlations





Some issues:

- **Gamma copula deltas are noisy**
- **Analyze the effects on bespoke pricing**
- **Stochastic Recovery slows down the model:**

  It would be useful to find efficient approximations in computing deltas

IV.2 Many dynamic models have been proposed in the literature but very few have actually reached the implementation stage

1. Multi-name default barrier models: Black & Cox (1976), Hull-White (2001, 2004), Zhou (2002)

- **Use the representation of default time as first exit from a barrier of a state process**
- **Cons:**
  - **Complex formulas even defaultable discount factors**
  - **Computation of CDO spreads computationally intensive (Monte-Carlo)**

1. *Random intensity models in the spirit of Duffie & Garleanu (2001)*

- **Introduce randomness in spreads by introducing random default intensities**
- **Choose random intensity process such as to obtain simple expressions for conditional default probabilities, e.g., affine processes**
- **Pros:**
  - **Closed form expressions for CDS spreads and allow for calibration of parameters to CDS curves**
- **Cons:**
  - **Pricing of multiname products done by Monte-Carlo and computationally intensive**
  - **Calibration to CDO tranche quotes not simple**

Approaches based on portfolio losses

Bottom-up approach

- **Calibrate implied default probabilities for portfolio components to credit default swap term structures**
- **Add extra ingredient (copula or factor structure) to obtain joint distribution of default times $F(t_1,\ldots t_n)$ (n-dimentional probability distribution)**
- **Use numerical procedure to compute the risk-neutral distribution of portfolio loss L_t from F: recursion methods, FFT, quadrature, Monte-Carlo,…**
- **Imply correlation parameters from tranche spreads**

Approaches based on portfolio losses (cont.)

The top-down approach

The idea: view portfolio credit derivatives as options on the total portfolio loss L_t and build a pricing model based on the risk-neutral/market-implied dynamics of L_t

- *Approach proposed by Schonbucher which models directly the term-structure of conditional distribution of the total portfolio loss*

Pros:

- **Calibration to initial "base correlation skew" is automatic**
- **Standardized tranches are calibrated so model prices are consistent with tranche-based hedging**
- **Provide a joint model for spread and default risk**
- **No Copulas**

- ***Approach proposed by Schonbucher (Cont.)***

  Cons:

  - **At present: Just a framework – specification needs to be done**
  - **No deltas / sensitivities to individual names**
  - **Applicable to indices only, not to bespoke portfolios.**

- ***Approach in the spirit of Giesecke & Goldberg where you model the spot loss process*** L_t

  - **The portfolio loss process is specified directly in terms of an intensity and a distribution for the loss at an event**
  - **The complete loss surface is described by one set of parameters, and so is the term structure of index and tranche spreads for all attachment points**
  - **Analytical and simulation methods are available to efficiently calculate credit derivative prices**
  - **Single name hedges are generated by thinning the loss process**

Other Open Problems

- ***Hedging of a CDO tranche***
  - **Micro-hedging with single-name CDSs vs. macro-hedging with credit indices**

V. Some New Issues

Multi-Dimensional Models (Hybrids)

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- **Modelling of a single asset is not too badly understood.**
- **Demands from clients raise the need for mixing assets from the same class (baskets) or assets from different classes (hybrids).**
  - Example is an option on a basket of stocks (or currencies or commodities).
  - Another example is a swap paying a FX option => needs modelling domestic and foreign swap rates/volatilities and FX volatilities.
- **Traders like ‘lego’ multi-asset models where each asset could be calibrated separately.**
  - For basket products (equity, credit), a static approach enabling for separate calibration of marginal distributions used to be copulas.
  - But correlation smiles (iTraxx or 2Y/10Y CMS spread) are hard to recover with classic copulas (Gauss, Student, Archimedean). New trends are local correlation and Lévy copulas.
  - Static approaches do not give a satisfactory hedge.
- **Most hybrid diffusions are Gaussian models. What else?**
- **Stochastic correlation (Wishart processes).**

Quasi-Analytical Prices for Vanilla

- **To speed the calibration procedure (when auto-calibration is not possible) an issue is to have quasi-analytical formulas for standard options.**
  - Quasi-analytical formulas can be exact : quadratures/Fourier transforms.
  - Other are approximations : SABR's formula through asymptotic expansion.

Two main approaches:

  - Find new models (SABR/Heston-like, Lévy-driven, Hyperbolic BM-driven) providing exact formulas together with a satisfactory smile.
  - For existing models, improving approximation formulas (ex Hagan's formula does not provide a well-defined probability density).
- **This issue is also critical for calibrating hybrid models.**
- **Relating qualitative properties of smile and asymptotic behavior of the asset p.d.f. can be useful (R. Lee, P. Friz).**

Projection Techniques for Calibration

- **One technique used by practitioners for calibration purposes is the Markov Projection Techniques.**
  - The idea is to replace a complex (stochastic volatility) model by an ‘equivalent’ (meaning having the same margins) local volatility model. Since the local volatility model can be auto-calibrated, we hope to simplify the calibration of the full model.
  - This idea has been used for calibrating hybrid models (Piterbarg, Antonov) and for calibrating top-down approaches for credit derivatives (Lopatin).
  - The theoretical support for this technique is the Gyongy lemma. Improvements in this direction would be helpful.

Numerical Techniques

- **Old problems but still critical.**
  - **Need for efficient PDE solvers (dimension > 3, jumps, cross-derivatives).**
    - Quantification techniques (G. Pagès)
  - **Improve Monte-Carlo techniques**
    - Pseudo-random or quasi-MC generators (Sobol with many dimensions).
    - American Monte-Carlo techniques better than the Longstaff-Schwartz algorithm and able to provide an upper bound.
    - Generic variance reduction methods for large classes of pay-offs.
    - Importance sampling method with stochastic algorithms applied to path-dependent products.