Second SMAI European Summer School in Financial Mathematics

Paris, 24-29 August 2009

Abstracts of contributed talks

Smoothness of densities and Tail Estimates for SDEs with locally smooth coefficients and applications

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We study smoothness of densities for solutions of SDEs whose coefficients are smooth and nondegenerate only on an open domain D. Under these assumptions we prove that a smooth density exists on D, and formulate upper bounds suitable for tail estimates under some additional conditions, mainly dealing with the growth of coefficients and their derivatives. This specifies and extends some results by Kusuoka and Stroock in [1], but our approach is substantially different and based on a technique to estimate the Fourier transform inspired from [2] and [3]. The study is motivated by models for financial securities which rely on SDEs with singular (non-lipschitzian or non-elliptic) coefficients. We apply our results to a square-root type diffusion (CIR or CEV type) with local coefficients, giving exponential estimates on the density tail and studying the behaviour at zero.

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Dynamic Risk Indifference Pricing and Hedging in Incomplete Markets

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In this work, we investigate dynamic pricing and hedging formulas in incomplete markets based on the risk indifference principle : we replace the criterion of maximizing utility by minimizing risk exposure because the latter is more often used in practice and because it is a natural extension to the idea of pricing and hedging in complete markets. Using a dual characterization of dynamic risk measures coming from BMO martingales (see [3]), the risk indifference pricing problem reduces to two (zero-sum) stochastic differential games, which we solve by means of backward stochastic differential equation (BSDE) theory. We find an explicit formula for the dynamic risk indifference price in terms of solutions of BSDEs. We follow the spirit of [4] who study a similar risk indifference pricing problem using PDE techniques. Importantly, our stochastic analysis approach does not impose Markovian assumptions on the coefficients and it encompasses the case of dynamic time-consistent risk measures (as, e.g., risk measures coming from g-expectations ; see [2]). The work will also include the extension to the jump diffusion case as well as a Malliavin calculus derivation of a (quasi) hedging formula, as in [1].

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Connecting Continuous and Discrete Path-Dependent Options under Exponential Levy Model

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We consider continuous path-dependent options which depend on the extremal price of the underlying asset during the life of the options. Perhaps, in exponential Lévy model closed-form formulae are not, in general, available for pricing these options. Then we need to use a discrete numerical method for valuating them. In this context, we study the best way to price a continuous path-dependent option using a discrete one. The reverse issue also arises when we have an explicit formula for a continuous option, and that we want to evaluate its discrete version. This is joint work with D. Lamberton.

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A Probabilistic Numerical Method for Fully Nonlinear Parabolic Nonlocal PDEs

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In this work, the results for probabilistic numerical scheme for fully nonlinear PDEs of [2] is generalized to work for nonlocal PDEs. The first step and the most important step of this generalization, is to find a Monte Carlo approximation of the integral operator of Lévy type for some Lévy measure ν :

$$\int_{R^d_*} \big(\varphi(t, x + \eta(t, x, z)) - \varphi(t, x) - \mathbb{1}_{\{|z|\leq 1\}} D\varphi(t, x)\eta(t, x, z)\big)\, d\nu(z)$$

which appears in a large class of nonlinear equations (H-J-B equations). Then the results of convergence and rate of convergence are followed directly by generalizing some results of [1] to nonlocal PDEs.

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Optimal liquidation in limit order books with stochastic liquidity

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We consider the problem of optimally placing market orders so as to minimize the expected liquidity costs from buying a given amount of shares. The liquidity price impact of market orders is described by an extension of a model for a limit order book with resilience that was proposed by Obizhaeva and Wang (2006). We extend their model by allowing for a price impact which is linear in the number of shares, but is described by a rather general stochastic differential equation instead of being constant in time. This is our main contribution. It turns out that the optimal buying strategy is not deterministic anymore as in Obizhaeva and Wang, but adapts to the order book height. We proof that it can be described by a no trading and a discrete trading region. Scaling properties of the value function and convexity arguments are being used. The barrier is numerically analyzed in the case of a general geometric Brownian motion and a Cox-Ingersoll-Ross process. Situations where trading is 'passive' respectively 'aggressive in the liquidity' arise. This is joint work with Torsten Schöneborn and Mikhail Urusov.

Çetin-Jarrow-Protter model of liquidity in a Binomial market

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We study the Binomial version of the illiquid market model introduced by Çetin, Jarrow and Protter [1] for continuous time. We develop efficient numerical methods for the analysis of this model. In particular, we study the liquidity premium that results from the model. In [1] the arbitrage free price of a European option traded in this illiquid market is equal to the classical value. However, the corresponding hedge does not exists and the price is obtained only in L^2-approximating sense. Çetin, Soner and Touzi [2] investigated the super-replication problem using the same supply curve model but under some restrictions on the trading strategies. They showed that the super-replicating cost differs from the Black-Scholes value of the claim, thus proving the existence of liquidity premium. In this paper, we study the super-replication problem in discrete time but with no assumptions on the portfolio process. We recover the same liquidity premium as [2] by passing to the continuous time limit. This is an independent justification of the restrictions introduced in [2]. Moreover, we also propose an algorithm to calculate the option's price for a Binomial market. This is joint work with Halil Mete Soner.

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Viscosity and Principal-Agent Problem

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We develop a stochastic control system from a continuous-time Principal-Agent model in which both the principal and the agent have imperfect information and different beliefs about the project. We attempt to optimize the agent's utility function under the agent's belief. Via the corresponding Hamilton-Jacobi-Bellman equation we prove that the value function is jointly continuous and satisfies the Dynamic Programming Principle. These properties directly lead to the conclusion that the value function is a viscosity solution of the HJB equation. Uniqueness is then also established.

Implied Lévy volatility

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The concept of implied volatility under the Black-Scholes model is one of the key points of its success and its widespread use since it allows to perfectly match model prices and market prices. In fact it gives another, more convenient and robust, way of quoting plain vanilla European option prices. Rather than quoting the premium in the relevant currency, the options are quoted in terms of Black-Scholes implied volatility. Over the years, option traders have developed an intuition in this quantity. As it turns out, this model parameter depends on the characteristics of the contract. More precisely, it depends on the strike price and the remaining lifetime of the option. The precise functional form is called the volatility surface and follows its own dynamics in the market. This model parameter needs to be adjusted separately for each individual contract given the inadequacy of the underlying Black-Scholes model. separately for each individual contract, is because the underlying Black-Scholes model is not adequate. By analyzing empirical historical data, it is not hard to see that stock returns tend to be more skewed and have fatter tails than those the normal distribution can provide. Hence blind trust in a single implied volatility number and all the numbers derived from that, like deltas and other hedge parameters could be dangerous. Here a similar concept is developed but now under a Lévy framework and therefore based on distributions that match more closely historical returns.

We introduce the concept of implied Lévy volatility, hereby extending the intuitive Black-Scholes implied volatility into a more general context. The Lévy models are obtained by replacing the Wiener distribution modeling the diffusion part of the log-return process by a more empirically founded Lévy distribution. under the Black-Scholes model, the logarithm of the stock return follows a normal distribution whereas the empirical log-returns exhibit some skewness, excess kurtosis and a fatter tail behavior. the Gaussian copula model has become the common model in practice. The extension of this classical model to the class of one factor Lévy models allows to consider distributions which fit better the empirical return distribution characterized by asymmetry, high peak and fatter tails than the Gaussian distribution. The Lévy space volatility model will arise by multiplying volatility with the underlying Lévy process, whereas the Lévy time volatility model will arise by multiplying volatility squared with time. Lévy implied time and space volatility are introduced and a study of the resulting skew-adjustment is made.

The price and Greeks of vanilla options are computed by making use of the COS method proposed by Fang and Oosterlee. This method rests on Fourier-cosine series expansions and can be applied for any model if the characteristic function of the log-price process at maturity T is available.

By switching from the Black-Scholes world to the Lévy world, we introduce additional degrees of freedom (i.e. parameters that can be set freely) which can be used in order to minimize the curvature of the volatility surface. We look how Black-Scholes curves are translated into implied Lévy volatility curves and vice versa. It is shown that any smiling or smirking Black-Scholes volatility curve can be transformed into a flatter Lévy volatility curve under a well chosen parameter set. This gives some evidence to the fact that the implied Lévy models could lead to flatter volatility curve for more practical datasets. Hence, implied Lévy volatility model can be of a particular interest for practitioners facing the problem of pricing barrier options since for the Black-Scholes model, it is not clear which volatility one should use (the one of the barrier or the one of the strike).

Model performance is studied by analyzing delta-hedging strategies for short term ATM vanilla under the Normal Inverse Gaussian and the Meixner model, both qualitatively and on historical time-series of the S&P500. The Lévy degrees of freedom can thus be determined such that the absolute value of the mean and the square root of the variance of the daily hedging error are minimized. It is shown that using the historical optimal parameters leads to a significant reduction of the variance of the hedging error (amounting to more than 50 percents), which is particularly attractive for option hedging.

Moreover, we investigate the Delta hedging performance of a portfolio of liquid vanilla options with different strike prices and times to maturity written on the Nasdaq and traded from the 2nd of January 2000 on. It is shown that making use of the Lévy models with appropriate degrees of freedom leads to a significant improvement of the mean and the variance of the Profit and Loss. This is joint work with José Manuel Corcuera, Peter Leoni and Wim Schoutens.

A time-and price-continuous order book model

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Nowadays almost all major stock exchanges such as NYSE and London Stock Exchange either fully or partially trade via electronic limit order books. Market participants can choose to submit *limit orders* which are stored in the limit order book or *market orders* which are immediately executed against the best available prices. Outstanding limit orders can also be *cancelled*. The limit order book is publicly available to all market participants and is an important indicator for the current liquidity of the asset.

In this talk I will set up a microstructure model for a limit order book which includes dependence of incoming limit/market orders and cancellation on key parameters such as *spread* and *volume imbalance*. The model is determined by the total amount of incoming limit/market orders and cancelled orders. These quantities are given by continuous (stochastic) processes. I will analyse conditions under which this infinite-dimensional system can be reduced to a finite-dimensional model. In this setting, I will give some specific examples of limit/market order inflow rates and analyse the resulting models. In particular, I shall analyse key quantities of interest such as the behaviour of the spread, mid-quote price and time-to-fill.

The results can be used in a variety of applications such as micro-traders, high-frequency proprietary trading strategies and smart order routing.

A Robust Regression Monte Carlo Method for Pricing High-Dimensional American-Style Options

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We propose a new class of regression-based Monte Carlo methods for pricing high-dimensional American-style options. On the basis of the dynamic programming principle in terms of the optimal stopping time, we fit the continuation value at every exercise date by robust regression rather than by ordinary least squares. By using robust regression, we are able to get a more accurate approximation of the continuation value due to detection of outliers and leverage points. To prove convergence of our robust regression Monte Carlo method, we use techniques of the statistical learning theory. Our method runs with few basis functions, and it turns out that our suggested estimator is unbiased. Moreover, we extend approaches for variance reduction by importance sampling for American-style options, and we focus on both stochastic approximation and optimization methods. In comparison to existing Monte Carlo methods, we improve convergence significantly.

Constructing time-homogeneous diffusions consistent with perpetual option prices

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Consider an optimal stopping problem (e.g. the pricing of, and optimal exercise strategy for, a perpetual option) with a one-parameter objective function (e.g. a call payoff), and suppose we are given the expected discounted values for the problem (e.g. option prices) for a continuous range of parameter values (e.g. a range of strikes). Under mild regularity conditions on the payoff function we show how to construct a time-homogeneous diffusion consistent with the given values. The forward problem of determining the expected values given a process is related to the inverse problem through a generalized duality relation with respect to the log-transformed payoff function.

Backward stochastic dynamics on a filtered probability space

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We consider the following backward stochastic dynamics based on a general filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$:

$$\begin{cases} dY_t = -f_0(t, Y_t, L(M)_t)dt - \sum_{i=1}^N f_i(t, Y_t)dB_t^i + dM_t, \\ Y_T = \xi \in L^2(\Omega, \mathcal{F}_T, P) \end{cases}$$

where L is a given operator to be introduced in the main text, B is an N-dimensional Brownian motion as given, and M is a square-integrable martingale to be determined. Under *adaptedness* constraints on Y, we prove that the equation admits a solution pair (Y, M), which is unique in the sense of strict solutions to be introduced in the main text. The martingale representation is not required in order to prove the existence and uniqueness, and instead we establish the existence and uniqueness of a functional differential equation, in a form $V = \mathbb{L}(V)$, where $\mathbb{L}$ is a non-linear functional. Finally we indicate a connection between the backward stochastic dynamics discussed here and a class of non-linear PDEs, namely semi-linear PDEs with non-local integral term. This is a joint work with Terry Lyons and Zhongmin Qian.

Conditional Certainty Equivalent

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In a general dynamic framework we study the conditional version of the classical notion of the certainty equivalent of a risky position X, defined as the implicit solution $C_{s,t}$ of the equation

$$u(C_{s,t}(X)(\omega), s, \omega) = E\left[u(X, t, \cdot)|\mathcal{F}_s\right](\omega).$$

where $u(x, t, \omega)$ is a stochastic dynamic utility satisfying natural conditions. For this analysis we propose a dynamic version of the theory of Musielak-Orlicz space, which fits very well with stochastic dynamic utilities. It turns out that for large classes of stochastic dynamic utilities $C_{s,t}$ is a quasi concave operator between two Orlicz spaces induced respectively by $u(x, t, \omega)$ and $u(x, s, \omega)$. This concept leads to the investigation of quasi concave maps and their dual representation : to this aim we extend the representation formulas of quasi concave conditional maps to the dynamic case. This is joint work with Marco Frittelli.

A Malliavin calculus approach to general stochastic differential games with partial information

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In this paper we consider a general partial information stochastic differential game where the state process is a controlled Itô-Lévy process. Suppose the dynamics of a state process $X(t) = X^{(u_0,u_1)}(t,\omega)\,;\, t \geq 0, \omega \in \Omega$, is a controlled Itô-Lévy process has the form

$$\begin{cases} dX(t) = b(t, X(t), u_0(t), \omega)dt + \sigma(t, X(t), u_0(t), \omega)dB(t) \\ \qquad\qquad\quad + \int_{\mathbb{R}_0} \gamma(t, X(t^-), u_0(t^-), u_1(t^-, z), z, \omega)\tilde{N}(dt, dz); \\ X(0) = x \in \mathbb{R}. \end{cases} \tag{1}$$

where the coefficients $b : [0, T] \times \mathbb{R} \times U \times \Omega \to \mathbb{R}$, $\sigma : [0, T] \times \mathbb{R} \times U \times \Omega \to \mathbb{R}$ and $\gamma : [0, T] \times \mathbb{R} \times U \times K \times \mathbb{R}_0 \times \Omega$ are given $\mathcal{F}_t$-predictable processes and U, K are given open convex subsets of $\mathbb{R}^2$ and $\mathbb{R} \times \mathbb{R}_0$ respectively. Here $\mathbb{R}_0 = \mathbb{R} - \{0\}$, $B(t) = B(t, \omega)$, and $\eta(t) = \eta(t, \omega)$, given by

$$\eta(t) = \int_0^t \int_{\mathbb{R}_0} z\tilde{N}(ds, dz);\ t \geq 0,\ \omega \in \Omega,$$

are a 1-dimensional Brownian motion and an independent pure jump Lévy martingale, respectively, on a given filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. Thus

$$\tilde{N}(dt, dz) := N(dt, dz) - \nu(dz)dt$$

is the *compensated Poisson jump measure* of $\eta(\cdot)$, where $N(dt, dz)$ is the *Poisson jump measure* and $\nu(dz)$ is the *Lévy measure* of the pure jump Lévy process $\eta(\cdot)$. The processes $u_0(t)$ and $u_1(t, z)$ are the control processes and have values in a given open convex set U and K respectively for a.a. $t \in [0, T]$, $z \in \mathbb{R}_0$ for a given fixed $T > 0$. Also, $u_0(\cdot)$ and $u_1(\cdot)$ are càdlàg and adapted to a given filtration $\{\mathcal{E}_t\}_{t\geq 0}$, where

$$\mathcal{E}_t \subseteq \mathcal{F}_t, \qquad t \in [0, T].$$

$\{\mathcal{E}_t\}_{t\geq 0}$ represents the information available to the controller at time t. For example, we could have

$$\mathcal{E}_t = \mathcal{F}_{(t-\delta)^+}; \quad t \in [0, T],\ \delta > 0 \text{ is a constant},$$

meaning that the controller gets a delayed information compared to $\mathcal{F}_t$. Let $f : [0,T] \times \mathbb{R} \times U \times K \times \Omega \to \mathbb{R}$ and $g : \mathbb{R} \times \Omega \to \mathbb{R}$ are given $\mathbb{F}$-adapted stochastic processes. Suppose there are two players in the stochastic differential game and the given performance functionals for players are as follows :

$$J_i(u_0, u_1) = \mathbb{E}^x \Big[\int_0^T \int_{\mathbb{R}_0} f_i(t, X(t), u_0(t), u_1(t,z), z, \omega) \mu(dz) dt + g_i(X(T), \omega) \Big],$$

for $i = 1, 2$, where μ is a measure on the given measurable space $(\Omega, \mathcal{F})$ and $\mathbb{E}^x = \mathbb{E}^x_P$ denotes the expectation with respect to P given that $X(0) = x$. Suppose that the controls $u_0(t)$ and $u_1(t,z)$ have the form

$$u_0(t) = (\pi_0(t), \theta_0(t)); \quad t \in [0,T]$$

$$u_1(t,z) = (\pi_1(t,z), \theta_1(t,z)); \quad (t,z) \in [0,T] \times \mathbb{R}_0.$$

Let $\mathcal{A}_\Pi$ and $\mathcal{A}_\Theta$ denote the given family of controls $\pi = (\pi_0, \pi_1)$ and $\theta = (\theta_0, \theta_1)$ such that they are contained in the set of $\mathcal{E}_t$-adapted controls, (1) has a unique strong solution up to time T and

$$\mathbb{E}^x \Big[\int_0^T \int_{\mathbb{R}_0} |f_i(t, X(t), \pi_0(t), \pi_1(t,z), \theta_0(t), \theta_1(t,z), z, \omega)| \mu(dz) dt + |g_i(X(T), \omega)| \Big] < \infty,$$

for $i = 1, 2$. The partial information non-zero-sum stochastic differential game problem we consider is the following :

Find $(\pi^*, \theta^*) \in \mathcal{A}_\Pi \times \mathcal{A}_\Theta$ *(if it exists) such that*

(i) $J_1(\pi, \theta^*) \leq J_1(\pi^*, \theta^*)$ *for all* $\pi \in \mathcal{A}_\Pi$

(ii) $J_2(\pi^*, \theta) \leq J_2(\pi^*, \theta^*)$ *for all* $\theta \in \mathcal{A}_\Theta$.

By using Malliavin calculus, we derive a maximum principle for this general stochastic differential game. (This is a joint work with Bernt Øksendal and Ta Thi Kieu An)

Analysis of Fourier transform valuation formulas and applications

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The aim of this article is to provide a systematic analysis of the conditions required such that Fourier transform valuation formulas are valid in a general framework : i.e. when the option has an arbitrary payoff function and depends on the path of the asset price process. An interplay between the conditions imposed on the payoff function and on the process arises naturally. We also extend these results to the multi-dimensional case, and discuss the calculation of Greeks by Fourier transform methods. As an application, we price options on the minimum of two assets in Lévy and stochastic volatility models. This is joint work with Ernst Eberlein and Kathrin Glau.

Differentiability of BSDE driven by continuous martingales and hedging in incomplete markets

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In this talk we consider quadratic growth BSDE driven by continuous local martingales. First we derive the Markov property of a forward-backward system when the driving martingale is a strong Markov process. Then we establish the differentiability of a FBSDE with respect to the initial value of its forward component. It enables us to obtain the main result of this talk that is to describe the control process of the BSDE in terms of a differential operator of the solution process and the correlation coefficient of the forward process. This formula generalizes the results obtained by several authors in the Brownian setting, designed to represent the optimal delta hedge in the context of cross hedging insurance derivatives that generalizes the derivative hedge in the Black-Scholes model. It involves Malliavin's calculus which is not available in the general martingale setting. Consequently, we propose a new method based on stochastic calculus techniques. This is a joint work with Peter Imkeller and Anthony Réveillac.

Optimal trading strategies under arbitrage

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Explicit formulas for optimal trading strategies in terms of minimal required initial capital are derived to replicate a given terminal wealth in a continuous-time Markovian context. To achieve this goal this talk does not assume the existence of an equivalent local martingale measure. Instead a new measure is constructed under which the dynamics of the stock price processes simplify. It is shown that delta hedging does not depend on the "no free lunch with vanishing risk" assumption. However, in the case of arbitrage the problem of finding an optimal strategy is directly linked to the non-uniqueness of the partial differential equation corresponding to the Black-Scholes equation. The recently often discussed phenomenon of "bubbles" is a special case of the setting in this talk. Several examples at the end illustrate the techniques described in this work.

Minimal Sufficient conditions on a primal optimizer in utility maximization

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We continue the study of utility maximization in the nonsmooth setting and give a counterexample to a conjecture made by Deelstra, Pham and Touzi on the optimality of random variables valued in an appropriate subdifferential. We use this as the basis for proving minimal sufficient conditions on a random variable for it to be a primal optimizer in the case where the utility function is neither strictly concave nor differentiable.

A Penalty based Finite Element Method for the Pricing of American Options

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In the Black-Scholes framework, the valuation of an American option can be formulated as a linear complementarity problem (LCP). A certain variational formulation of this LCP is considered and a finite element method is applied. A proof of stability and convergence of the finite element method is given. The finite element discretisation yields a discrete LCP, which is solved using a penalty method. Regularity properties of the original problem and the penalised problem are studied.

Optimal Stock Selling/Buying Strategy with reference to the Ultimate Average

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We are concerned with the optimal decision to sell or buy a stock in a given period with reference to the ultimate average of the stock price. More precisely, we aim to determine an optimal selling (buying) time so as to maximize (minimize) the expectation of the ratio of the selling (buying) price to the ultimate average price over the period. This is an optimal stopping time problem which can be formulated as a variational inequality problem. The problem gives rise to a free boundary that corresponds to the optimal selling (buying) strategy. We provide a partial differential equation approach to characterize the free boundary (or equivalently, the optimal selling (buying) region). It turns out that the optimal selling strategy is bang-bang, which is the same as that obtained by Shiryaev, Xu and Zhou (2008) taking the ultimate maximum of the stock price as the benchmark, while the optimal buying strategy can be a feedback one subject to the type of averaging and parameter values. Moreover, by a thorough characterization of free boundary, we reveal that the bang-bang optimal selling strategy heavily depends on the fact that the averaging period starts from time zero, and a feedback optimal selling strategy is possible if the averaging period starts from a time horizon earlier than time zero.