Black-Box Optimization (Search)

Minimize an objective function (also: cost, loss, error, or fitness function)

$$f: \mathcal{X} \subset \mathbb{R}^n \to \mathbb{R}, \quad x \mapsto f(x)$$

in a black-box scenario (direct search, no gradients)

$$x \longrightarrow \int f(x)$$

where the black box can be

- non-linear, non-convex, discontinuous, dynamic, stochastic
- from milli-seconds to hours to evaluate

Objective:

- convergence to a global essential infimum of f as fast as possible
- (informally, time-finite) find $x \in \mathcal{X}$ with small f(x) value using as few back-box calls (function evaluations) as possible

Why Do We Need to Measure Performance?

- putting algorithms to a standardized test
 - simplify judgement
 - simplify comparison
 - regression test/quality check under algorithm changes
- algorithm selection

understanding of algorithms

How do we measure performance?

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Birds View

We can measure performance on

- real world problems
 - expensive
 - comparison is typically limited to certain domains
 - experts have limited interest to publish
- "artificial" benchmark functions
 - cheap
 - data acquisition is comparatively easy
 - problem of representativity
- caveat: parameter of algorithms

Measuring Performance

...empirically...

- convergence graphs is all we have to start with
- the right presentation cannot be overestimated

the details are important

Displaying Three Runs (three trials)



not like this (it's unfortunately a common picture)

Displaying Three Runs (three trials)



better like this (shown are the same data), caveat: fails with negative f-values

Displaying Three Runs (three trials)



even better like this: subtract minimum value over all runs

Displaying 51 Runs



observation: three different "modes", which would be difficult to represent or recover in single statistics

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mean/average function value

• tends to emphasize large values



geometric average function value $exp(mean_i(log(f_i))) = (\prod_{i=1}^N f_i)^{1/N}$

- reflects "visual" average
- depends on offset
- artefact due to adding 1e-11

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average iterations

- reflects "visual" average
- here: incomplete



- unique for uneven number of data
- independent of log-scale, offset...

median(log(data))=log(median(data))

same when taken over x- or y-direction

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Implication

- use the median as summary datum
- more general: use quantiles as summary data

for example out of 15 data: 2nd, 8th, and 14th value represent the 10%, 50%, and 90%-tile

unless there are good reasons for a different statistic

Examples





Comparison of 4 algorithms using the "median run" and the 90% central range of the final value on two different functions (Ellipsoid and Rastrigin)

caveat: this range display with simple error bars fails, if, e.g., 30% of all runs "converge"

Examples: Plotting All Data



Statistical Assessment

- Don't be scared!
- 1) Assess the meaning/relevance of a difference first (the only difficult part)
- 2) Apply rank-sum test (Wilcoxon, Mann-Whitney U)
 - only assumption: no equal data values as usual: useful even if assumptions do not hold, for categorical data: χ^2 -test
 - hypothesis: $p(x > y) \neq p(x < y) \neq 1/2$
 - compares sum of ranks in a combined ranking
 - two-sided 1%-significance *p*-value needs only 2x5 data values
- For the same p-value, fewer significant data are better

using enough data, *any* difference can be made significant

Generally: non-parametric tests, Kolmogorov-Smirnov test for ECDFs, no need to use the t-test

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Performance Measure(s)

Runtime

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Three Convergence Graphs



recall: convergence graphs is all we have

(recall) Black-Box Optimization

Two objectives:

- Find solution with small(est possible) function value
- With the least possible search costs (number of function evaluations)
- For measuring performance: fix one and measure the other

Two objectives



convergence graph is a plot in objective space

fixed-cost versus fixed-target



number of function evaluations (time)

fixed-cost versus fixed-target



number of function evaluations (time)

fixed-cost versus fixed-target



number of function evaluations (time)

Evaluation of Search Algorithms Behind the scene

a performance should be

- quantitative on the ratio scale (highest possible)
 - "algorithm A is two *times* better than algorithm B" is a meaningful statement
- can assume a wide range of values
- meaningful (interpretable) with regard to the real world possible to transfer from benchmarking to real world

runtime or first hitting time is the prime candidate (we don't have many choices anyway)

The performance measure we use

Run length or runtime or first hitting time to a given target function value measured in number of fitness function evaluations

equivalent to first hitting time of a sublevel set in search space

How can we deal with "missing values"?

fixed-cost versus fixed-target



number of function evaluations (time)

Fixed-target: Measuring Runtime

- 1. Fix a target *f*-value (most difficult part)
- 2. Compute the success rate \hat{p} as

 $\widehat{p} = \frac{\text{\# of successful runs (that reached the target)}}{\text{\# of all runs}} \in [0, 1]$ $\widehat{R} = \frac{1 - \widehat{p}}{\widehat{p}} = \frac{\text{\# of unsuccessful runs}}{\text{\# of successful runs}} \in [0, \infty]$

- \widehat{R} is the odds ratio to be unsuccessful
- \widehat{R} is the number of unsuccessful runs observed *for each single successful* run (i.e. normalized by # of successful runs)

Fixed-target: Measuring Runtime

 $\widehat{R} = \frac{1 - \widehat{p}}{\widehat{p}} = \frac{\text{\# of unsuccessful runs}}{\text{\# of successful runs}} \in [0, \infty]$

\widehat{R} is the odds ratio to be unsuccessful \widehat{R} is the number of unsuccessful runs observed *for each single successful* run (i.e. normalized by # of successful runs)

3. Compute "expected runtime" to hit the target

average runtime for a single successful run

$$\mathsf{ERT} := \overline{\mathsf{RT}}_{\mathrm{succ}} + \widehat{R} \times \overline{\mathsf{RT}}_{\mathrm{unsucc}}$$

average runtime spent in unsuccessful runs to achieve one successful run

- $\begin{array}{l} \mathrm{SP1} \ := \overline{\mathsf{RT}}_{\mathrm{succ}} + \widehat{R} \times \overline{\mathsf{RT}}_{\mathrm{succ}} = \overline{\mathsf{RT}}_{\mathrm{succ}}(1 + \widehat{R}) \text{ disregarding} \\ \text{ runlength of unsuccessful runs} \end{array}$
- if $\widehat{R} < \infty$, else we can assume $\mathsf{ERT} \ge \sum \mathsf{RT}_{unsucc}$

Fixed-target: Measuring Runtime

 \widehat{R} is the number of unsuccessful runs observed *for each single successful* run (i.e. normalized by # of successful runs)

3. Compute "expected runtime" to hit the target

average runtime for a single successful run

 $\mathsf{ERT} := \overbrace{\mathsf{RT}}^{}_{\mathrm{succ}} + \underbrace{\widehat{R} \times \overline{\mathsf{RT}}}_{\mathrm{unsucc}}$

average runtime spent in unsuccessful runs to achieve one successful run

 $\begin{array}{l} \mathrm{SP1} \ := \overline{\mathsf{RT}}_{\mathrm{succ}} + \widehat{R} \times \overline{\mathsf{RT}}_{\mathrm{succ}} = \overline{\mathsf{RT}}_{\mathrm{succ}}(1 + \widehat{R}) \text{ disregarding} \\ \text{ runlength of unsuccessful runs} \end{array}$

We can simulate a single runtime by "restarting" until the first success

$$\mathsf{RT} = \mathsf{RT}_{\mathrm{succ}} + \sum \mathsf{RT}_{\mathrm{unsucc}}$$

 \Longrightarrow distribution of runtimes incorporating unsuccessful runs \Longrightarrow display the distribution or a statistic of it

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Break

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Summary

- plot carefully
- display all data
- use the median as summary datum

unless for runtimes or you know exactly what you do

- more general: use quantiles as summary data
- assess a performance difference *before* to worry about statistical significance
- vertical vs. horizontal view-point
- run"time" RT and
 - ERT (expected RT)
 - runtime ECDF (empirical cumulative distribution fct)





ECDF:

Empirical Cumulative Distribution Function of the Runtime
A Convergence Graph



First hitting time is monotonous



 first hitting time: a monotonous graph

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another convergence graph



another convergence graph with hitting time



a target value delivers two data points



ECDF with four data points









50 equally spaced targets





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15 runs



15 runs





15 runs



15 runs 50 targets

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Performance Evaluation of Anytime Black Box Optimizers



15 runs 50 targets



15 runs 50 targets ECDF with 750 steps



15 runs integrated in a single graph

Target budgets/run-lengths



1) define reference target budgets

Target budgets on the reference algorithm



1) define reference target budgets

2) compute best function value achieved by a reference algorithm

Target budgets on the reference algorithm



1) define reference target budgets

2) compute best function value achieved by a reference algorithm

Run-length based target f-values



Run-length based target *f*-values



Example for ECDFs



Empirical cumulative distribution functions (ECDFs) of running lengths (left) and function values (right)

Overview results 2012



Results of 2012 (20-D)



Results of 2010 (20-D)



Results of 2009 (20-D)



ECDF: Summary

Empirical Cumulative Distribution Functions

- recover a single convergence graph (and generalize)
- can aggregate over any set of functions and target values

they display a set of run lengths or runtimes (RT)

- for RT on a single problem (function & target value) allow to estimate any statistics of interest from them, like median, expectation (ERT),... in a meaningful way
- AKA data profile [Moré&Wild 2009]
- Performance profile [Dolan&Moré 2002]: ECDFs of run lengths divided by the smallest observed run length

Different Displays of Runtimes

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Performance Evaluation of Anytime Black Box Optimizers

Scaling Behaviour with Dimension



- slanted grid lines: quadratic scaling
- horizontal lines: linear scaling
- light brown: artificial best 2009

Example: Scaling Behaviour



- slanted grid lines: quadratic scaling
- horizontal lines:
 linear scaling
- light brown: artificial best 2009

Experiments in >40-D are more often than not virtually superfluous
ERT scatter plots, all dimensions&targets



- estimated Expected Run Time (ERT), two algorithms
- 2-10 D: first algorithm "dominates"
- 20 & 40 D: second algorithm "dominates"

ERT scatter plots, all dimensions&targets



Single Function Table

Table 6: 20-D, running time excess ERT/ERT_{best} on f_6 , in italics is given the median final function value and the median number of function evaluations to reach this value divided by dimension

					6 Attract	tive sector	•				
Δ ftarget	1e + 03	1e + 02	1e+01	1e + 00	1e-01	1e-02	1e-03	1e-04	1e-05	1e-07	∆ftarget
ERThest/D	4.03	26	64.7	87.2	123	152	184	219	248	309	ERThest/D
ALPS	59	25	34	54	64	78	100	150	370	14e-7/2e5	ALPS [17]
AMaLGaM IDEA	26	22	19	22	21	22	22	21	22	22	AMaLGaM IDEA [4]
avg NEWUOA	2.3	1.1	1	1	1	1	1	1	1	1	avg NEWUOA [31]
BayEDAcG	46	41	60e+0/2e3								BayEDAcG [10]
BFGS	2.2	2.7	3.6	4.7	4.7	4.9	5	4.8	4.9	61	BFGS [30]
Cauchy EDA	6200	1500	1e3	1700	17e-1/5e4						Cauchy EDA [24]
BIPOP-CMA-ES	2.9	2.2	1.5	1.7	1.6	1.6	1.6	1.5	1.6	1.6	BIPOP-CMA-ES [15]
(1+1)-CMA-ES	1.9	4.5	13	180	1200	13e-1/1e4					(1+1)-CMA-ES [2]
DASA	12	6.8	9.9	19	25	33	49	58	63	74	DASA [19]
DEPSO	11	7.5	12	64	13e-1/2e3						DEPSO [12]
DIRECT	18	31	40e+0/5e3			-	-	-			DIRECT [25]
EDA-PSO	27	46	40	45	44	44	44	44	44	44	EDA-PSO [6]
full NEWUOA	5	1.9	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.4	full NEWUOA [31]
G3-PCX	4.1	1.4	1.4	2	2.1	2.1	2.2	2.2	2.3	2.4	G3-PCX [26]
simple GA	320	130	2e3	11e + 0/1e5		1.4		-	-		simple GA [22]
GLOBAL	5	2.9	3.6	4.9	8.5	42e-3/2e3					GLOBAL [23]
iAMaLGaM IDEA	5.1	5.6	5.4	6.8	7.1	7.7	7.8	7.7	8	8.3	iAMaLGaM IDEA [4]
LSfminbnd	9	31	160	760	1100	960	72e-1/1e4				LSfminbnd [28]
LSstep	140	260	2300	59e+0/1e4							LSstep [28]
MA-LS-Chain	11	4.9	7.5	8.9	8	7.7	7.2	6.7	6.5	6	MA-LS-Chain [21]
MCS (Neum)	1.8	33	42e+0/4e3								MCS (Neum) [18]
NELDER (Han)	2.2	2.4	2.7	3.3	3.2	3.5	3.5	3.5	4	7.4	NELDER (Han) [16]
NELDER (Doe)	1.5	2.3	9.1	20	28	65	110	430	46e-5/2e4		NELDER (Doe) [5]
NEWUOA	1	1	1	1.3	1.4	1.5	1.6	1.6	1.7	1.7	NEWUOA [31]
(1+1)-ES	2	2.2	2.1	2.8	3.9	5.2	6.1	6.5	6.4	6.7	(1+1)-ES [1]
POEMS	89	26	31	37	36	36	36	35	36	37	POEMS [20]
PSO	6.4	280	1100	1400	980	820	710	620	570	790	PSO [7]
PSO_Bounds	9.5	45	120	150	140	140	140	130	160	220	PSO_Bounds [8]
Monte Carlo	2.4e5	48e + 1/1e6									Monte Carlo [3]
Rosenbrock	2.1	3.9	31	76	210	230	810	21e-2/1e4			Rosenbrock [27]
IPOP-SEP-CMA-ES	3.2	2.1	1.7	1.9	1.9	1.9	1.9	1.9	2	2	IPOP-SEP-CMA-ES [29]
VNS (Garcia)	5	2.8	1.9	1.9	1.7	1.7	1.7	1.6	1.6	1.6	VNS (Garcia) [11]

Questions?

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Python

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Python

- a general-purpose, well-designed, modern high-level programming language
- dynamically-typed, highly object-oriented (not enforced), highly modularized
- for scripting, for programming, for interactive usage
- comes with thousands of packages
- the Python programming language is much better designed than Matlab/Octave
- IPython can replace Matlab/Octave for interactive usage
- (I)Python is free and available on almost every computer

Popularity of Programming Languages (TIOBE)

← → C 🏫 🗋 www.tiobe.com/index.php/content/paperinfo/tpci/index.... 🔂 🤆

Position May 2013	Position May 2012	Delta in Position	Programming Language	Ratings May 2013	Delta May 2012	
1	1	=	С	18.729%	+1.38%	
2	2	=	Java	16.914%	+0.31%	
3	4	t	Objective-C	10.428%	+2.12%	
4	3	Ļ	C++	9.198%	-0.63%	
5	5	=	C#	6.119%	-0.70%	
6	6	=	PHP	5.784%	+0.07%	
7	7	=	(Visual) Basic	4.656%	-0.80%	
8	8	=	Python	4.322%	+0.50%	
9	9	=	Perl	2.276%	-0.53%	
10	11	t	Ruby	1.670%	+0.22%	
11	10	Ļ	JavaScript	1.536%	-0.60%	
12	12	=	Visual Basic .NET	1.131%	-0.14%	
20	22	tt	MATLAB	0.563%	0.00%	
24	Nikolaus Ha	nsen	R Pertormance Evaluation	0.480%	Black Roy (Intimizer

BBOB with COCO in practice (for dummies)

COCO (COmparing Continuous Optimizers): a tool for black-box optimization benchmarking

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Bugs in version 11.05:	bbob-2010-results

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Matlab script (exampleexperiment.m):

```
dimensions = [2, 3, 5, 10, 20, 40]; % small dimensions first, for CPU reasons
functions = benchmarks('FunctionIndices'); % or benchmarksnoisy(...)-
instances = [1:5, 31:40]; % 15 function instances
for dim = dimensions-
  for ifun = functions \neg
    for iinstance = instances\neg
      fgeneric('initialize', ifun, iinstance, datapath, opt); -
     MY_OPTIMIZER('fgeneric', dim, fgeneric('ftarget'), eval(maxfunevals) - f
     disp(sprintf([' f%d in %d-D, instance %d: FEs=%d with %d restarts, fbes
     fgeneric('finalize');-
   end-
   disp([' date and time: ' num2str(clock, ' %.0f')]);¬
 end-
 disp(sprintf('---- dimension %d-D done ----', dim));-
end-
```

Running the experiment at an OS shell:

\$ nohup nice octave < exampleexperiment.m > output.txt &
\$ less output.txt

GNU Octave, version 3.6.3 Copyright (C) 2012 John W. Eaton and others. This is free software; see the source code for copying conditions. [...] Read http://www.octave.org/bugs.html to learn how to submit bug reports.

For information about changes from previous versions, type `news'.

```
f1 in 2-D, instance 1: FEs=242, fbest-ftarget=-8.1485e-10, elapsed time [h]: 0.00
f1 in 2-D, instance 2: FEs=278, fbest-ftarget=-6.0931e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 3: FEs=242, fbest-ftarget=-9.2281e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 4: FEs=302, fbest-ftarget=-4.5997e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 5: FEs=230, fbest-ftarget=-9.8350e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 6: FEs=284, fbest-ftarget=-7.0829e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 7: FEs=278, fbest-ftarget=-6.5999e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 8: FEs=272, fbest-ftarget=-8.7044e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 9: FEs=248, fbest-ftarget=-2.6316e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 10: FEs=302, fbest-ftarget=-4.6779e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 11: FEs=272, fbest-ftarget=-5.1499e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 12: FEs=260, fbest-ftarget=-8.8635e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 13: FEs=266, fbest-ftarget=-2.5484e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 14: FEs=218, fbest-ftarget=-9.9961e-09, elapsed time [h]: 0.00
f1 in 2-D, instance 15: FEs=248, fbest-ftarget=-7.5842e-09, elapsed time [h]: 0.00
   date and time: 2013 3 29 19 59 26
f2 in 2-D, instance 1: FEs=824, fbest-ftarget=-7.0206e-09, elapsed time [h]: 0.00
f2 in 2-D, instance 2: FEs=572, fbest-ftarget=-9.2822e-09, elapsed time [h]: 0.00
```

. 1

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Post-processing at the OS shell:

\$ python codepath/bbob_pproc/rungeneric.py datapath
[...]

\$ pdflatex templateACMarticle.tex

[...]



Post-processing at the OS shell:

\$ python codepath/bbob_pproc/rungeneric.py datapath
[...]

\$ pdflatex templateACMarticle.tex

[...]



Black-Box Optimization Benchmarking Template for Noiseless Function Testbed

Draft version *

Forename Name

ABSTRACT

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation

1. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1 and 2 and in Table 1.

*Camera-ready paper due April 17th.

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GECCO'09, July 8–12, 2009, Montréal Québec, Canada. Copyright 2009 ACM 978-1-60558-505-5/09/07 ...\$5.00.



Figure 1: Expected Running Time (ERT, •) to reach $f_{opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-3}, 10^{-5}, 10^{-5}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The ERT(Δf) equals to $\#FEs(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{opt} + \Delta f$ was surpassed during the trial. The $\#FEs(\Delta f)$ are the total number of function evaluations while $f_{opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of function $\#FEs(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials, and quadratic scaling.

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Δf f1 in 5-D, N=5, mFE=883 f1 in 20-D, N=5, mFE=2991 Δf $\#$ ERT 10% 90% RT _{ence} $\#$ ERT 10% 90% RT _{ence}	f_2 in 5-D, N=5, mFE=2467 Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
10 5 60el 4.8el 7.3el 6.0el 5 3.4e2 3.0e2 3.8e2 3.4e2	10 5 1.2e3 1.1e3 1.4e3 1.2e3 5 1.4e4 1.4e4 1.5e4 1.4e4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1 5 1.3e3 1.2e3 1.5e3 1.3e3 5 1.5e4 1.5e4 1.5e4 1.5e4 1.5e4 1.5e4 1.6e4 1.5e4
1e-3 5 $3.8a2$ $3.6a2$ $4.1a2$ $3.8a2$ 5 $1.4a3$ $1.4a3$ $1.5a3$ $1.4a3$	1e-3 5 $1.7e3$ $1.6e3$ $1.8e3$ $1.7e3$ 5 $1.8e4$ $1.7e4$ $1.8e4$ $1.8e4$
1e-5 5 $5.4e2$ 5. $1e2$ 5. $7e2$ 5 $4e2$ 5 $2.0e3$ 1. $9e3$ 2. $1e3$ 2. $0e31e-8$ 5 $7.9e2$ 7. $6e2$ 8. $2e2$ 7. $9e2$ 5 $2.9e3$ 2. $7e3$ 2. $9e3$ 2. $9e3$	le=5 5 1.9e3 1.8e3 2.0e3 1.9e3 5 1.8e4 1.8e4 1.9e4 1.8e4 le=8 5 2.1e3 2.0e3 2.2e3 2.1e3 5 1.9e4 1.9e4 2.0e4 1.9e4
f3 in 5-D, N=5, mFE=1.83e6 f3 in 20-D, N=5, mFE=6.74e6	f4 in 5-D, N=5, mFE=1.89e6 f4 in 20-D, N=5, mFE=5.73e6
Δf # ERT 10% 90% RT _{SIEC} # ERT 10% 90% RT _{SIEC}	$\Delta f = \pm \text{RET} = 10\% = 90\% = \text{RT}_{\text{SBCC}} = \pm \pm \text{RET} = 10\% = 90\% = \text{RT}_{\text{SBCC}}$
1 5 7.8e3 4.6e3 1.1e4 7.8e3 0 40e-J 30e-J 60e-J 2.5e5	1 1 7.1e6 1.2e6 >7e6 7.8e2 0 J3e+0 99e-J J4e+0 6.8e5
le -1 4 5.9e5 1.1e5 1.4e5 1.3e5	1e - 1 0 $20e - 1$ $99e - 2$ $27e - 1$ $3.3e5$
le-5 4 5.9e5 53e4 1.4e5 1.3e5	10-5
10-8 4 5.945 9.044 1.465 1.465 [] f5 in 5-D, N=5, mFE=83 [f5 in 20-D, N=5, mFE=327	20-8 f6 in 5-D, N=5, mFE=2243 f6 in 20-D, N=5, mFE=12651
Δf # ERT 10% 90% RT succ # ERT 10% 90% RT succ	$\Delta f \# \text{ERT} = 10\% - 90\% \text{RT}_{RBCC} \# \text{ERT} = 10\% - 90\% \text{RT}_{RBCC}$
10 5 4.3 e1 3.6 e1 5.0 e1 4.3 e1 5 2.1 e2 1.9 e2 2.3 e2 2.1 e2 1 5 6.3 e1 5.3 e1 7.2 e1 6.3 e1 5 2.6 e2 2.4 e2 2.8 e2 2.6 e2	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
1e-1 5 63e1 5.3e1 7.0e1 6.3e1 5 2.7e2 2.5e2 2.9e2 2.7e2	1e-1 5 5.9 <i>e</i> 2 5.6 <i>e</i> 2 6.2 <i>e</i> 2 5.9 <i>e</i> 2 5 4.3 <i>e</i> 3 3.8 <i>e</i> 3 4.7 <i>e</i> 3 4.3 <i>e</i> 3
1e-3 5 63e1 5.4e1 7.2e1 63e1 5 2.7e2 2.5e2 2.9e2 2.7e2 1e-5 5 63e1 5.8e1 7.3e1 63e1 5 2.7e2 2.5e2 2.9e2 2.7e2	le-3 5 9.0e2 8.2e2 9.7e2 9.0e2 5 6.2e3 5.8e3 6.7e3 6.2e3 le-5 5 1.3e3 1.2e3 1.4e3 1.3e3 5 8.1e3 7.7e3 8.5e3 8.1e3
1e-8 5 6.3e1 5.8e1 7.0e1 6.3e1 5 2.7e2 2.4e2 2.9e2 2.7e2	le-8 5 1.9e3 1.7e3 2.0e3 1.9e3 5 1.1e4 1.1e4 1.2e4 1.1e4
$\Delta f = 10\% - 10\% $	$\Delta f = \text{ERT} = 10\% = 90\% = \text{RT}_{\text{succ}} = \# = \text{ERT} = 10\% = 90\% = \text{RT}_{\text{succ}}$
10 5 7.6e1 5.4e1 1.0e2 7.6e1 5 3.5e3 2.6e3 4.5e3 3.5e3	10 5 2.1e2 1.7e2 2.4e2 2.1e2 5 7.3e3 6.7e3 7.9e3 7.3e3
le -1 5 1.9e3 1.1e3 2.5e3 1.9e3 5 2.9e4 2.5e4 3.4e4 2.9e4	le -1 5 1.4e3 1.1e3 1.7e3 1.4e3 5 1.7e4 1.5e4 1.8e4 1.7e4
le -3 5 2.1e3 1.5e3 3.0e3 2.1e3 5 3.1e4 2.6e4 3.4e4 3.1e4	le -3 5 1.8e3 1.5e3 2.0e3 1.8e3 5 1.9e4 1.8e4 2.0e4 1.9e4
le-8 5 2.3e3 1.7e3 3.0e3 2.3e3 5 3.2e4 2.7e4 3.6e4 3.2e4	le-8 5 2.2e3 1.9e3 2.5e3 2.2e3 5 2.1e4 2.0e4 2.1e4 2.1e4
fg in 5-D, N=5, mFE=2507 fg in 20-D, N=5, mFE=49613	A f 10 in 5-D, N=5, mFE=2251 f10 in 20-D, N=5, mFE=19839
10 5 2.0e2 1.7e2 2.3e2 2.0e2 5 8.0e3 7.3e3 8.7e3 8.0e3	10 5 1.0c3 8.4c2 1.2c3 1.0c3 5 1.4c4 1.3c4 1.5c4 1.4c4
1 5 9.0e2 7.6e2 1.0e3 9.0e2 5 2.1e4 1.5e4 2.6e4 2.1e4	1 5 1.3e3 1.2e3 1.4e3 1.3e3 5 1.5e4 1.5e4 1.6e4 1.5e4
le-3 5 1.8e3 1.7e3 1.9e3 1.8e3 5 2.3e4 1.9e4 2.9e4 2.4e4	1e-1 5 1.5e3 1.3e3 1.5e3 1.5e3 5 1.7e4 1.5e4 1.7e4 1.7e4 1.7e4 1.8e4 1.8e4
le -5 5 2.0e3 1.8e3 2.1e3 2.0e3 5 2.5e4 1.9e4 3.1e4 2.5e4	le-5 5 1.9e3 1.8e3 1.9e3 1.9e3 5 1.8e4 1.8e4 1.9e4 1.8e4
f11 in 5-D, N=5, mFE=2305 f11 in 20-D, N=5, mFE=16503	f12 in 5-D, N=5, mFE=7403 f12 in 20-D, N=5, mFE=36471
$\Delta f \neq \text{ERT} 10\% 90\% \text{RT}_{\text{SBCC}} \neq \text{ERT} 10\% 90\% \text{RT}_{\text{SBCC}}$	Δf # ERT 10% 90% RT _{gucc} # ERT 10% 90% RT _{gucc}
1 5 1.4e3 1.3e3 1.6e3 1.4e3 5 1.2e4 1.2e4 1.2e4 1.2e4	10 5 6.762 5.362 8.162 6.762 5 2.263 2.163 2.363 2.2631 5 1.663 1.363 1.963 1.663 5 5.963 3.563 9.363 5.963
le-1 5 1.6e3 1.5e3 1.7e3 1.6e3 5 1.3e4 1.2e4 1.3e4 1.3e4	le-1 5 2.2e3 1.7e3 2.6e3 2.2e3 5 1.1e4 7.1e3 1.5e4 1.1e4
1e-5 5 2.0e3 1.9e3 2.0e3 2.0e3 5 1.5e4 1.4e4 1.5e4 1.5e4	le-5 5 3.3e3 2.0e3 4.1e3 3.3e3 5 2.2e4 1.9e4 2.6e4 2.2e4
le-8 5 2.2c3 2.1c3 2.3c3 2.2c3 5 1.6c4 1.5c4 1.6c4 1.6c4 1.6c4 1.6c4 1.6c4 1.6c4	le -8 5 3.9e3 2.9e3 5.3e3 3.9e3 5 2.8e4 2.5e4 3.1e4 2.8e4
$\Delta f \# \text{ ERT } 10\% \ 90\% \ \text{RT}_{SDCC} \# \text{ ERT } 10\% \ 90\% \ \text{RT}_{SDCC}$	Δf # ERT 10% 90% RT _{SUCC} # ERT 10% 90% RT _{SUCC}
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10 5 1.3e1 6.0e0 2.0e1 1.3e1 5 2.7e2 2.5e2 3.0e2 2.7e2 1 5 1.2e2 1.1e2 1.4e2 1.2e2 5 6.7e2 6.4e2 7.1e2 6.7e2
1e-1 5 $1.6e3$ $1.5e3$ $1.7e3$ $1.6e3$ 5 $2.1e4$ $1.5e4$ $2.6e4$ $2.1e4$	1e-1 5 2.3e2 2.2e2 2.5e2 2.3e2 5 1.1e3 1.0e3 1.1e3 1.1e3
1e-3 5 2.1e3 2.1e3 2.1e3 2.1e3 5 2.3e4 1.7e4 2.7e4 2.3e4 1e-5 5 2.9e3 2.7e3 3.2e3 2.9e3 5 4.8e4 3.8e4 5.8e4 4.8e4	1e - 3 5 0.4e2 0.1e2 0.8e2 0.4e2 5 3.8e3 3.7e3 4.0e3 3.8e3 1e - 5 5 1.4e3 1.3e3 1.4e3 1.4e3 5 9.7e3 9.1e3 1.0e4 9.7e3
1e-8 5 4.6a3 3.8e3 5.5a3 4.6e3 5 9.6e4 5.9e4 1.2a5 9.6e4	le-8 5 2.4e3 2.2e3 2.5e3 2.4e3 5 2.3e4 2.1e4 2.4e4 2.3e4
Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}	Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
10 5 1.0e3 5.6e2 1.5e3 1.0e3 5 2.7e4 2.6e4 2.7e4 2.7e4	10 5 3.3e2 2.6e2 4.1e2 3.3e2 5 5.9e3 2.6e3 1.1e4 5.9e3
1 5 7.5e3 5.0e3 9.3e3 7.5e3 5 2.7e5 2.3e5 3.1e5 2.7e5 1e-1 5 2.3e4 1.2e4 3.1e4 2.3e4 5 3.1e5 2.5e5 3.7e5 3.1e5	1 5 2.7e3 9.5e2 4.4e3 2.7e3 5 3.8e4 2.4e4 5.7e4 3.8e4 le-1 5 8.8e3 8.2e3 9.6e3 8.8e3 5 8.3e4 7.5e4 9.1e4 8.3e4
1e-3 5 2.3e4 1.4e4 3.4e4 2.3e4 5 3.1e5 2.5e5 3.7e5 3.1e5	le -3 5 9.3e3 8.6e3 9.9e3 9.3e3 5 2.7e5 2.3e5 3.1e5 2.7e5 le -5 5 9.5e3 9.1e3 1.0e4 9.5e3 5 2.7e5 2.4e5 5.0e5 3.7e5
1e-8 5 2.5e4 1.4e4 3.7e4 2.5e4 5 3.3e5 2.7e5 3.9e5 3.3e5	le-8 5 1.0e4 9.6e3 1.1e4 1.0e4 5 3.8e5 2.4e5 5.1e5 3.8e5
f ₁₇ in 5-D, N=5, mFE=12567 f ₁₇ in 20-D, N=5, mFE=225555	f 18 in 5-D, N=5, mFE=27313 f 18 in 20-D, N=5, mFE=262843
447 L 26 14 16 1 17 1 17 17 17 16 17 16 16 16 16 16 17 17 17 17 16 17 17 17 17 17 17 17 17 17 17 17 17 17	A L & FRT 100 000 BT & FRT 100 000 BT
10 5 1.8el 8.4e0 3.2e1 1.8e1 5 1.6e2 1.3e2 1.9e2 1.6e2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
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Table 1: Shown are, for a given target difference to the optimal function value Δf : the number of successful trials (#); the expected running time to surpass $f_{opt}+\Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf -value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of functions executed in one trial. See Figure 1 for the names of functions.



Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^{-k} (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-k} (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-k} for running times of D, 10D, 100D... function evaluations (from right to left cycling black-cyan-magenta). Top row: all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

Test Functions

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Test Functions

Test functions

• define the "scientific question"

the relevance can hardly be overestimated

- should represent "reality"
- are often too simple?

remind separability

a number of testbeds are around

1 Separable fi	unctions
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- 1.2
 Empsolution Function

 1.3
 Rastrigin Function
- 1.4 Büche-Rastrigin Function
- 1.5 Linear Slope

2 Functions with low or moderate conditioning

2.6	Attractive Sector Function	
2.7	Step Ellipsoidal Function	
2.8	Rosenbrock Function, original	
2.9	Rosenbrock Function, rotated	,

3 Functions with high conditioning and unimodal

3.10	Ellipsoidal Function	•
3.11	Discus Function	•
3.12	Bent Cigar Function	•
3.13	Sharp Ridge Function	•
3.14	Different Powers Function	•

4 Multi-modal functions with adequate global structure

4.15	Rastrigin Function		
4.16	Weierstrass Function		
4.17	Schaffers F7 Function		
4.18	Schaffers F7 Function, moderately ill-conditioned		
4.19	Composite Griewank-Rosenbrock Function F8F2		

5 Multi-modal functions with weak global structu								
	5.20 Schwefel Function							
	5.21 Gallagher's Gaussian 101-me Peaks Function							
	5.22 Gallagher's Gaussian 21-hi Peaks Function	•						
	5.23 Katsuura Function							

GECCO-BBOB

Test Functions

Questions?

Nikolaus Hansen