On modelling of electricity spot price

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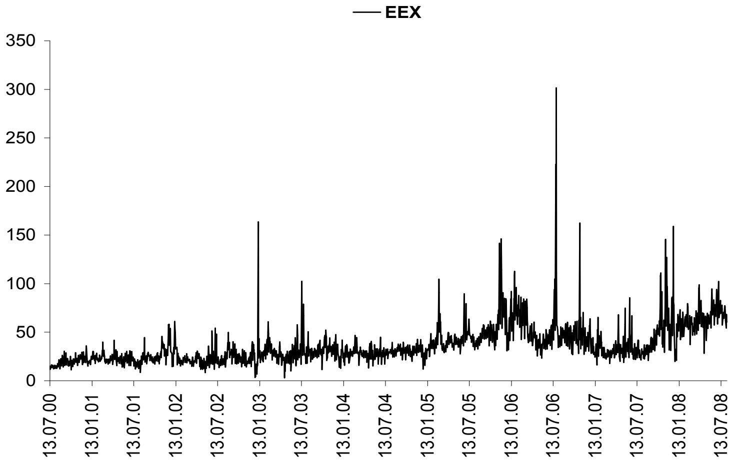
Problems

Spot prices demonstrate such typical features as:

- *seasonality*: daily, weekly, monthly;
- *mean-reversion* or *stationarity*;
- *spikes*: may occur with some seasonal intensity;
- *high volatility*.

For our empirical analysis we use a data set of the Phelix Base electricity price index at the European Power Exchange (EEX).

EEX electricity spot price dynamics



Basics of the models

1. **Jump-diffusion model**, [1]. It proposes a one-factor mean-reversion jump-diffusion model, adjusted to incorporate seasonality effects.
2. **Threshold model**, [9] and [6]. It represents an exponential Ornstein-Uhlenbeck process driven by a Brownian motion and a state-dependent compound Poisson process.
3. **Factor model**, [3]. It is an additive linear model, where the price dynamics is a superposition of Ornstein-Uhlenbeck processes driven by subordinators to ensure positivity of the prices.

Basics of the models

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a filtered probability space:

- time horizon $t = 0, \ldots, T$ is fixed;
- in general, electricity spot price at time $0 \leq t \leq T$ by $S(t)$ takes the form:

$$S(t) = e^{\mu(t)} X(t)\,; \tag{1}$$

- $\mu(t)$ is a deterministic function modelling the seasonal trend;
- $X(t)$ is some stochastic process modelling the random fluctuation.

Basics of the models

Spot prices may vary with seasons:

$$\mu(t) = \alpha + \beta t + \gamma \cos(\epsilon + 2\pi t) + \delta \cos(\zeta + 4\pi t)\,, \qquad (2)$$

where the parameters $\alpha, \beta, \gamma, \delta$, ϵ and ζ are all constants:

- α is fixed cost linked to the power production;
- β drives the long run linear trend in the total production cost;
- γ, δ, ϵ and ζ construct periodicity by adding two maxima per year with possibly different magnitude.

Specification $X(t)$ for the jump-diffusion model

$$S(t) = e^{\mu(t)} X(t)\,,$$
$$d\ln X(t) = -\alpha \ln X(t)\, dt + \sigma(t)\, dW(t) + lnJ\, dq(t)\,,$$

- α is one mean-reversion parameter;
- $\sigma(t)$ is the time-dependent volatility;
- J is the proportional random jump size, $\ln J \sim N(\mu_j\,, \sigma_j^2)$;
- $dq(t)$ is a Poisson process such that:

$$dq(t) = \begin{cases} 1, \text{ with probability} & ldt \\ 0, \text{ with probability} & 1 - ldt, \end{cases}$$

- l is the intensity or frequency of spikes.

Specification $X(t)$ for the threshold model

$$S(t) = e^{\mu(t)} X(t)\,,$$

$$d \ln X(t) = -\theta_1 \ln X(t)\, dt + \sigma\, dW(t) + h(\ln(X(t-)))\, dJ(t)\,, \quad (3)$$

- θ_1 is one mean-reversion parameter, positive constant;
- σ is Brownian volatility parameter, positive constant.

The Brownian component models the normal random variations of the electricity price around its mean, i.e., *the base signal*.

Specification $X(t)$ for the threshold model

$$S(t) = e^{\mu(t)} X(t)\,,$$

$$d \ln X(t) = -\theta_1 \ln X(t)\, dt + \sigma\, dW(t) + h(\ln(X(t-)))\, dJ(t)\,,$$

where J is a time-inhomogeneous compound Poisson process:

$$J(t) = \sum_{i=1}^{N(t)} J_i\,,$$

and $N(t)$ counts the spikes up to time t and is a Poisson process with time-dependent jump intensity.
$J_1, J_2, \ldots$ model the magnitude of the spikes and are assumed to be i.i.d. random variables.
The function h attains two values, ± 1, indicating the direction of the jump.

Specification $X(t)$ for the factor model

$$S(t) = e^{\mu(t)} X(t) \,,$$

where $X(t)$ is a stochastic process represented as a weighted sum of n independent non-Gaussian Ornstein-Uhlenbeck processes $Y_i(t)$

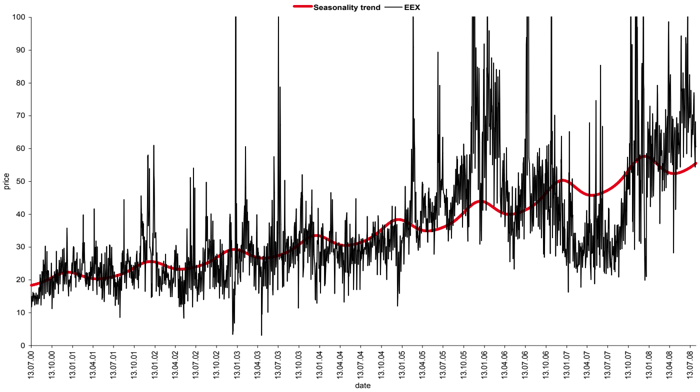
$$X(t) = \sum_{i=1}^{n} w_i Y_i(t) \,, \tag{4}$$

where each $Y_i(t)$ is defined as

$$dY_i(t) = -\lambda_i Y_i(t) dt + dL_i(t) \,, \;\; Y_i(0) = y_i, i = 1, \ldots, n \,. \tag{5}$$

w_i are weighted functions; λ_i are mean-reversion coefficients; $L_i(t)$, $t = 1, \ldots n$ are independent *càdlàg* pure-jump additive processes with increasing paths.

Seasonality function $\mu(t)$ is common for both the factor and the threshold model. The method of *non-linear least squares* (OLS) is used for estimating the parameters.



Estimation parameters

- seasonality function, mean-reversion α
- filtering spikes characteristics: size distribution and frequency
- rolling historical volatility $\sigma(t)$

Estimation parameters

- mean-reversion speed α is estimated by using linear regression, i.e. in the discrete version representation:

$$x_t = \theta_t + \beta x_{t-1} + \eta_t, \tag{6}$$

  - θ_t is the mean-reverting level;
  - β is the modified mean-reversion speed;
  - η is the Brownian motion and jumps.

Estimation parameters

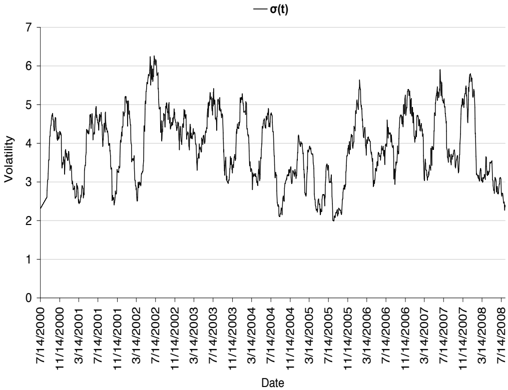
- extraction of the spikes from the original series of returns by simple iterative procedure;
- procedure repeats as long as no more outliers are filtered out;
- it gives the standard deviation of the jumps σ_j and the cumulative frequency of jumps l;

Estimation parameters

- rolling historical volatility is taken from *Eydeland and Wolyniec* [5]. $m = 30$ days, i.e. the width of the window:

$$\sigma(t_k) = \sqrt{\frac{1}{m-1}\sum_{i=k-m+1}^{k}\left(\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \sum_{i=k-m+1}^{k}\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}}\right)^2} \quad (7)$$

Estimation parameters



Estimation of the model parameters

- jump threshold Γ is set to filter out the jump and continuous paths.
- then we estimate:
  - θ_1 - the smooth mean-reversion force;
  - θ_2 - the maximal expected number of jumps;
  - θ_3 - the reciprocal average jump size;
  - σ - the Brownian local volatility.

Estimation of the model parameters

The approximative logarithmic likelihood function is constructed:

$$\begin{aligned}\mathcal{L}(\Theta \mid \Theta^0, E) \quad = \quad & \sum_{i=0}^{n-1} \frac{(\mu(t_i) - E_i)\theta_1}{\sigma^2} \Delta E_i^c - \frac{\Delta t}{2} \sum_{i=0}^{n-1} \left(\frac{(\mu - E_i)\theta_1}{\sigma} \right)^2 \\ & -(\theta_2 - 1) \sum_{i=0}^{n-1} s(t_i)\Delta t + N(t) \ln \theta_2 \\ & + \sum_{i=0}^{n-1} \left[-(\theta_3 - 1) \frac{\Delta E_i^d}{h(E_i)} \right] + N(t) \ln \left(\frac{1 - e^{-\theta_3 \psi}}{\theta_3 (1 - e^{-\psi})} \right), \end{aligned} \tag{8}$$

it is possible to split it up into three independent parts and maximize them separately:

$$L(\Theta \mid \Theta^0, E) = F_1(\theta_1) + F_2(\theta_2) + F_3(\theta_3). \tag{9}$$

Procedure includes:

- deseasonalization
- identification the number of OU processes or factors involved
- filtering of the spike process and the base signal
- estimating of the base signal parameters
- analysis of the spike process

Assessment the number of factors required

Compare in the L^2 norm the empirical autocorrelation function (ACF) with theoretical one $\rho(k)$, see *Barndorf-Nielsen and Shephard*[2]:

$$\rho(k) = \widetilde{w}_1 e^{-k\lambda_1} + \widetilde{w}_2 e^{-k\lambda_2} + \cdots + \widetilde{w}_n e^{-\lambda_n}\,, \tag{10}$$

where k is a lag number; $\widetilde{w}_i$ are positive constants summing up to 1; λ_i are mean-reversion parameters. The larger λ, the faster the process comes back to its mean level, therefore it refers to the spike mean-reversion speed.

We obtained $n = 2$ as the optimal number of factors: one for spike and one for base signal.

Hard thresholding procedure

- is taken from Extreme Value Theory and helps to filter out the spikes;
- uses the methods from non-parametric statistics and provides as output both the base signal and the spike process;
- is reliable in the context of return distribution characteristics;

For details see *Meyer-Brandis and Tankov* [7] and *Nazarova* [8].

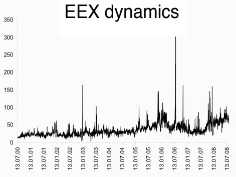
Base signal parameters estimation

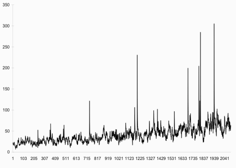
- The main task here is to find the so-called *background driving Lévy process* $L_1(t)$ such that OU $Y_1(t)$ process has the same stationary distribution. A reasonable choice is the Gamma distribution, which is motivated by the opportunity to obtain an explicit analytical expression for the moments, otherwise compute them numerically, see *Barndorf-Nielsen and Shephard* [2].
- Evaluation of parameters of Gamma distribution is done by implementing a prediction-based estimating functions method developed by *Sørensen* [10] and *Bibby et al.* [4].

Prediction-based estimating functions method

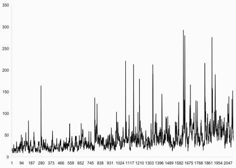
The method is closely related to the method of prediction error estimation that is used in the stochastic control literature. Here the method is applied to sums of Ornstein-Uhlenbeck processes. The estimating functions are based on predictors of functions of the observed process. We focus on a finite-dimensional space of predictors. For the optimal estimating functions this allows one to only involve unconditional moments.

Simulations comparison

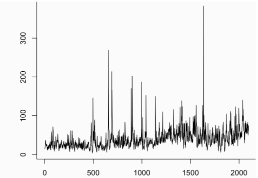




Factor model



Threshold model



Jump-diffusion model

Method of moments

How do we assess the performance of the models?

- by simulating calibrated models dynamics;
- by computing the returns and their descriptive statistics;
- by comparing the moments of the returns with empirical ones.

Method of moments

Tabelle: Comparative descriptive statistics results for the threshold, factor and jump-diffusion models.

Moment	Average	Std. Dev	Skewness	Kurtosis
EEX	**0.0006**	**0.2985**	**0.4050**	**6.6179**
Jump-diffusion model (Normal)	0.0007	0.3191	0.8343	10.3935
Threshold model (trunc. exp)	0.0006	0.2935	0.8336	5.9783
Factor model (Pareto)	0.0006	0.1595	1.6749	10.5308
Modified threshold model (Gamma)	0.0006	0.2822	0.5566	2.9946
Modified factor model (Gamma)	0.0006	0.1465	1.2414	5.7399

General concluding remarks

- We have analysed and discussed the empirical performance of three continuous-time electricity spot price models.
- Further investigation on the derivatives pricing.

Thank you for your attention!

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