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Abstracts of contributed talks

Impulse control problem with switching technology

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We consider an impulse control problem in infinite horizon applied with switching technology. We suppose that the firm decides at certain moments (impulse moments) to switch technology, leading to a jump of the firm value. We show that the value function for such problems satisfies a dynamic programming principle version. Our objective is to look for an optimal strategy which maximizes the value function associated with a switching problem.

Optimal multiple stopping times problems of jump diffusion processes

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In this work, we extend the result of Carmona and Touzi for optimal multiple stopping problem to a market where the price process follows a jump diffusion process. We construct a sequence of new reward functions where the pay-off is not a priori right continuous adapted process. We prove the existence of multiple exercise policies under weaker assumptions than in the classical case. Next, we derive a sequence of dynamic programming variational inequality satisfied by the value function in the sense of viscosity solutions. We prove a comparison theorem which will be useful for numerical purpose, postponed in a further study.

A functional differential equation approach to the numerical simulation of BSDEs

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Backward stochastic dynamics have been introduced by Liang, Lyons and Qian in order to provide a generalization of BSDEs for non-Brownian filtrations by the use of a Doob-Meyer decomposition argument. In particular, this approach allows a representation of the solution of the classical BSDE introduced by Pardoux and Peng in terms of the solution of a functional differential equation (which completely describes the finite variation part arising in the Doob-Meyer decomposition for the first component of the solution of the BSDE). In this work, this functional differential equation approach is applied in order to numerically simulate solutions of BSDEs with Lipschitz driver. An analysis of the error induced by the use of approximations of conditional expectations is also included. This is joint work with Gechun Liang.

Work effort, consumption, and portfolio choice : When the occupational decision matters

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We analyze a highly-qualified individual with respect to the choice between two distinct career paths. She can choose between a mid-level management position in a large company paying a rather high salary and an executive position within a smaller listed company where less salary is offered but the possibility to directly affect the company's performance. The decision and stochastic control problem is studied in an environment where the individual can invest in the financial market including the share of the smaller listed company. Thus she can participate indirectly from a higher work effort leading to an increasing expected return. The utility maximizing strategy from consumption, investment, and work effort is derived in closed form for the case of logarithmic utility. Aspects of the power utility setting are discussed as well. Further, we calculate the appropriate compensation of the individual by comparing the utility she gains by accepting the offer to work for the smaller listed company with the utility by taking up the job offer of the large company. This gives insight in some aspects of optimal contract design. Our framework is for example applicable to the pharmaceutical industry, the IT sector, and the financial industry.

Refined volatility expansion in the Heston model

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It is known that Heston's stochastic volatility model exhibits moment explosion, and that the critical moment can be obtained by solving (numerically) a simple equation. This yields a leading order expansion for the implied volatility at large strikes (Roger Lee's moment formula). Motivated by recent tail-wing refinements of this moment formula, we first derive a novel tail expansion for the Heston density, and then show the validity of a refined volatility expansion. Our methods and results may prove useful beyond the Heston model : the entire analysis is based on affine principles ; at no point do we need knowledge of the (explicit, but cumbersome) closed form expression of the Fourier transform of log-spot. This is joint work with P. Friz, A. Gulisashvili, and S. Sturm.

Option pricing and the cost of risk

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The problem of pricing and hedging options is well understood in the context of the Black-Scholes model. In this model, a perfect hedge is always possible, meaning, there exists a dynamic strategy such that trading in the underlying asset replicates the payoff of the option. However, the possibility of a perfect hedge is restricted to certain models and restrictive assumptions. In more realistic models a perfect hedge is not possible and thus an option bears a residual risk that cannot be hedged away completely. Therefore, pricing an option consist of two parts : the cost of a hedging strategy that reduces the risk, and a premium to cover the residual risk.

As an example, we assume that a trader wishes to minimize the price of a given option. To avoid that she chooses hedging strategies which are too risky, the trader is punished when taking excessive risks. To do so, we introduce an extra capital reserve bank account, which earns a smaller rate of return then a standard deposit bank account. The reserve account should always contain a minimal amount of money, which depends on the residual risk that the trader's portfolio is exposed to. The residual risk is measure by a convex risk measure and the problem leads to a convex optimization problem which can be solved and has an explicit solution.

Implied volatility asymptotics of affine stochastic volatility models with jumps

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In this paper, we propose a unified approach for the implied volatility asymptotic under the general class of affine stochastic volatility models with jumps (which includes one-dimensional Levy processes). Under mild conditions on the jump measures, we derive semi-closed form formulae for the implied volatility as the maturity gets large or small. We also aim at obtaining a classification of jumps for calibration purposes.

Conditional density method in the computation of the delta with application to power market

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We consider assets modelled by a multi-factor dynamics in which we have Brownian motion part and jump part. We study the computation of the delta of options written in such models. For this purpose, we apply the conditional density method in which the knowledge of the density of one factor is enough to derive expressions for the delta not involving any differentiation of the pay-off function. We will also study the robustness to the model and we will give some examples in applications to power and commodity markets, including numerical examples. This talk is based on joint work with : Fred E. Benth and Giulia Di Nunno.

Model independent bounds for variance swaps

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If asset price processes are continuous, the model independent price of a variance swap is equal to twice the price of a log contract. Otherwise, the range of variance swap prices consistent with 'no arbitrage' and call prices is bounded by the prices of variance swaps on two related jump martingales.

Utility maximization with additive habits : Optimal consumption and wealth

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Utility function with additive habits assigned to a financial agent, takes into consideration her addiction to certain levels of past consumption, and constitutes a significant neo-classical preference device, that enables to measure appropriately the satisfaction from consumption. Despite of its extensive recent study, almost nothing is known about the structure of the optimal consumption paths in the setting of incomplete markets and random endowments. We derive a recursive procedure for solving this utility maximization problem and uncover various economical features of the optimal consumption stream, such as monotonicity, concavity and asymptotics for large levels of wealth.

On modelling of electricity spot price

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Energy-related spot prices demonstrate various behaviour characterised by mean-reversion, seasonality effects, regime-switching, spikes and exceptionally high volatility which are far different from standard financial counterparts. In our research we will focus on the electricity market modelling. We conduct an empirical analysis of three recently proposed and widely used models for electricity spot price data, see F. E. Benth, R. Kiesel and A. Nazarova (2010). The jump-diffusion model, proposed by A. Cartea and M. G. Figueroa (2005) and the threshold model, proposed by A. Roncoroni (2002) and further developed by H. Geman and A. Roncoroni (2006) suggest a one-factor Ornstein-Uhlenbeck process driven by a Brownian motion with jumps. The models describe the dynamics of the log-prices, automatically ensuring positive spot prices. The novel twist in the threshold model is twofold. First, the authors introduce a state-dependent sign of the jump component. Secondly, the estimation process makes use of a threshold, which is set iteratively, so that the estimated parameters are calibrated to the empirical kurtosis. The factor model, proposed by F. E. Benth, J. Kallsen and T. Meyer-Brandis (2007) is an additive linear model, where the price dynamics is given as a superposition of Ornstein-Uhlenbeck processes driven by subordinators to ensure positivity of the prices. It separates the modelling of spikes and base components. We calibrate all the models to German spot price data observed at the German power exchange EEX by using various techniques. While calibrating the threshold model, we implement several statistical methods described by H. Geman and A. Roncoroni (2006). Also we apply the prediction-based estimation functions technique suggested by M. Sørensen (2000) to calibrate the factor model, together with filtering techniques for spike identification adopted from T. Meyer-Brandis and P. Tankov (2008). We also use some estimating procedures from A. Cartea and M. G. Figueroa (2005) to calibrate the jump-diffusion model. The contribution of the paper is at first to compare the models of different level of complexity at one data set. At second, to answer the question which model performs better according to chosen criteria. We have a critical view on the estimation and the properties of the three models, point out several deficiencies in all the models, and suggest various improvements.

A general maximum principle for anticipative stochastic control and applications to insider trading

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In this paper we suggest a general stochastic maximum principle for optimal control of anticipating stochastic differential equations driven by a Lévy type of noise. We use techniques of Malliavin calculus and forward integration. We apply our results to study a general optimal portfolio problem of an insider. In particular, we find conditions on the insider information filtration which are sufficient to give the insider an infinite wealth. We also apply the results to find the optimal consumption rate for an insider.

Hedging under arbitrage

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It is shown that delta hedging provides the optimal trading strategy in terms of minimal required initial capital to replicate a given terminal payoff in a continuous-time Markovian context. This holds true in market models where no equivalent local martingale measure exists but only a square-integrable market price of risk. In order to ensure the existence of the delta hedge, sufficient conditions are derived for the necessary differentiability of expectations indexed over the initial market configuration.

Limit theory for heavy-tailed models on a lattice

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Let $\{Z_{i,j} : i, j \in \mathbb{Z}\}$ be i.i.d. random variables where the $Z_{i,j}$'s have regularly varying tail probabilities. By establishing a limit theory for point processes based on $\{Z_{i,j}\}$, we explore the extremal behavior of several lattice models based on $\{Z_{i,j}\}$, e.g. spatial averages $\{X_{i,j} := \sum_{k,l\in\mathbb{Z}} c_{k,l} Z_{i+k,j+l} : (i, j) \in \mathbb{Z}^2\}$ and moving maxima $\{Y_{i,j} := \bigvee_{k,l\in\mathbb{Z}} \varphi_{k,l} Z_{i+k,j+l} : (i, j) \in \mathbb{Z}^2\}$. By establishing a limit theory for point processes based on $\{X_{i,j}\}$ and $\{Y_{i,j}\}$ we obtain approximations for probabilities of extremal events. The theory builds on results of Davis and Resnick for extremes of linear processes with heavy-tailed distributions.

The distribution of portfolio payoffs and increases in risk aversion

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Extending the work of Phil Dybvig and Yajun Wang 2009, we consider a risk averse agent A, a risk loving agent L and their optimal payoff X_A, resp. X_L for the maximal expected utility problem in a semimartingale market. In complete markets one can prove that X_L stochastically dominates X_A, which in turn is equivalent to X_L being distributed as X_A plus a nonngegative random variable Z ("risk premium") plus a noise term ϵ. If additionally at least one agent A, L has non increasing risk aversion, the result in fact holds with constant $Z = \mathbb{E}[X_L - X_A] \geq 0$. Numerical evidence suggests that this phenomenon still holds in incomplete markets with one risk-free asset, and we discuss its validity in stochastic volatility models.
This is work in progress (jointly with Mathias Beiglböck and Johannes Muhle-Karbe).

A direct proof of the Bichteler-Dellacherie theorem and connections to arbitrage

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We give an elementary proof of the celebrated Bichteler-Dellacherie Theorem which states that the class of stochastic processes S allowing for a useful integration theory consists precisely of those processes which can be written in the form $S = M + A$, where M is a local martingale and A is a finite variation process, i.e., which are semi-martingales. Our argument does not rely on the continuous time Doob-Meyer decomposition nor any change of measure techniques. Instead we succeed to construct the desired representation rather directly from a discrete time Doob-Meyer decomposition. By passing to convex combinations we obtain a direct construction of the continuous time Doob-Meyer decomposition, which then yields the desired decomposition. As a by-product of our proof we obtain a characterization of semi-martingales in terms of a variant of *no free lunch with vanishing risk*, thus extending a result of Delbaen and Schachermayer (1994). This is a joint work with Mathias Beiglböck and Walter Schachermayer.

Multilevel path simulation for jump diffusion SDEs

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We investigate the extension of the multilevel Monte Carlo path simulation method (Giles 07, 08) to jump-diffusion SDEs. The first part considers models with finite rate activity (e.g. Merton 76), using a jump-adapted discretisation in which the jump times are computed and added to the standard timestepping discretisation times. The key component in multilevel analysis is the calculation of an expected payoff difference between a coarse path simulation and a fine path simulation with twice as many timesteps. If the Poisson jump rate is constant, the jump times are the same on both paths and the multilevel extension is relatively straightforward, but the implementation is more complex in the case of path-dependent jump rates for which the jump times naturally differ (Glasserman and Merener 04).

The second part we consider the variance gamma model (Madan and Seneta 90) as an example of a jump diffusion process with infinite activity. Here the benefits of the multilevel method arise in applications with path-dependent payoffs such as lookback and barrier options, which requires approximate interpolation of the numerical solution within each timestep.