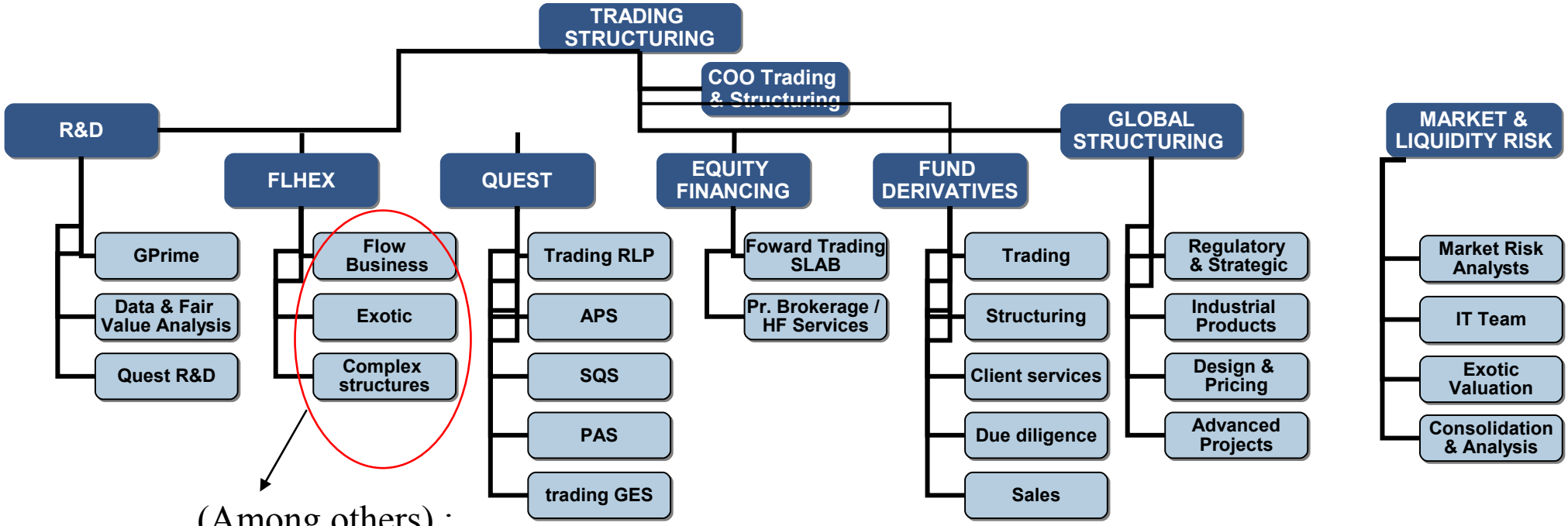
Credit equity modelling

Alexandre Davroux

September 10, 2008

BNP Paribas Equities and Derivatives Organisation



(Among others) :

Proprietary trading desk : 15 people

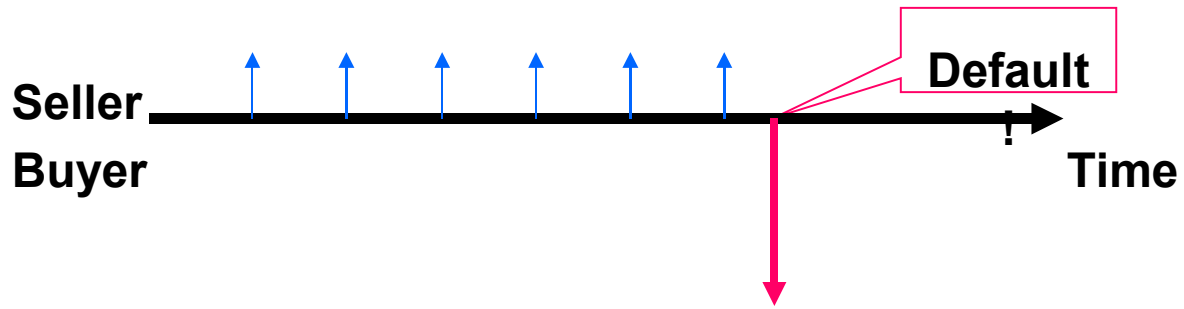
- Half is dedicated to volatility arbitrage
- The other half is dedicated to CB / Credit equity business
- Generating 5% of EQD PnL

Contents

Corporate & Investment Banking

The simplest and most actively traded credit instrument : the Credit default swap contract

- The Credit default swap contract is a contract that protects against a default from the counterparty. It can basically be seen as an insurance.
- The payoff involves two legs (like a usual FI swap) :
  - The premium leg : every three months, the protection buyer pays to the protection seller a fees (or spread) proportional to the guaranteed notional of the contract
  - The default leg : in the event of a default, the protection seller should pay for the loss of capital the protection seller.
    The settlement of the contract can be either physical or cash. For a physical settlement, the protection buyer will deliver a bond to the protection seller, who, in exchange, will give 100% of the notional.



Credit default swap contract - valuation

- Valuation of the premium leg :

$$JP = s(T) \sum_{i=1}^{N} B(0, t_i')(t_i' - t_{i-1}')\mathbb{Q}(\tau \geq t_i')$$

- Valuation of the default leg :

$$JD = (1 - R) \int_0^T B(0, s)\mathbb{Q}(s \leq \tau \leq s + \mathrm{d}s)\mathrm{d}s$$

- Where B(t,T) is the zero coupon of maturity T, $\mathbb{Q}(\tau \geq t_i')$ is the probability that the default occurs after ti
- The most liquid contract is a 5Y CDS, the maturity of which depends on the striking date.
- Following the FI terminology, the fair spread is defined as the one for which the contract value is zero.
- Default probability calibration : this model is the standard market. It is now used as a tool to imply the default probabilities (which would be hard to calibrate on an historical basis) (like the BS formula in the equity world)

Mono underlying credit equity arbitrage (1/3)

- When trying to setup an arbitrage between the credit and the equity market, one should look for contracts which are pricing more or less the same risk.
- The credit leg is not very hard to find : we will use a Credit Default Swap
  - As an alternative, one could have consider options on CDS for example
- For the equity leg, one should now look for a combination of contracts which pays in the event of a default. A large amount of products are available such as :
  - Stocks,
  - Vanilla,
  - Variance Swap,
  - Dividends swap,
  - Equity Default Swap …
- But one should remind that the more exotic the product is, the larger the upfront margin will be

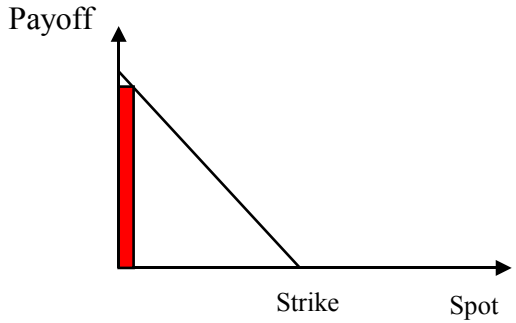
Mono underlying credit equity arbitrage (2/3)

One of the simplest credit equity arbitrage setup is as follow :

- Credit leg : Credit default swap
  - Payoff in the case of a default : 100%-Recovery Rate
  - Premium can be paid upfront
- Equity leg : plain vanilla
  - One can use a deep out-of-the-money put of strike K and same maturity as the CDS
  - Payoff in the case of a default : K-S_recovery
  - An american put can be used if one wants to get rid of the potential maturity mismatch
- Assumptions : S_recovery=0.0, Recovery=40%, K=60%
- The position reads :
  - Sell the protection on a CDS with a notional (K-S_recovery)
  - Buy a put of strike K with a notional (1-R)

Mono underlying credit equity arbitrage (3/3)

- Why does this arbitrage not work ?
  - Because of the mismatch between the two payoffs :
    - The CDS pays only if there is a default,
    - The put may pay even if there is no default
  - Because of the simplified assumptions :
    - Constant recovery,
    - Recovery spot,
  - Because of the liquidity
    - Vanilla of strike 60% maturity 5Y is hard to find
    - Same story for a CDS with a shorter maturity



- What can be done ?
  - One side of the trade can be used anyway (protection selling),
  - This setup could be used to dynamically assess the cheapness / dearness of the credit leg versus the equity leg.
  - But the most efficient way would be to be able to price this position within a unified framework, ie with an hybrid equity credit model

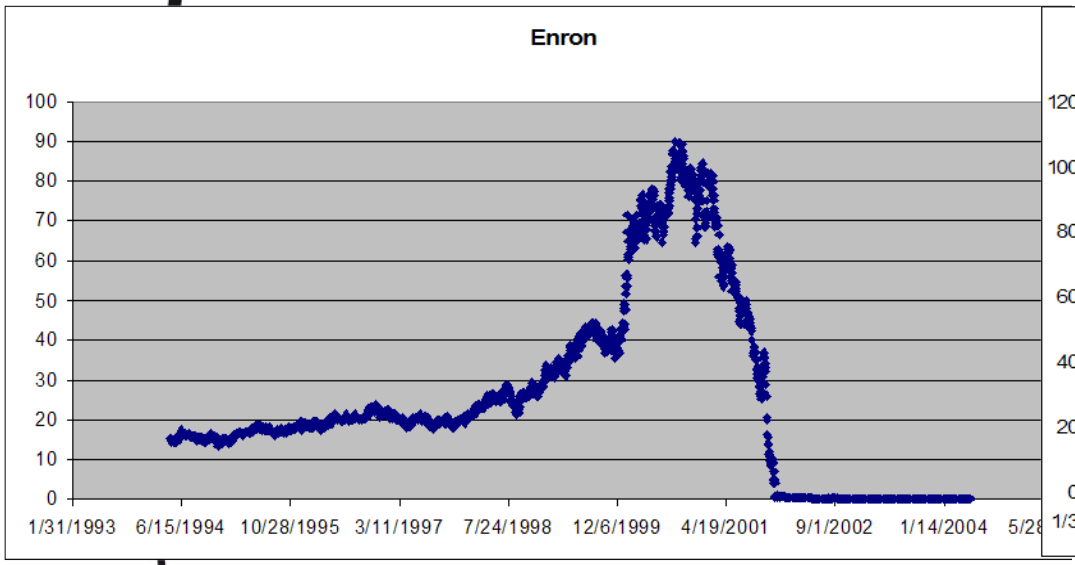
Contents

Corporate & Investment Banking

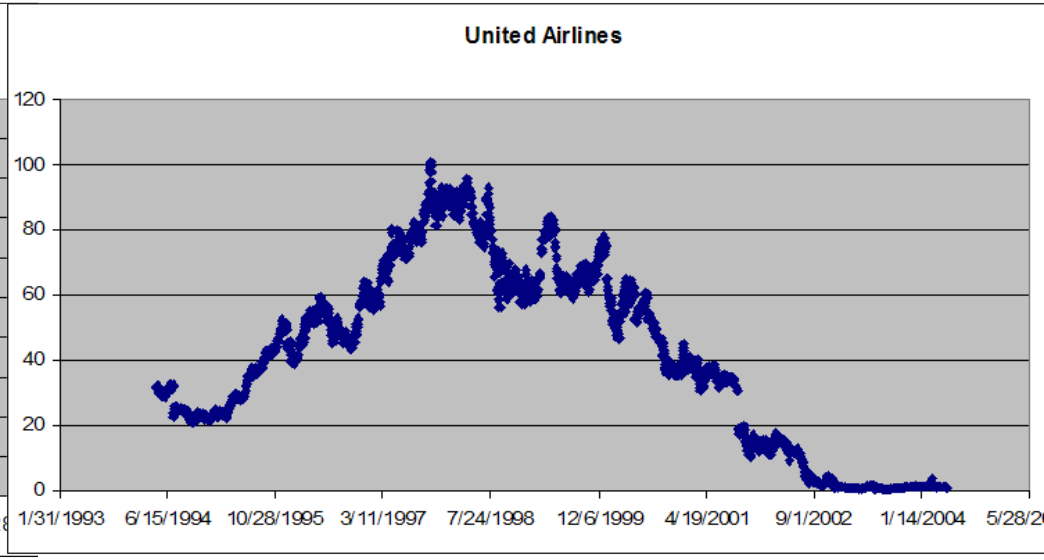
How does a stock goes to default ?

The process of going to default for a stock is dependent on the particular story of the company

•Jump to default (ENRON)



“Smooth” default (CAL)



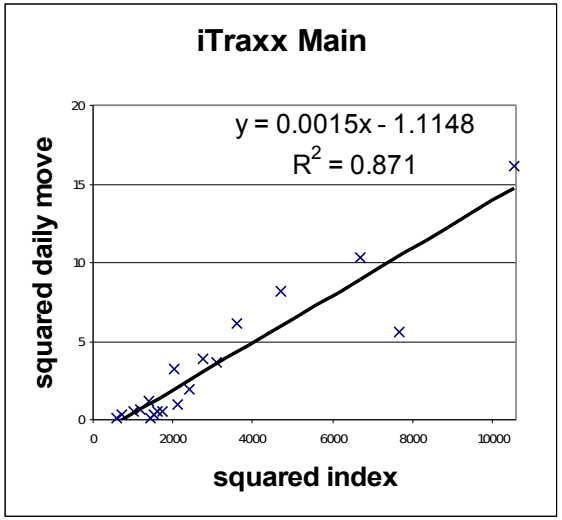
•One has to keep in mind that in most case, the spot in the case of a default is very low (of the order of 1€ or 1$). The usual modelisation of the stock as a diffusion process makes it almost impossible to reach those values.

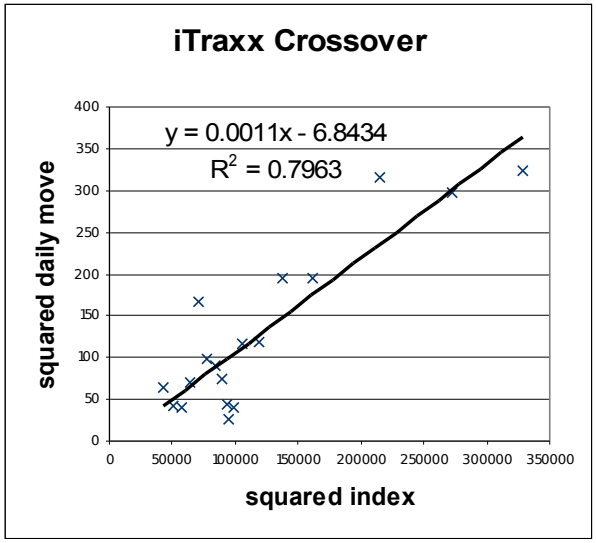
•Different behaviors in the US and in Europe due to the Chapter 11 law

•For a 1Y volatility of 40%, with two standard deviations, the spot is divided by 50 in two years

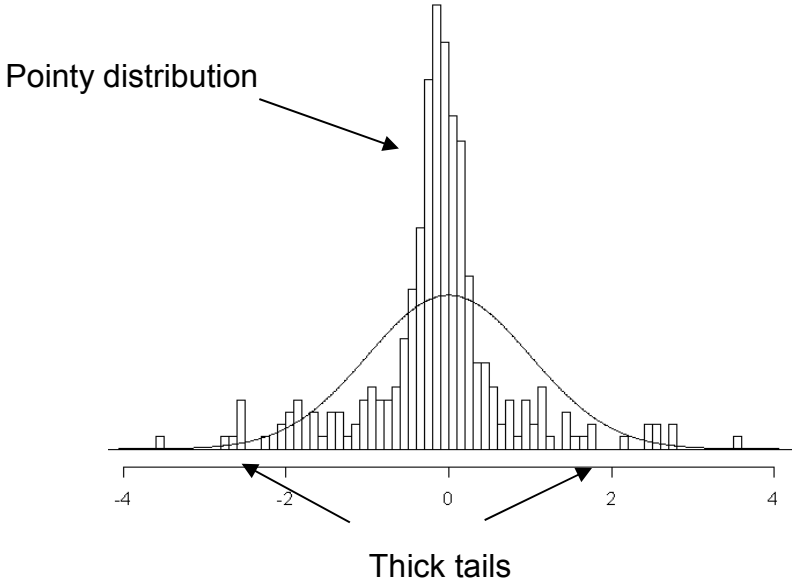
Credit spread modelisation

- Log normal ? Plot of $\mathbb{E}\left[(S_{t+1}-S_t)^2 \middle| S_t^2\right]$





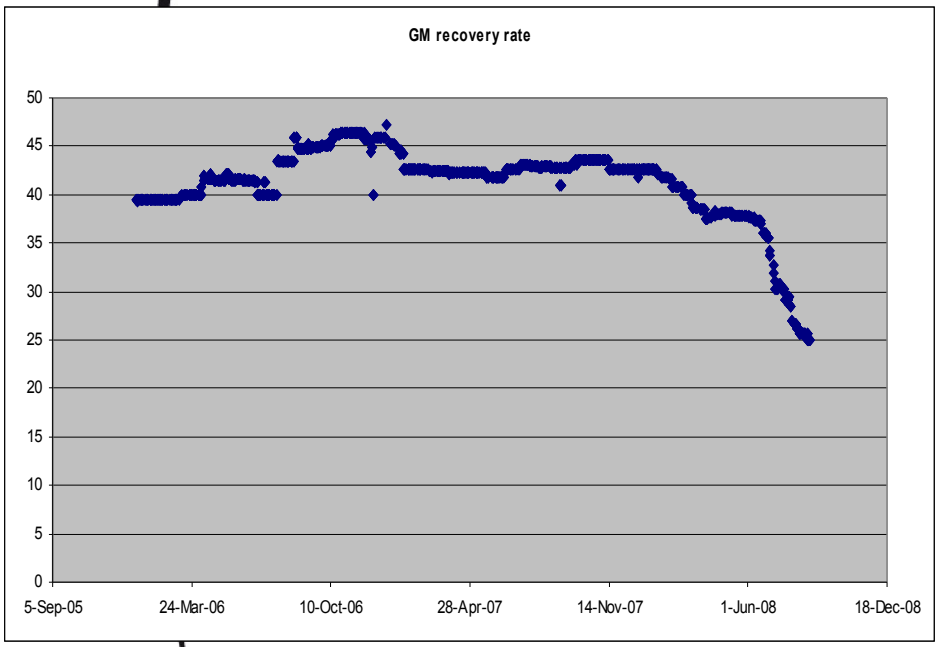
- Law of the daily returns :



- Jump characteristics : (jump = return > 3 stddev)
  - Frequency : between 6 and 12 per year (twice as much as the equity)
  - Amplitude : larger than the equity

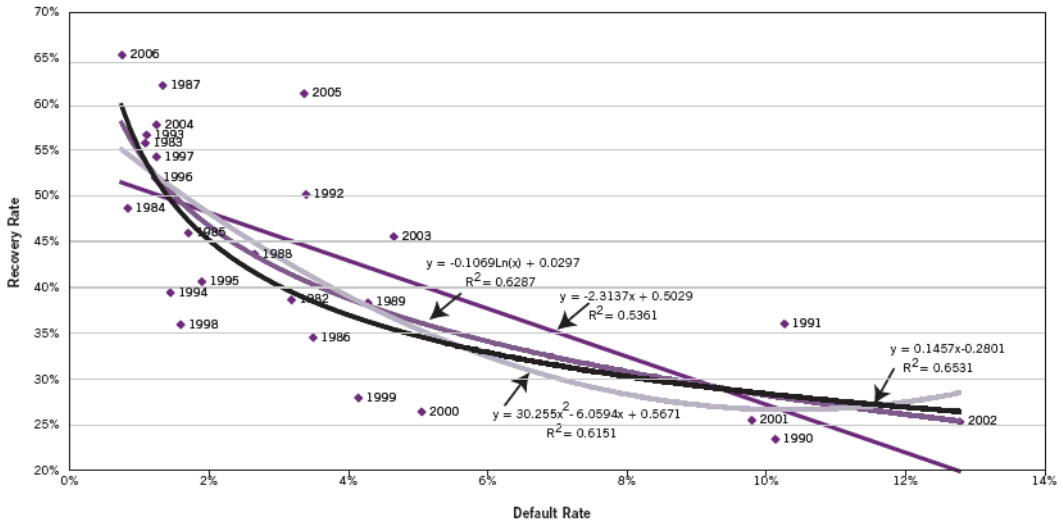
Recovery asumption

“Constant” Recovery rate ?



Recovery skew

Figure 11 **Recovery Rate/Default Rate Association**
Defaulted Bonds Data Set (1982–2006), Dollar Weighted Average Recovery Rates to Dollar-Weighted Average Default Rates

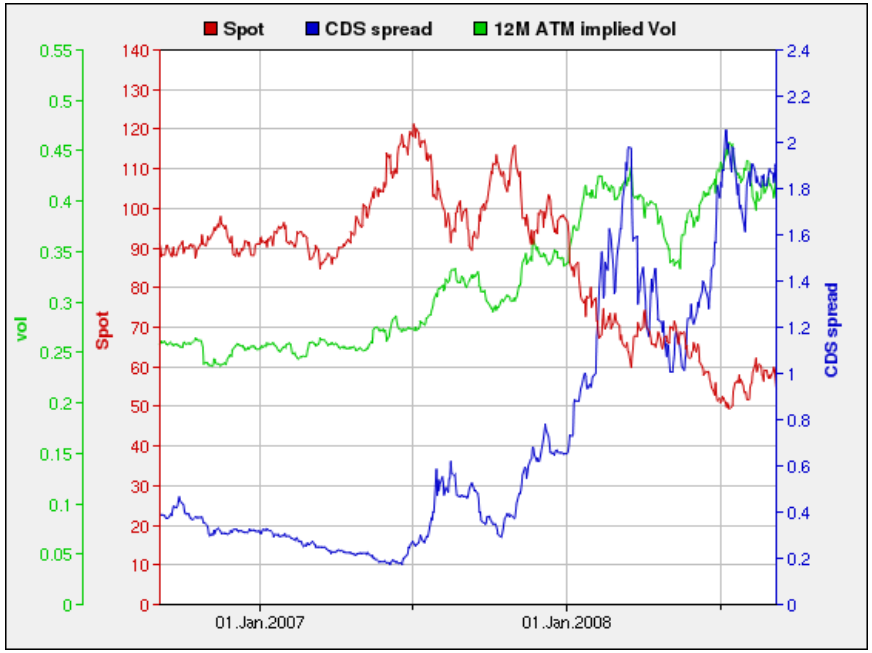


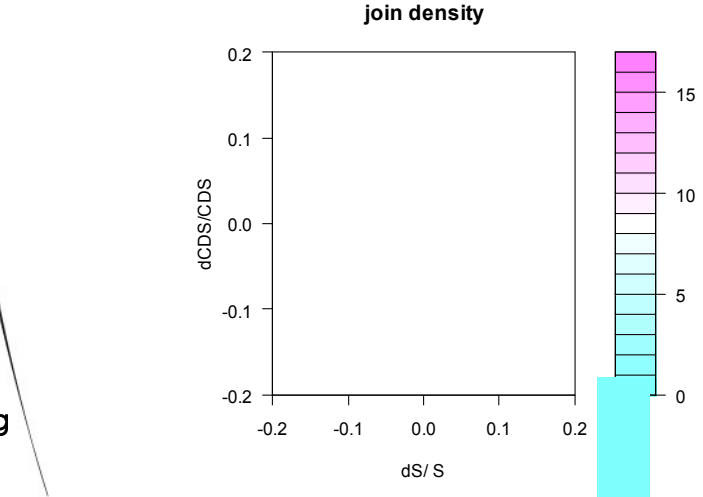
Source: The Link Between Default and Recovery Rates, E. Altman, B. Brady, A. Rest, and A. Sironi, NYU Salomon Center, working paper S-03-4, 2003 and *Journal of Business*, November 2005.

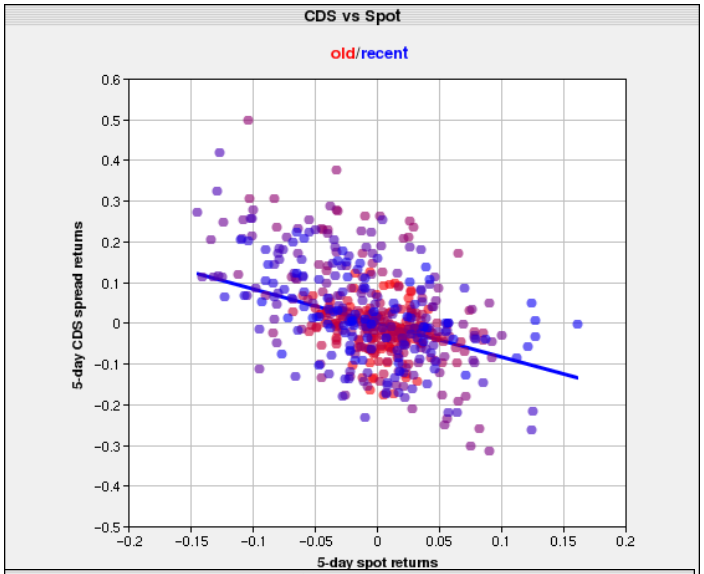
Corporate & Investment Banking

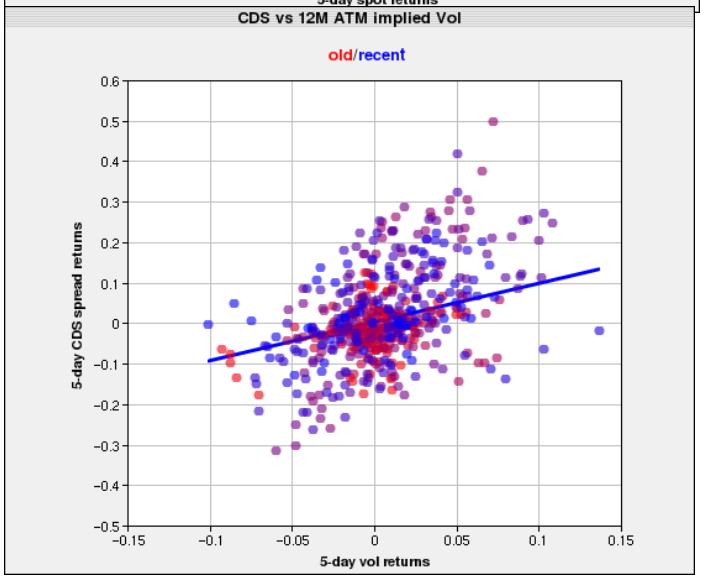
Credit equity join distribution

- Spot, 1Y ATM volatility and spreads join distribution of Renault





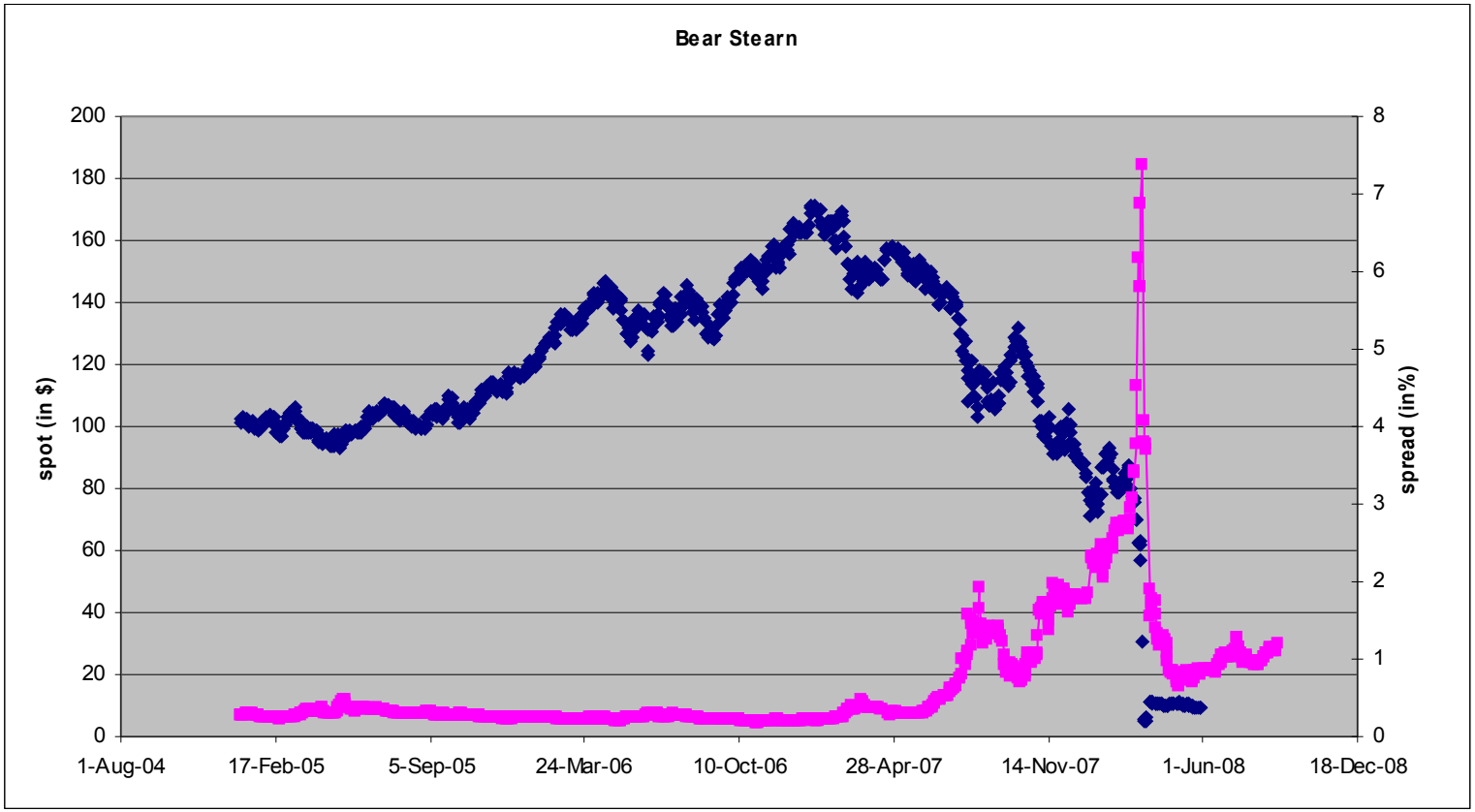




Corporate & Investment Banking

A few words about the Bear stearns story

- On the 14th of March, 2008, Bear Stearns was bought by JPMorgan Chase
  - The stock dropped from 60$ the 12thof March to 2$



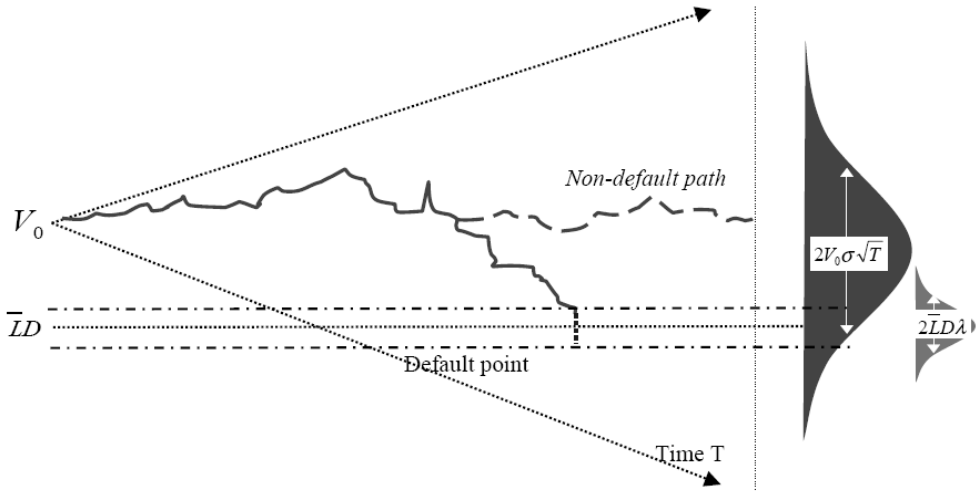
Corporate & Investment Banking

Contents

Corporate & Investment Banking

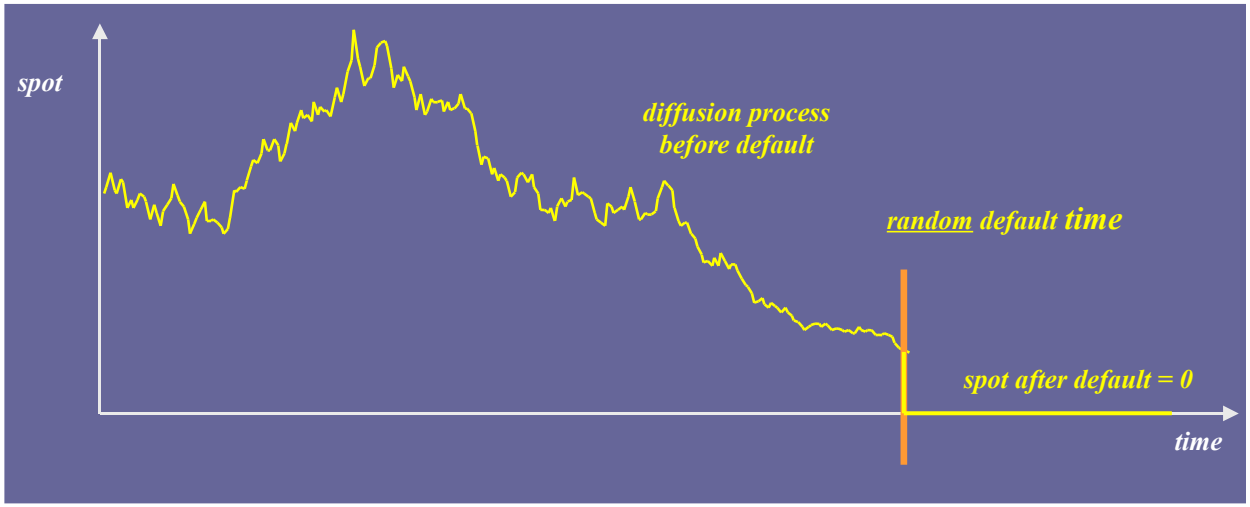
The structural models

- The credit grades model (based on the Merton model (1975)) www.creditgrades.com
  - A default is modelled as the first time when a prescribed barrier level is hitten
  - The barrier level is specified from the company fundamental data (debt, volatility)



  - Usually, it is difficult to calibrate the whole CDS curve with this kind of model (especially the short maturities). But the sensitivities are good.

The reduced form models



- The default is seen as an unpredictable event : it is modelled as the first occurrence of a Poisson process

$$\frac{dS}{S} = \mu_t dt + \sigma(t,T) dW_t + dQ_t$$

Where Q_t is a jump process and the instanteneous jump probability is given by a prescribed function p(t,S)

e.g. p(t,S)=(Sref/S)^alpha

Convertible bonds overview

Introduction

- Corporate security, convertible into shares by the holder
  - Limited lifetime (maturity)
  - Face value (par amount)
  - Coupon rate
  - Conversion ratio
    - Eg: 1 bond , par amount $1000 →50 shares
  - Conversion periods may be restricted
    - Euro / american / bermudean style
    - Lockout period
    - Mandatory conversion
- Features are detailed in the prospectus
- Strong secondary market, but exchanged OTC in Europe
- Benefit for the issuer: reduced coupon rate (because of time value)
- Payoff=Max(Face Value, CR S)

  = FV + CR Max(K, S) with K=Face Value / CR

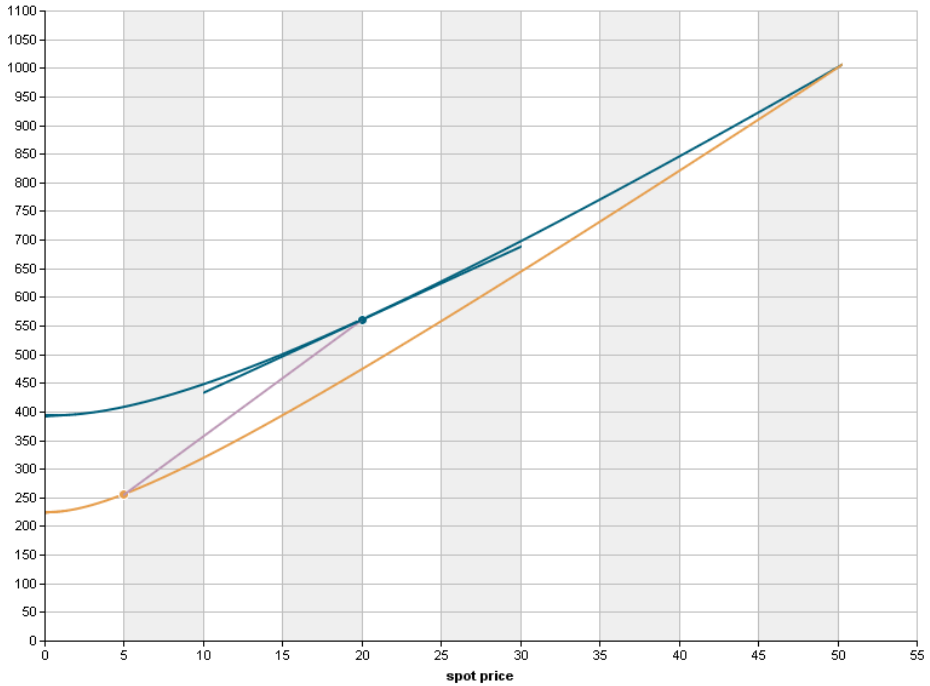
The p(S) model

Why an equity-credit linked model ?

- Scenario: spot goes from $20 to $5
- Credit spread is very likely to have risen (say from 50bp to 200bp)
- If we price with the credit model with each market condition:

Spot price	Credit spread	Premium	Delta
20	50bp	560	64%
5	400bp	255	50%

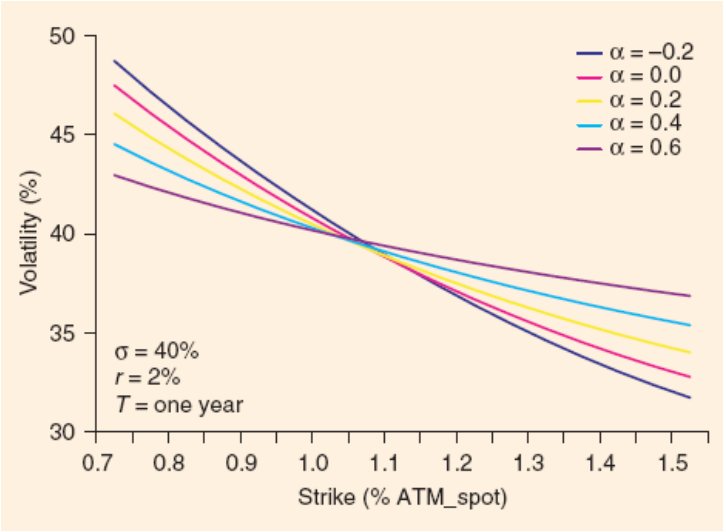
- Realized delta is above 100% !
- Correlation between stock price and credit spread affect delta because of variations in bond floor



A few words about the CEV model

$$\frac{dS_t}{S_t} = rdt + \sigma S_t^{\alpha-1} dW_t \quad \text{if} \quad t < \tau$$
$$S_t = 0 \quad \text{if} \quad t \geq \tau$$

- The CEV model is one of the few models for which the spot can goes to zero with a non zero probability.
- There is only two parameters to fit (sigma, alpha)
  => one can fit the ATM volatility and the skew, and then compute the implicit credit spread level

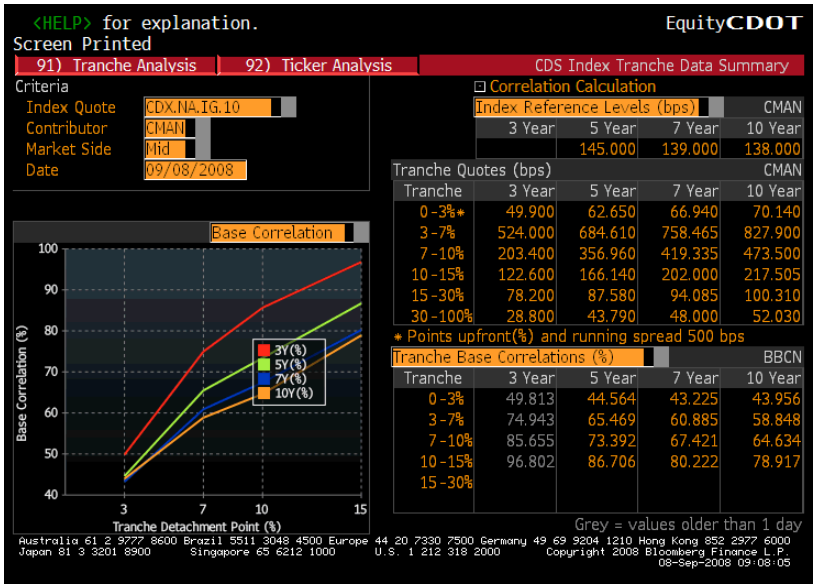


- For more details, see B.Leblanc and M.Atlan (Risk magazine, August 2005)

Contents

Corporate & Investment Banking

Equity credit multi underlyings payoff



- Credit market
  - Credit Indexes
    - Europe : Itraxx (Main and Xover)
    - US : CDX Investment Grade and High Yield
  - Tranches (or CDO on the indexes)
    - Standardized tranches (0-3%, 3-7% …)
    - Correlation skew
  - The standard market model is a gaussian copula to model the dependencies between the defaults
  - The correlation level is not an observable value, therefore it is not easy to calibrate.
  - A bespoke CDO basket typically involves more than 100 names
- Equity market :
  - Multi underlying basket : best of, worst of
  - Typical number of underlyings involved : 3 to 5 names

Corporate & Investment Banking

Thank you

Questions ?

Corporate &
Investment
Banking