

Hedging and Vertical Integration in Electricity Markets*

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Abstract

This paper analyzes the interactions between competitive (wholesale) spot, retail, and forward markets and vertical integration in electricity markets. We develop an equilibrium model with producers, retailers, and traders to study and quantify the impact of forward markets and vertical integration on prices, risk premia and retail market shares. We point out that forward hedging and vertical integration are two separate mechanisms for demand and spot price risk diversification that both reduce the retail price and increase retail market shares. We show that they differ in their impact on prices and firms' utility due to the asymmetry between production and retail segments. Vertical integration restores the symmetry between producers' and retailers' exposure to demand risk while linear forward contracts do not. Vertical integration is superior to forward hedging when retailers are highly risk averse. We illustrate our analysis with data from the French electricity market.

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1 Introduction

Corporate risk management has long been viewed as a prominent motive for vertical integration. In particular, managers have long perceived the supply and/or demand insurance rationale for vertical integration as most important.¹ In the 1970s, for example, oil refining industries produced up to 90% of their crude oil needs in order to avoid the significant costs that a stop in deliveries would entail (Teece, 1976). More generally, uncertainty in demand, lack of market flexibility and risk aversion constitute rationale for vertical integration.² Increasingly, managers have also benefited from financial hedging instruments that have developed tremendously in the past few decades (Hull, 2003). Yet the interactions between these two risk management approaches have been overlooked so far.

In the electricity industry, these questions are essential. Indeed, risk management and the vertical boundaries of utilities are central to the ongoing debate on the energy market regulatory reforms (Joskow, 2005, Gilbert and Newberry, 2006). Throughout these reforms, a number of firms went bankrupt, including Enron (trader) and Pacific Gas & Electricity (retailer) in 2001, and British Energy (nuclear producer) in 2002. Among the many countries that have liberalized electricity markets, New Zealand illustrates well retailers' risk management problems. This market, which has not been subject to the regulatory restrictions on the retail segment that were instrumental to the financial disaster in California, has suffered from insufficient generation assets and from limited forward markets. Interestingly, most non-integrated retailers in New Zealand have quickly exited the market as integrated producers could diversify their risks more efficiently than retailers, which has provided them with an important strategic advantage (Bertram, 2005).

To which extent could other risk management techniques, such as electricity derivatives, have helped avoid these problems? In early stages of most electricity market reforms, production capacity has been sufficient if not excessive so new players have usually entered the retail segment only. In most cases, liberalization has not entailed the splitting of the historical monopolies into independent firms (Littlechild, 2005). Instead, entrants have had to rely on financial markets, in particular forward markets, to source electricity. This has generally led to poor switching rates. Some believe this is due to the market power the incumbent enjoys in generation. In this paper, we argue that, even absent market power, non-integrated retailers will find it hard to gain a significant market share unless their risk aversion is limited and forward markets are well-developed.³

We develop a two-date equilibrium model of retail, (wholesale) spot and forward electric-

¹See Chandler (1964), Carlton (1979), Porter (1980), Hendrikse and Peters (1989), Perry (1989), Bolton and Whinston (1993), Emons (1996), and Joskow (2005).

²Most of the existing literature, typically abstracting from issues related to risk and uncertainty, view vertical integration mostly as a response to problems caused by contractual incompleteness (Grossman and Hart (1986)), Hart and Tirole (1990), as a way to acquire valuable private information about the production process (Arrow (1975)), to weaken rivals (Rey and Tirole (2009), but also Bolton and Whinston (1993) that consider a parsimonious model of risk management) or as a bargaining tool with vertically-related segments (Chemla (2003)). On vertical integration, see also Sekkat (2006) and Lafontaine and Slade (2007).

³Conventional wisdom holds that corporate financial arrangements such as vertical integration are motivated by long term risk management while financial instruments are used to manage short term risks. In our paper, vertical integration provides players with a short term risk management benefit. Hence, the difference between vertical integration and forward and future markets does not rely on a horizon effect.

ity markets to study and quantify risk management through forward markets and vertical integration. The model captures three important characteristics of electricity markets: demand is broadly inelastic, retailers cannot curtail demand, and output is non storable.⁴ In the first stage, downstream retailers and downstream subsidiaries of integrated firms choose both the number of accounts they open, which in our setting boils down to retail market shares, and forward positions under demand uncertainty. In the second stage, demand is realized, upstream producers produce electricity, sell it to retailers on the wholesale spot market, and retailers sell it to their clients. Since electricity is non-storable, no production can occur at the initial date to serve the market later on. The only ways to satisfy demand are to own a production asset, to buy on the spot market, or to buy a forward contract. We derive equilibrium prices and market shares and we examine to which extent a competitive forward market can be a substitute to productive assets in managing risk.⁵

Our setting also includes traders that can be either independent or affiliated to producers, retailers, and integrated firms. An independent trader is exposed to the risk stemming from the difference between the forward price and the realized spot price. The risk premium in the forward price compensates traders for this risk. In contrast, traders affiliated to producers and/or retailers can also diversify their risk with productive and commercial activities. We derive the equilibrium prices and quantities on the three markets in closed forms. We show that vertical integration and forward hedging are substitute risk management techniques. Both of them lead to a decrease in the retail price. Intuitively, they both enable retailers to hedge, which they value because of their risk aversion, and they reduce the retail segment's expected cost of access to electricity. However, we show them to be imperfect substitutes. A producer always benefits from trading forward contracts: Whether or not she trades on the forward market, she will produce electricity at the same cost and then sell it at a wholesale price equal to its marginal cost corresponding to the realized demand. Hence, she will only trade forward contracts if she expects that this will benefit her and forward markets can only make her better off. In contrast, retail firms do not necessarily obtain a higher utility when forward contracts become available: Since the forward market leads to a decrease in the retail price, it also affects retail profit. Not taking a position on the forward market cannot guarantee retailers to obtain the same utility as in the absence of a forward market.

This difference between the effect of forward markets on production and retail segments is due to the asymmetry between retailers and producers. Retailers decide their market shares under demand uncertainty, while producers set production after demand uncertainty is revealed. Producers, who make money by selling electricity at infra-marginal

⁴An earlier draft, Aid et al (2009), shows that our main results remain when the demand curve is elastic. The online supplement includes the main derivations with elastic demand.

⁵Non-storability prevents firms from benefiting from yet another risk management tool. To highlight that risk management is an important motive for retailers to obtain access to production assets, we also abstract from considerations related to strategic behavior or market power, and instead we assume price-taking firms that disregard any influence they could have on equilibrium prices or on other agents' decisions. In his empirical analysis of vertical integration in electricity markets, Mansur (2007) finds that vertical integration had both anti-competitive effects (for two large net selling firms) and pro-competitive (on other firms) effects. See also Gilbert and Newberry (2006) for electricity markets, Hastings and Gilbert (2005) for the gasoline industry and Hortacsu and Syverson (2005) for cement and concrete.

costs, are less sensitive to demand risk than retailers.⁶ We show that vertical integration can eliminate this asymmetry while linear forward instruments cannot.

Apart from our contribution to the risk management features of vertical integration, our paper is related to Allaz's (1992) and Bessembinder and Lemmon's (2002) equilibrium models of forward markets. We add a retail activity to their settings, and we consider jointly forward hedging and vertical integration as risk management tools. We also contribute to the recent literature on risk management through market versus non-market mechanisms. While markets enable investors to diversify (Doherty and Schlesinger (2002)) and are less sensitive to moral hazard (Doherty (1997)), institutions such as reinsurance companies keep an important role. In Gibson, Habib and Ziegler (2007), the importance of non-market mechanisms stems from excessive information gathering from investors in financial markets. In our setting, vertical integration arises as a response to market incompleteness as firms with concave objectives can only trade a small set of linear instruments. This implies that when risk aversion is sufficiently high, forward instruments cannot help firms to hedge their risk, while vertical integration can.

The paper is organized as follows. We develop our model and we present the equilibrium problem in Section 2. We then compare two different environments. First, in Section 3, we derive the equilibrium and the effect of vertical integration on retail prices in the absence of a forward market. In Section 4, we derive the equilibrium with the forward market and we analyze the effect of both forward contracts and vertical integration on prices, risk premia, and retail market shares. We then illustrate our results in Section 5 through the French electricity market. Section 6 concludes.⁷

2 The model

In this section we first describe price-taking retail, (wholesale) spot and forward markets for an electricity market, i.e. a market in which the output is non-storable, demand is inelastic, and retailers are required to meet demand from their clients. Then, we define an equilibrium on these markets.

2.1 The markets

We consider a set \mathcal{P} of producers that produce a homogenous, non-storable good that they can sell to a set \mathcal{R} of retailers. After sourcing on the wholesale markets, retailers compete in a market for consumers whose demand D is random. Demand D is described by a random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We assume that demand is inelastic. Section 5 shows that this assumption is broadly consistent with electricity markets. In addition, all firms are assumed to be price-takers, i.e. they disregard any influence they could have on prices, or on the other agents' behavior.

⁶This is consistent with the fact that Californian retailers became financially distressed shortly after peaks in spot and forward prices. This also suggests that retailers may have a higher incentive for vertical integration than producers.

⁷The appendix contains the proofs. The online supplement extends our analysis to an elastic demand curve.

We denote by \mathcal{K} the set of all agents: Producers, retailers and traders. Agents are not necessarily specialized in a single segment. For simplicity, all agents have access to wholesale markets and are allowed to trade, including speculative agents that play no role in production or retail segments but who can settle any position at market prices on financial markets.⁸ Hence the subsets \mathcal{P} and \mathcal{R} of \mathcal{K} are possibly intersecting. We thus have four different types of agents:

- Upstream producers who produce and sell on the wholesale markets;
- Downstream retailers who buy on the wholesale markets and sell goods to consumers;
- Integrated firms who produce and deliver outputs to consumers;
- Traders who trade on all markets.

Retailers can obtain electricity from three sources: Wholesale markets, production if they are integrated firms, and forward markets where they agree to buy or sell electricity at the next date for price q . In particular, forward markets turn out to be linear contracts.

In our setting, vertical integration can be thought of as a profit-sharing scheme as in Rey and Tirole (2006) and Chemla (2003).⁹ It is also a mechanism that enables agents to diversify industry-specific risk that will dominate other forms of mergers, and in particular horizontal integration, in diversifying risk that channels throughout vertically-related segments.

There are two dates:

- At $t = 0$, retailers choose how many client accounts to open (out of a publicly known total number), which boils down to market shares $\alpha_k \in [0, 1]$, $k \in \mathcal{R}$. In addition, agents take forward positions f_k , $k \in \mathcal{K}$ (where $f_k > 0$ represents a purchase).
- At $t = 1$, demand uncertainty is revealed. Agents take positions S_k , $k \in \mathcal{K}$, on the wholesale spot market (where $S_k > 0$ denotes a purchase) and producers also choose their generation levels G_k , $k \in \mathcal{P}$. Since electricity is non-storable, production can only occur at that time $t = 1$ when the demand uncertainty is observed and in which consumers buy the good.¹⁰

At $t = 1$, the market-clearing constraint requires that demand be satisfied:

$$1 = \sum_{k \in \mathcal{R}} \alpha_k . \tag{2.1}$$

In addition, generation levels must meet demand D :

$$D = \sum_{k \in \mathcal{P}} G_k . \tag{2.2}$$

⁸Enron or Dreyfus are examples of such traders.

⁹As in these papers and many of those summarized in Rey and Tirole (2006), vertical integration may then arise as a response to contractual incompleteness where output and profits are difficult to contract upon.

¹⁰Our results are robust to definitions where decisions on the retail and forward markets are not taken simultaneously.

The market-clearing constraints on the wholesale spot and the forward markets can be written

$$0 = \sum_{k \in \mathcal{K}} S_k \quad (2.3)$$

and

$$0 = \sum_{k \in \mathcal{K}} f_k . \quad (2.4)$$

Agent k obtains a total payoff from its activity on the retail, forward and spot markets (net of its production costs):

$$\pi_k = p\alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - qf_k - wS_k - c(G_k) \mathbf{1}_{\{k \in \mathcal{P}\}} ,$$

where p , w , and q denote the retail price, the wholesale spot price and the forward price, respectively, and c is the cost function to any producer $k \in \mathcal{P}$.¹¹ The cost function is defined on \mathbb{R}_+ , it is continuously differentiable, strictly convex, and it satisfies the Inada conditions $c'(0+) = 0$, $c'(+\infty) = +\infty$.

Non-storability requires that the net volume sold or produced by agent k at $t = 1$ is zero:

$$0 = \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - f_k - S_k - G_k \mathbf{1}_{\{k \in \mathcal{P}\}} ,$$

This allows us to discard variable S_k and to write:

$$\pi_k = (p - w)\alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} + (w - q)f_k + (wG_k - c(G_k)) \mathbf{1}_{\{k \in \mathcal{P}\}} .$$

The payoff has three possible ingredients: The payoff to a retailer that satisfies demand $\alpha_k D$ at retail price p by sourcing on the wholesale spot market at price w ; the payoff to a trader buying a volume f_k on the forward market at price q and selling it on the spot market at price w ; and the profit made by a producer who generates a volume G_k at cost $c(G_k)$ and sells it on the spot market at price w .

We further assume that agent k 's preferences over random profits are described by a mean-variance utility function.¹² The utility function is denoted:

$$MV_{\lambda_k}[\xi] := \mathbb{E}[\xi] - \lambda_k \text{Var}[\xi] .$$

¹¹An earlier draft included heterogeneous cost functions. The analysis was essentially unchanged.

¹²We follow a large strand of the asset pricing literature (Elton and Gruber, 1995) in using mean-variance utility functions. It is well-established that although MV_{λ_k} is not monotonic, which allows for negative equilibrium prices, it can be seen as a second order expansion of a monotonic Von Neumann-Morgenstern utility function (Markowitz, 1979). Given our concern for equilibrium prices and financial hedging, this setting is more convenient here than the risk-neutral approach followed by a number of existing corporate risk management models. These consider risk-neutral shareholders whose concern for risk stems from market frictions that are sufficient to depart from Modigliani and Miller's irrelevance results. Examples of such frictions include bankruptcy costs and lost tax shields (Grinblatt and Titman, 2002, Chapter 21, Smith and Stulz, 1984). Debt and non-debt tax shields can be lost (Titman and Wessels, 1988, Grinblatt and Titman, 2002), and bankruptcy becomes more likely (Graham, 2000), when cash flows are volatile. See also Froot, Scharfstein and Stein (1993) and Mello and Ruckes (2007). Hence, one may attempt to reconcile both settings by thinking of the risk aversion coefficient λ_k as a cost of market frictions that is proportional to cash flow volatility and that prompts firms to care about corporate risk management. It should be noted that this setting enables us to combine asset pricing, industrial organization and corporate finance.

2.2 Spot market equilibrium

We proceed by backward induction and we start our analysis by determining the spot market equilibrium at $t = 1$. At that time, when they enter the spot market, agents know the realization of demand uncertainty D , and decisions on the retail and forward markets have already been made. The outcome turns out to be independent of any decision taken at time $t = 0$. Producer k 's generation payoff, $wG_k - c(G_k)$, leads to an equilibrium spot price:

$$G_k^* \equiv G = (c')^{-1}(w^*), w^* = C'(D), \quad (2.5)$$

where the superscript -1 denotes the inverse function, and the aggregate cost function C is the sum of the production costs over the entire generation segment and satisfies

$$C(D) = \sum_{k \in \mathcal{P}} c(G^*(w^*)).$$

In equilibrium, all upstream firms produce up to the point where the spot price equals the marginal cost. This is consistent with both perfectly competitive markets (Green, Mas Colell, and Whinston, 2005) and regulated industries in which prices are set at marginal cost by the regulator (Laffont and Tirole, 1993).

The spot market equilibrium only depends on (exogenous) demand D and is therefore independent of any other equilibrium prior to time $t = 1$. This results from the non-storability condition and the assumption that D is inelastic.¹³

Both the equilibrium spot price w^* and the generation payoff

$$\Pi_k^g := (w^* G^* - c(G^*)) \mathbf{1}_{\{k \in \mathcal{P}\}} \quad (2.6)$$

are exogenous random variables, the distribution of which is assumed to be known by all agents. We can then substitute w^* and G^* for variables w and G , and we define the payoff to agent k as

$$\Pi_k(p, q, \alpha_k, f_k) := \Pi_k^r(p, \alpha_k) + \Pi_k^t(q, f_k) + \Pi_k^g, \quad (2.7)$$

where Π_k^g is defined in (2.6) and

$$\begin{aligned} \Pi_k^r(p, \alpha_k) &:= (p - w^*) \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} \\ \Pi_k^t(q, f_k) &:= (w^* - q) f_k. \end{aligned}$$

Here, Π_k^r is the net retail payoff derived from supplying a retail demand by sourcing on the spot market, and Π_k^t is the net trading profit earned by buying f_k units of goods on the forward market and selling them on the spot market. Finally, Π_k^g is the net generation payoff obtained by producing electricity and selling it on the spot market.

As a consequence, any agent's utility function satisfies:

$$\begin{aligned} MV_{\lambda_k} &= MV_{\lambda_k}^r + MV_{\lambda_k}^t + MV_{\lambda_k}^g \\ &\quad - 2\lambda_k (\text{Cov}(\Pi_k^r, \Pi_k^t) + \text{Cov}(\Pi_k^r, \Pi_k^g) + \text{Cov}(\Pi_k^t, \Pi_k^g)). \end{aligned}$$

¹³In Allaz (1992), the elasticity of demand to the spot price implies that the spot price is a function of forward positions and it reduces the market power of producers.

When her retail and generation profit are negatively correlated, an integrated agent has a higher utility than the sum of two activities.

2.3 Competitive Equilibrium

In order to define a competitive equilibrium, we introduce the following two sets:

$$\begin{aligned} \mathbf{A} &:= \left\{ (\alpha_k)_{k \in \mathcal{K}} \in [0, 1]^{|\mathcal{K}|} : \forall k \notin \mathcal{R}, \alpha_k = 0 \text{ and } \sum_{k \in \mathcal{K}} \alpha_k = 1 \right\} \\ &:= \left\{ (f_k)_{k \in \mathcal{K}} \in \mathbb{R}^{|\mathcal{K}|} : \sum_{k \in \mathcal{K}} f_k = 0 \right\}. \end{aligned}$$

Definition 2.1. *A retail-forward equilibrium is a quadruple $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbf{A} \times \mathbf{F}$ such that:*

$$(\alpha_k^*, f_k^*) = \operatorname{argmax}_{\alpha_k, f_k} \operatorname{MV}_{\lambda_k} [\Pi_k(p^*, q^*, \alpha_k, f_k)], \forall k \in \mathcal{K}.$$

This defines a simultaneous competitive equilibrium on both markets. Each agent submits a supply function that specifies her position on the forward market and her market share for each price level. Each agent chooses her supply function taking prices as given. Then, the auctioneer collects all supply functions and sets prices so that markets clear and demand is satisfied.

3 Analysis in the absence of a forward market

In this section, we derive the equilibrium without a forward market as a benchmark. We define the profit function without a forward position:

$$\Pi_k^0(p, \alpha_k) := \Pi_k(p, 0, \alpha_k, 0) = \Pi_k^r(p, \alpha_k) + \Pi_k^g.$$

In this simplified setting, Definition 2.1 reduces to:

Definition 3.1. *A retail equilibrium is a pair $(p^*, \alpha^*) \in \mathbb{R}_+ \times \mathbf{A}$ such that:*

$$\alpha_k^* = \operatorname{argmax}_{\alpha_k} \operatorname{MV}_{\lambda_k} [\Pi_k^0(p^*, \alpha_k)], \forall k \in \mathcal{K}.$$

3.1 Characterization of the equilibrium

Let

$$\Pi_I^g := \sum_{k \in \mathcal{R} \cap \mathcal{P}} \Pi_k^g$$

be the aggregate generation profit realized by all integrated firms, and let

$$\Pi^r := \sum_{k \in \mathcal{R}} \Pi_k^r(p^*, \alpha_k^*) = (p^* - w^*)D$$

be the aggregate retail profit to all retailers. We also define

$$\Lambda := \left(\sum_{k \in \mathcal{K}} \lambda_k^{-1} \right)^{-1}, \quad \Lambda_{\mathcal{R}} := \left(\sum_{k \in \mathcal{R}} \lambda_k^{-1} \right)^{-1}, \quad (3.1)$$

the aggregate risk aversion coefficients for the set of all agents and for the set of all retailers, respectively. Parameter λ_k^{-1} corresponds to Agent k 's risk tolerance, as in Wilson (1979) and Gollier (2004). We focus on interior equilibria where constraints $\alpha^* \in [0, 1]$ and $p^* \geq 0$ are not binding, by discarding cases where there are retailers with zero market shares. We obtain:

Proposition 3.1. $(p^*, \alpha^*) \in \mathbb{R}_+^* \times \text{int}(\mathbf{A})$ defines an equilibrium of the retail problem without a forward market iff:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]} - \frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}, \quad (3.2)$$

and p^* solves the second order polynomial equation:

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, (p^* - w^*)D + \Pi_I^g]. \quad (3.3)$$

Proof. See Appendix A \square

Before we discuss the properties of market shares in Section 3.3, we rewrite 3.2 as

$$\alpha_k^* = \frac{\mathbb{E}[\Pi^r] - 2\lambda_k \text{Cov}[\Pi^r, \Pi_k^g]}{2\lambda_k \text{Var}[\Pi^r]}.$$

Intuitively, firm k 's market share increases with its expected retail profits and decreases with the covariance between retail profit and its own generation profit. The higher the market share, the higher its risk exposure. The lower the covariance aforementioned, the greater the diversification benefits firm k obtains through production.

3.2 The retail price

We now examine the properties of the retail price. A priori, solutions to (3.3) may or may not exist and be unique.¹⁴ We will point out that, if there exist two solutions, only one is relevant. To this end, we first determine the risk-neutral price p^0 , i.e. the price when there are risk-neutral retailers:

$$p^0 = \frac{\mathbb{E}[w^*D]}{\mathbb{E}[D]}. \quad (3.4)$$

¹⁴This is a common feature in such mean-variance settings.

Since $w^* = C'(D)$ is a non-decreasing function of D , w^* and D are positively correlated, and the risk neutral retail price is greater than the expected spot price: $p^0 \geq \mathbb{E}[w^*]$.

When all retailers are risk-averse, we expect the equilibrium retail price to tend to the risk neutral price when the risk aversion coefficient of at least one retailer tends to zero. We show that there can be only one such economically relevant retail price:

Proposition 3.2. *Only one equilibrium retail price can satisfy the property that the price converges to the risk-neutral price as one agent becomes risk neutral.*

Proof. See Appendix A \square

Since the market shares are a function of the correlation between retail profit and generation profit and the retail price is a function of the correlation between the retail profit and the integrated firms' production profit, one expects vertical integration to lead to interesting risk diversification properties. The following Proposition confirms this intuition:

Proposition 3.3. *When Π^r and Π_I^g are negatively correlated, the presence of integrated producers decreases the retail price.*

Proof. See Appendix A \square

The correlation between Π^r and Π_I^g tends to be negative: Since $w^* = C'(D)$, profit Π_k^g increases with D . In contrast, keeping p^* fixed and taking into account that w^* increases with D , the retail profit $\Pi^r = (p^* - w^*)D$ decreases with D provided p^* is not too high. It should be noted that the higher the slope and the more important the curvature of C , the more negative we expect this correlation, and the stronger the effect of vertical integration on retail prices. In the numerical application in Section 5, this correlation is indeed negative, and we predict that the higher the marginal costs of the portfolio of technologies (for example for non-nuclear technologies), the more important the effect of vertical integration on retail prices.

Vertical integration by at least one firm leads to a decrease in the retail price because it facilitates risk-sharing between the production and the retail segments. Since the production segment provides the retail segment with a natural hedge, integrated retailers require a lower risk premium and hence they charge a retail price that is lower than in the absence of vertical integration.

Intuitively, vertical integration enables firms to manage risk by reducing expected costs and increasing expected utility. To see this, we can compare two extreme cases: One in which the market is served by only one risk averse integrated monopolist, and another in which there are one non-integrated retailer and one non-integrated producer. Suppose that the integrated monopolist and the retailer have the same risk aversion coefficient. Hence market shares do not play any significant role, and the only relevant variable is the retail price. In the first case, the utility of the integrated firm is given by $\mathbb{E}[pD - C(D)] - \lambda \text{Var}[pD - C(D)]$, since the wholesale price is merely a transfer price between the production and the retail segments so that the retail price, say p_I , will be determined with respect to the total cost of production. In the second case, the retailer has to buy

electricity at its marginal cost and to resell it at retail price p_{NI} . Its utility is given by $\mathbb{E}[pD - wD] - \lambda \text{Var}[pD - wD]$ and the retail price will be determined with respect to the marginal cost. Hence,

$$p_I = \frac{\mathbb{E}[D]}{2\lambda \text{Var}[D]} + \frac{\text{Cov}[D, C(D)]}{\text{Var}[D]} \quad p_{NI} = \frac{\mathbb{E}[D]}{2\lambda \text{Var}[D]} + \frac{\text{Cov}[D, wD]}{\text{Var}[D]},$$

so that, the difference satisfies:

$$p_{NI} = p_I + \frac{\text{Cov}[D, wD - C(D)]}{\text{Var}[D]}.$$

Hence, vertical integration leads to a decrease in the retail price for a given level of risk aversion. Since the retail price has to be determined *before* the realization of demand, an integrated firm can decide to sell electricity at its average cost of production, whereas a retailer has to sell it at its marginal cost — which is always higher than the average cost.

3.3 Market shares

As a benchmark, note that when there are risk-neutral retailers, the market shares held by the risk averse retailers are given by:

$$\alpha_k^0 = -\frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}, \quad (3.5)$$

while the remaining demand is split among the risk neutral retailers. In particular, a risk averse retailer who has no generation unit ends up with a zero market share.

Note that from the non-negativity constraint for market shares, equation (3.5) implies $\text{Cov}[\Pi^r, \Pi_k^g] \leq 0$, the same condition that ensures that vertical integration leads to a decrease in the retail price.

In the absence of integrated producers and with risk-neutral suppliers, the equilibrium market shares are proportional to, and only depend on, risk tolerances:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}.$$

Proposition 3.4. *When Π^r is negatively correlated with Π_I^g , vertical integration leads a retailer to choose a higher market share than if she was not integrated.*

In the presence of integrated firms, the equilibrium market shares to non-integrated retailers satisfy

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]},$$

while the integrated firms have market shares:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]} - \frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}.$$

Intuitively, the integrated agents can choose a high investment in the retail market while benefiting from the natural hedge provided by its production segment. In addition, although the presence of integrated firms affects market shares, the relative market shares among the set of non-integrated retailers remain unchanged:

$$\frac{\alpha_i^*}{\alpha_j^*} = \frac{\lambda_j}{\lambda_i} .$$

4 Analysis with a forward market

We now allow firms to trade in the forward market. We first characterize the equilibria. Then, we analyse retail and forward prices and positions.

4.1 Characterization of the equilibria

Let

$$\Pi^e := \sum_{k \in \mathcal{K}} \Pi_k(p^*, q^*, \alpha_k^*, f_k^*) = p^* D - C(D) ,$$

be the aggregate profit to both the production and the retail segments, which coincides with the social surplus.

We still focus on interior equilibria such that the non-negativity constraints on market shares are not binding. The equilibrium with a forward market is characterized by the following Proposition:

Proposition 4.1. $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+^* \times \mathbb{R}_+^* \times \text{int}(\mathbf{A}) \times \mathbf{F}$ defines an equilibrium of the retail-forward equilibrium problem iff:

$$f_k^* = \frac{\Lambda}{\lambda_k} \frac{\text{Cov}[w^*, \Pi^e]}{\text{Var}[w^*]} - \frac{\text{Cov}[w^*, \Pi_k^g]}{\text{Var}[w^*]} - \alpha_k^* \frac{\text{Cov}[w^*, \Pi^r]}{\text{Var}[w^*]} \quad (4.1)$$

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}\left[w^*, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g\right] - \frac{\text{Var}[w^*]}{\Delta} \text{Cov}\left[\Pi^r, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g\right] \quad (4.2)$$

$$q^* = \mathbb{E}[w^*] - 2\Lambda \text{Cov}[w^*, p^* D - C(D)] , \quad (4.3)$$

and p^* is a root of the second order polynomial equation

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, (p^* - w^*)D + \Pi_I^g] \quad (4.4)$$

$$+ 2\Lambda_{\mathcal{R}} \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}\left[w^*, (p^* - w^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}}(p^* D - C(D))\right] ,$$

where

$$\Delta := \text{Var}[w^*] \text{Var}[\Pi^r] - \text{Cov}^2[w^*, \Pi^r] . \quad (4.5)$$

Proof. See Appendix A. \square

4.2 The forward price

From (4.3), the forward price equals the expected spot price corrected by a risk premium term that accounts for the correlation between the spot price and the aggregate profit Π^e , and the market aggregate risk aversion.¹⁵

We can rewrite q^* as:

$$q^* = \mathbb{E}[Zw^*] \text{ with } Z := 1 - 2\Lambda(\Pi^e - \mathbb{E}[\Pi^e]) .$$

If Λ is sufficiently small to ensure that Z is always strictly positive, Z defines a change in probability and q^* is given by the expected w^* under a risk-neutral probability.

The forward price only depends on retail and spot prices. It is independent of the distribution of market shares and of that of generation assets. Moreover, the risk-neutral forward price, i.e. the price if there are risk-neutral traders, boils down to the expected spot price $q^0 = \mathbb{E}[w^*]$.

When the cost function is quadratic, e.g. $c(x) := \frac{1}{2}x(ax+b)$, $a, b > 0$, q^* can be written:

$$q^* = \mathbb{E}[w^*] - \frac{2\Lambda}{A}\text{Var}[w^*](p^* - \mathbb{E}[w^*]) + \frac{\Lambda}{A}\text{Var}^{\frac{3}{2}}[w^*]\text{Skew}[w^*] ,$$

where $A^{-1} := \sum_{k \in \mathcal{P}} a^{-1}$, as in Bessembinder and Lemmon (2002).

The forward price increases with spot price skewness and, in the case where the retail price is higher than the expected spot price, the forward price decreases with spot price volatility. This shows that, in the case of electricity, where spot price volatility is high, forward prices that are lower than the expected spot price are common. Nevertheless, forward prices that are greater than the expected spot price can occur when spot price skewness is large and positive, i.e. when large upward peaks are possible.

In addition, the forward market equilibrium leads to the following relationship between retail and forward prices:

$$p^* = \frac{\mathbb{E}[w^*] - q^*}{2\Lambda\text{Cov}[w^*, D]} + \frac{\text{Cov}[w^*, C(D)]}{\text{Cov}[w^*, D]} .$$

In particular, higher forward prices correspond to lower retail prices and conversely.

4.3 The retail price

In the presence of a forward market, the equation for p^* is similar to that found in the absence of a forward market, with an extra term that corresponds to the hedging property of the forward market

$$2\Lambda_{\mathcal{R}} \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov} \left[w^*, (p^* - w^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}} (p^*D - C(D)) \right] .$$

¹⁵This equation has a form that is similar to that of other equilibrium models in a mean-variance setting, as shown in Allaz (1992).

As in Section 3, the risk-neutral retail price boils down to $p^0 = \frac{\mathbb{E}[w^*D]}{\mathbb{E}[D]}$. The reason for this is that in that case risk management is irrelevant. As in the preceding Section, we can write a Taylor expansion around $\Lambda_{\mathcal{R}} = 0$ and show that only one root for p^* is relevant, which ensures the uniqueness of the equilibrium retail price.

We can also exhibit the following properties of equilibrium retail prices:

- **The price in a fully integrated economy.** If all producers are integrated, i.e. $\mathcal{P} \subset \mathcal{R}$, we obtain:

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}}\text{Cov}[(p^* - w^*)D, p^*D - C(D)] \\ + 2\Lambda_{\mathcal{R}} \left(1 - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\right) \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, p^*D - C(D)].$$

In particular, in the absence of non-integrated traders, $\mathcal{R} = \mathcal{K}$ and $\Lambda = \Lambda_{\mathcal{R}}$, so that the above equation boils down to (3.3), i.e. the forward market has no impact on the retail price.

To see this, consider the following example: Suppose there are only N integrated producers, with the same risk aversion coefficient. By symmetry, $S_k^* = \frac{D}{N}$, $\alpha_k^* = \frac{1}{N}$ and $f_k^* = 0$ for all k . The agents do not take any position on the forward market. The retail price should therefore not be affected.

This result highlights the substitution effect between forward hedging and vertical integration. When the industry can diversify demand risk through vertical integration, forward hedging becomes irrelevant. Conversely, Section 5 illustrates that the presence of forward hedging reduces the impact of vertical integration on the retail price.

- **The effect of forward trading.** In a partially integrated economy, $p_F^* \leq p_{NF}^*$, i.e. the forward market reduces the retail price, iff

$$0 \leq \left(1 - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\right) \text{Cov}^2[w^*, (p_{NF}^* - w^*)D] \quad (4.6)$$

$$+ \text{Cov}[w^*, (p_{NF}^* - w^*)D] \text{Cov}[w^*, \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}}\Pi^g]. \quad (4.7)$$

In particular, this is verified if no producer is integrated and the retail income without a forward market is negatively correlated to the spot price. This is also verified if no retailer is integrated and $\Lambda = 0$ (e.g. if there are risk-neutral traders).

- **The effect of integration.** Let p_{NI}^* be the equilibrium retail price in the absence of integration. In order to compare this price with the equilibrium retail price in the presence of integrated firms, we substitute p_{NI}^* for p^* in the right hand side of (4.4) and we examine its sign, i.e. the sign of

$$\frac{\text{Cov}[w^*, (p_{NI}^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, \Pi_I^g] - \text{Cov}[(p_{NI}^* - w^*)D, \Pi_I^g]. \quad (4.8)$$

In particular, with a quadratic cost function, we obtain:

$$\begin{aligned} & \frac{\text{Cov}[w^*, (p_{NI}^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, \Pi_k^g] - \text{Cov}[(p_{NI}^* - w^*)D, \Pi_k^g] \\ &= \frac{A^3}{2a_k} \left(\text{Var}[D^2] - \frac{\text{Cov}^2[D^2, D]}{\text{Var}[D]} \right), \end{aligned}$$

which is always positive, so that the right side of (4.4) is positive for p_{NI}^* , i.e. partial vertical integration reduces the equilibrium retail price.

Section 5 illustrates that the presence of vertically integrated producers reduces the retail price. This effect is nonetheless drastically reduced in comparison to the case without a forward market.

We summarize our main results in the following Proposition:

Proposition 4.2. *A forward market (resp. partial vertical integration) reduces the retail price if and only if (4.7) is satisfied (resp. the sign of (4.8) is positive).*

4.4 Positions on the forward market

Equation (4.1) shows that in contrast to the forward price, forward positions depend on both p^* and α^* . Agent k 's forward position has three components. The first term, i.e. the fraction $\frac{\Lambda}{\lambda_k}$ of a constant term that involves the correlation between the global profit and the spot price, can thus be interpreted as the trading component.¹⁶ The second term, i.e. the fraction α_k^* of the correlation between the global retail profit and the spot price is the retail component. If the retail market revenue is negatively correlated with the spot price, as we argued in the previous section, then retailers will take long positions on the forward market to hedge against high spot prices. Finally, the last term corresponds to the generation component, which takes the form of the correlation between the generation payoff and the spot price. As generation profits are positively correlated to the spot price, producers will take short forward positions to hedge against low spot prices.

4.5 Positions on the retail market

We now turn to the market shares characterized in 4.2. When $\Pi_k^g = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g$ for all $k \in \mathcal{R}$, the market shares become $\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}$, as in a non-integrated economy without a forward market. This obtains, for instance, when there are no integrated firms, or all producers are integrated and generation profits are proportional to risk tolerances. Another formulation for α_k^* is:

$$\alpha_k^* = \alpha_k^0 + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \left(1 - \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}[w^*, \Pi_I^g] + \frac{\text{Var}[w^*]}{\Delta} \text{Cov}[\Pi^r, \Pi_I^g] \right),$$

¹⁶To see this, note that if k is only a trader, then the last two terms are zero.

where

$$\alpha_k^0 = \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}[w^*, \Pi_k^g] - \frac{\text{Var}[w^*]}{\Delta} \text{Cov}[\Pi^r, \Pi_k^g]$$

is retailer k 's risk-neutral market share. This enables us to analyse the deviation of market shares from the risk-neutral equilibrium.

In Section 5, we will observe two important characteristics of market shares. First, the presence of a forward market is a means for retailers to obtain larger market shares in the retail segment. Second, the higher the level of integration of an integrated producer, the higher its market share.

4.6 Utility functions and the strong asymmetry between retail and production

The asymmetry relative to risk between retailers and producers is due to several differences. First, in the absence of a forward market and vertical integration, retailers have to make market share decisions under uncertainty, while producers know the realization of demand when production takes place. Second, since final demand is inelastic, production profits are independent of the retail price, while retail profits depend on both retail and spot prices. This asymmetry is central to our analysis.¹⁷

Hence, a producer always benefits from trading forward contracts. Since, in our setting, the generation profit Π_k^g boils down to an exogenous random variable, a producer that chooses $\bar{f}_k(q) = 0$ for all forward price q obtains a utility $\text{MV}_{\lambda_k}[\Pi_k^g]$ identical to that obtained in the absence of a forward market. Therefore, since the strategy $\bar{f} = 0$ is admissible and yields the same utility as in the absence of a forward market, the presence of a forward market always increases the utility to producers.

In contrast, retailers do not necessarily obtain a higher utility when forward contracts become available.

Indeed, the retail profit Π_k^r depends on p^* , and thus on the other agents' decisions. If the retail price p^* in the presence of forward trading is different from the retail price without forward trading, agent k 's retail profit will also be different. Not taking a position on the forward market will not guarantee agent k to obtain the same utility as in the absence of a forward market.

Proposition 4.3. *Forward markets benefit producers, but not necessarily retailers, even though retailers benefit from trading forward contracts.*

In Subsection 5, the availability of forward contracts decreases the utility to retailers. When forward markets are available, each retailer is individually better off taking positions on them. Hence, all retailers trade forward contracts and they can offer lower retail prices. Nevertheless, the decrease in expected profits offsets that in variance and this risk hedging mechanism implies a decrease in utility in comparison to the environment without forward contracts.

¹⁷One illustration of this asymmetry is the Californian electricity crisis, in which retailers suffered large losses while producers could take advantage of high spot prices.

The effect of vertical integration on the agents' utility depends on the utility of vertically integrated firm. It turns out that our results are robust to various ways of handling this problem, in particular Wilson's (1979) approach to sum the risk tolerance of the integrated firms or an approach that would assume that all risk aversion coefficients, including those to vertically integrated firms, are equal.¹⁸ In Section 5, we observe that, like forward hedging, vertical integration decreases the agents' utility because of its negative effect on retail prices. Nevertheless, for large risk aversion coefficients this effect can be reversed and the gain from risk diversification through vertical integration can be higher than the loss in expected profit.

5 Numerical Application to the electricity industry

In order to gain further insight into the relationship between integration, forward trading and prices, we illustrate our analysis with data from the French electricity market.

The main purpose of this section is to assess broadly the variations of the different variables (prices, market shares and utilities) that were identified in our theoretical analysis. For instance, we have seen that the risk premium between the expected spot price and the forward price should increase with risk aversion. Our setting enables us to provide a quantitative estimate of this risk premium as well as the impact of different market structures on prices, market shares, and utilities. To do so, we build a simplified model of supply and demand in the French electricity market that captures the main features in our analysis.

5.1 Methodology

The French market is characterized by the presence of a partly regulated dominant agent, EDF, and competitors who entered the market at the beginning of the millenium. Transport and distribution are regulated activities conducted by EDF whereas production and retail are competitive. The spot market is a daily market where power is exchanged for every hour of the next day. These exchanges can be made either with bilateral (OTC) trades or on the non-mandatory organized spot market, EpexSpot.¹⁹ During the period we consider, around 70 firms were active, with an equal split between buyers and sellers. Around 20% of the daily consumption was exchanged on the spot market. In the retail market, after ten years of deregulation, 7% of industrial clients switched from the historical monopoly to a competitor whereas only 4% of residential clients followed the same path.²⁰ This situation is mainly due to the existence of regulated tariffs for competitive activities that are below market prices.

Our numerical analysis relies on five years of historical data of daily electricity consump-

¹⁸If we were to model the integration stage rather than focusing on the market structure, the change in utilities due to integration would also depend on the takeover game (see Tirole, 2006, Chapter 11, and Grinblatt and Titman, 2002, Chapter 20).

¹⁹All data and further information are available on <http://www.epexspot.com> and <http://clients.rte-france.com>.

²⁰See regulator 2009 activity report, p. 85 on www.cre.fr/fr/documents/publications/rapports_annuels.

tion starting on Jan 1st 2005 (source: RTE) and average production costs and capacities data (source RTE, EDF). This data is displayed on Figure 5.1 and Table 1. Data exhibit a clearly seasonal pattern that is well-known in electricity generation management. This pattern does not change our prediction since demand is unknown at time zero when firms have to decide whether or not to hedge. In our model, they consider demand as a random variable whose possible values are given by the time series provided on Figure 5.1. Their alternative is either to buy and sell on the spot market facing this demand volatility or to sell or buy a baseload forward contract.

Technology	Max. Capacity (MW)	Cost (EUR/MWh)
Nuclear	63130	15
Coal/Gas	14212	42
Fuel	8770	97

Table 1: Average Maximal Capacities and Costs by Technologies.

Capacities in Table 1 are maximal capacities. In the management of power production, power plants need to be regularly stopped for maintenance, leading to an available capacity below maximal capacity. From historical power production data we estimate the rate of unavailability to be 15%, leading to available capacities equal to 85% of the maximum capacities reported in Table 1.

In addition to nuclear, coal, gas and fuel, hydropower can be used to meet demand. Hydropower is produced by releasing water contained in dams through turbines. This is achieved at a negligible marginal cost source of electricity so we set its marginal cost to zero. The capacity of this source of electricity is limited by the volume of water available in the dams. Power producers have traditionally developed hydro resource management strategies to make an optimal use of this limited and inexpensive source of electricity. This strategy usually aims at using hydropower when demand is high in order to avoid using alternative and expensive sources of production. For the sake of this analysis, we consider that the producers manage their water resources to produce a fixed proportion of daily demand via hydropower. Based on historical hydropower production data, we estimate this proportion to be 15%.

Hence, all producers are assumed to follow the same simplified production management strategy: 15% of production is met via hydropower, then the nuclear, coal/gas and fuel technologies are used sequentially by order of increasing marginal cost with capacities equal to 85% of maximal capacities, until total demand is met. This production strategy leads to a global cost function and an equilibrium spot price illustrated on Figure 5.1.

Without loss of generality, the analysis can be performed with a maximum of 4 agents: a non-integrated retailer, a non-integrated producer, an outside trader and an integrated firm. Indeed, equations (4.1) and (4.2) are linear in λ_k^{-1} and Π_k^g while (4.4) and (4.3) only involve the aggregate risk aversion coefficients Λ and $\Lambda_{\mathcal{R}}$. This enables us to aggregate the behavior of all agents of a given type into a single agent. The degree of integration of the economy is then described by the distribution of production means between the non-integrated producer and the integrated firm. For the sake of simplicity we suppose that all

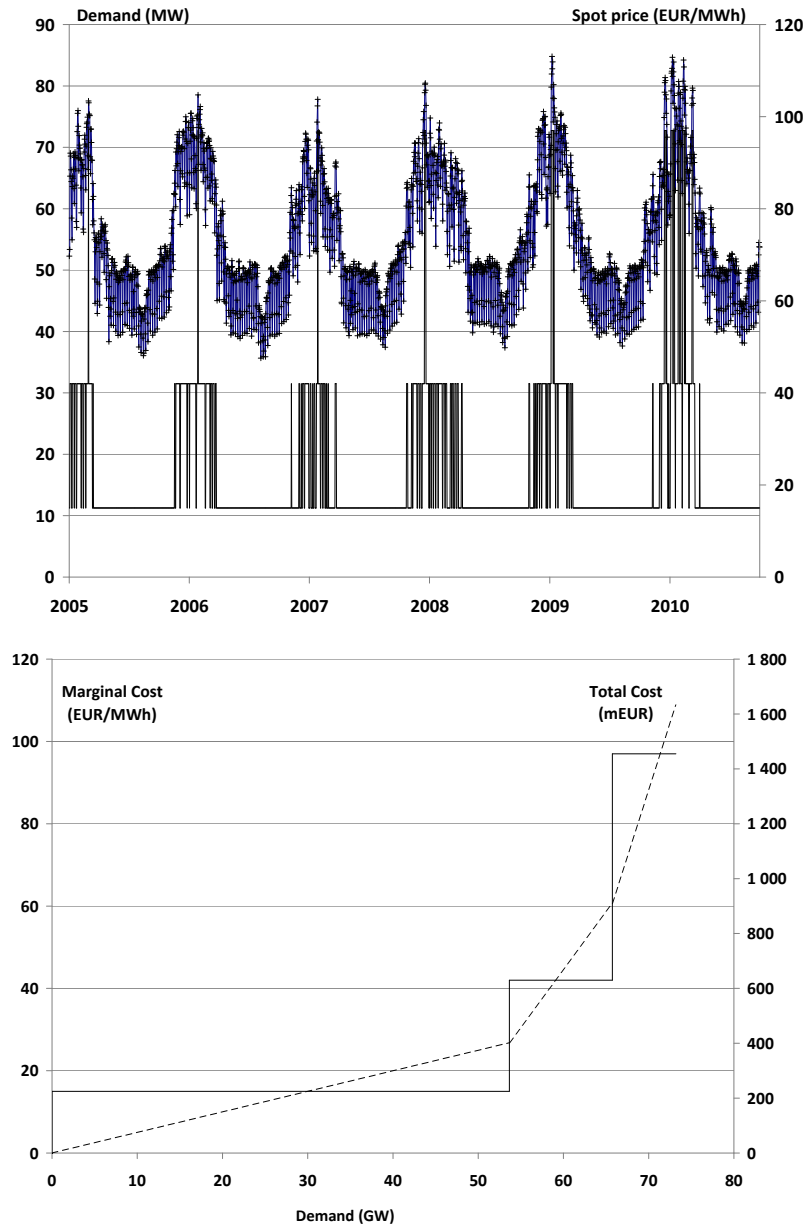


Figure 5.1: Above: Demand (line with crosses) and computed equilibrium spot prices (continuous line). Below: Marginal cost (continuous line) and total cost functions (breaking line).

types of productive assets are split between the two agents in equal proportions, i.e. the integrated agent owns $x\%$ of the hydro, nuclear, coal/gas and fuel total capacities while the non-integrated producer owns $(1 - x)\%$ of the capacities. Hence, the degree of integration of the economy is entirely described by the fraction x of productive assets owned by the integrated producer. In addition, the production profit of integrated producers is equal to $x\%$ of the total production profit.

Finally, the numerical analysis requires numerical estimates of the risk aversion coefficients. For simplicity, we set equal risk aversion coefficients for all agents. That coefficient needs to be sufficiently low so the forward prices remain positive.²¹ With the other parameter values set above, the model has solutions with risk aversion coefficients of about 10^{-7} and below.

5.2 Price Behavior

We first focus on forward and retail prices with the different market structures.

To analyze the behavior of forward prices, we run our numerical analysis with our 4 agents. We let the value of the risk aversion coefficient vary from 10^{-11} to 10^{-7} and the degree of integration of the economy vary from 0% to 100%.

We find that the risk premium $q^* - \mathbb{E}[w^*]$ is always negative, with forward prices always below the expected spot price. In addition, the intensity of the risk premium is very moderate: For risk aversion coefficients between 10^{-11} to 10^{-7} , the risk premium varies between 0 and -1 euro, while the expected spot price $\mathbb{E}[w^*]$ is equal to 23 euros. This confirms that the electricity forward market is a production driven market, but that the forward price risk premium is small compared to the expected spot for any market structure.

The pattern for the retail risk premium is different. The retail risk premium $p^* - \mathbb{E}[w^*]$ is always positive and varies significantly with both the level of risk aversion and integration than the forward risk premium. Figure 5.2 shows the retail price as a function of the level of integration in the economy for individual risk aversion coefficients of 10^{-7} , 10^{-8} and 10^{-9} , both without and with a forward market.

Without a forward market the retail risk premium can exceed 30 euros and it varies substantially with the market structure and the risk aversion coefficient. The highest risk premium is obtained in a market served by a non-integrated retailer buying her power to a non-integrated producer: The risk premium can reach 300% of the expected spot price for a reasonable level of risk aversion. The presence of a forward market reduces this dependency and makes the retail price less sensitive to both the level of integration and to the level of risk aversion. The retail risk premium is then considerably lower and very stable, with values varying between 2.00 and 2.40 euros.

5.3 Market Share Behavior

The retailer's market share depends on 3 factors: the availability of a forward market, the level of integration in the industry, and the risk aversion coefficient. Separately these parameters have a significant impact on market shares: The forward market increases substantially the market share of the non-integrated retailer, while the market share of

²¹This is due to our mean-variance setting. The intensity of the aggregate coefficients Λ and $\Lambda_{\mathcal{R}}$ and the intensity of the individual risk aversion coefficients for each type of agent (non-integrated retailer, non-integrated producer, outside trader, and integrated firm) .

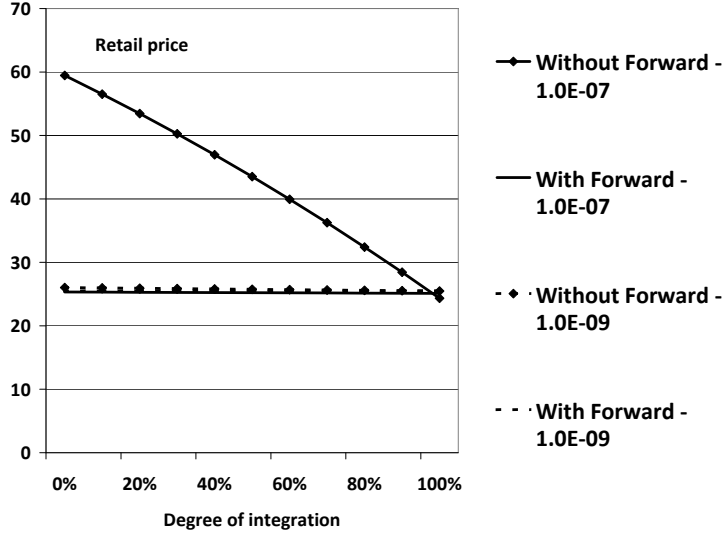


Figure 5.2: Variations of the retail price with the degree of integration for various risk aversions with and without a forward market.

the non-integrated retailer decreases with both the level of integration in the economy and the risk aversion coefficient. The joint impact of these 3 parameters is less obvious.

In a market where 100% of the production is integrated and there is a non-integrated retailer (1 non-integrated retailer and 1 integrated firm), the non-integrated retailer who cannot trade forward has a 0% market share whatever its risk aversion. However when there is a forward market it can obtain a market share of up to 30% if it has the same risk aversion as the integrated firm. This market share can be even higher (up to 65%) if the retailer is significantly less risk averse than the integrated firm. Similarly, in a market with both non-integrated producers and integrated agents (1 retailer, 1 producer, 1 outside trader and 1 integrated firm), the integrated firm as risk averse as the retailer obtains a larger market share than the retailer. The difference in the two market shares is all the more important as the level of integration is high. This advantage remains with a forward market but it is significantly reduced. If the producer is less risk averse than the integrated firm, she can even obtain a larger market share than the integrated agent.

Hence, this example shows that even with integrated firms, the forward market can help retailers obtain a significant market share.

5.4 Impact on Utility Functions

Figure 5.3 illustrates the impact of a forward market and of integration on utilities for a risk aversion coefficient of 10^{-7} . First, it illustrates that for any given level of integration of the economy, a non-integrated producer is always better off when there is a forward market, which is not true for the non-integrated retailer. Without a forward market, we have seen that the non-integrated retailer can charge a high retail risk premium. The

hedging benefit provided by a forward market leads to a lower volatility of her profit, but this does not compensate the loss in her expected profit due to an important decrease in the retail price.

Second, Figure 5.3 illustrates that the integrated firm obtains much of her utility from the retail market: For any given level of integration of the economy, a forward market always decreases her utility, which is the same qualitative behaviour as a non-integrated retailer.

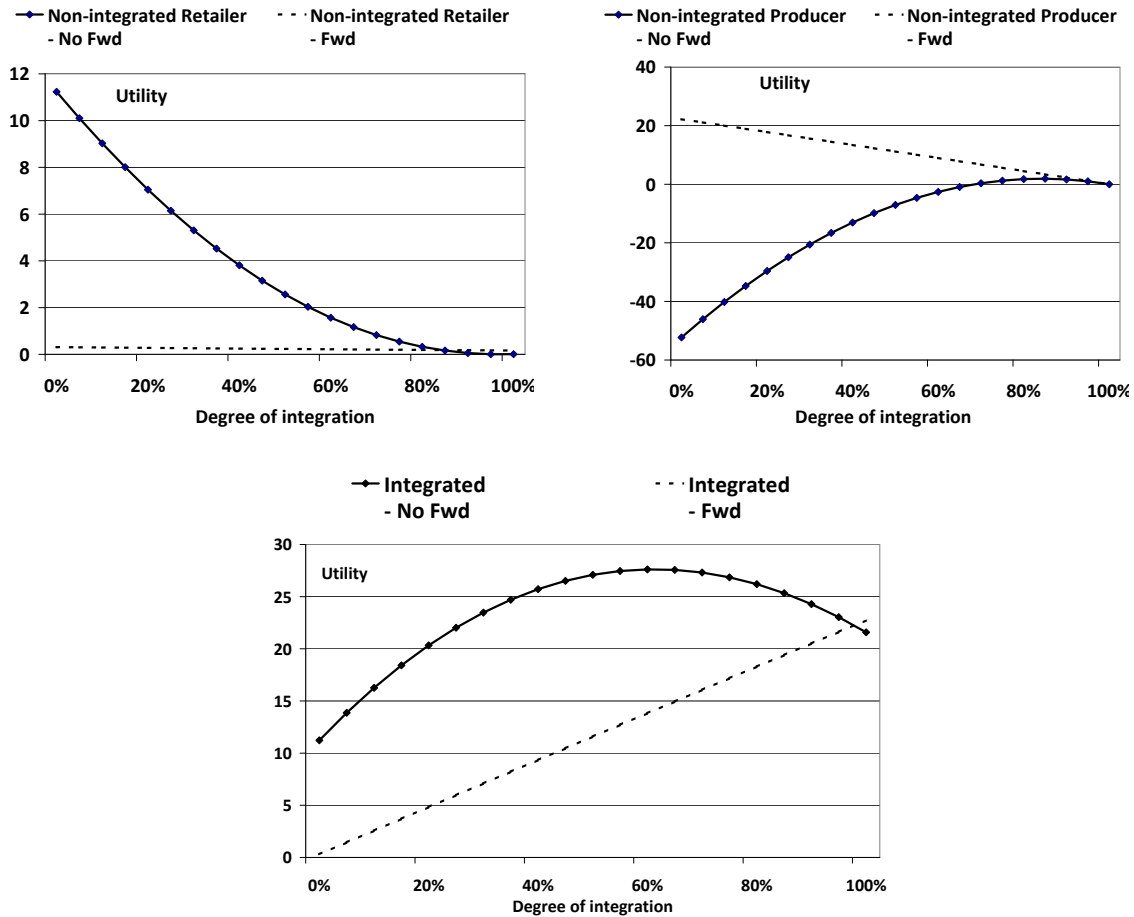


Figure 5.3: Variations of utility with the degree of integration for different agents with and without a forward market.

The impact of a forward market on utilities pertains for lower risk aversion coefficient, but its magnitude may vary. We performed a test with a risk aversion coefficient down to 10^{-9} . The impact of a forward market on the utility of a non-integrated retailer was essentially unchanged. Meanwhile the benefit of the forward market on the utility of a non-integrated producer was reduced drastically. She still benefits from a forward market but since she behaves nearly like a risk-neutral agent, the benefit is reduced by a factor of one hundred. Finally, the impact of a forward market on the integrated agent is a mix between the two effects above.

6 Empirical Predictions and Concluding Remarks

This paper has developed a model of an electricity industry with (wholesale) spot, retail, and forward markets in which both forward markets and vertical integration reduce retail prices. Vertical integration exhibits properties that linear instruments such as forward contracts cannot achieve. It eliminates the asymmetric risk structure between production and retail segments. We have shown throughout that some level of integration has a number of important risk diversification benefits even in the presence of forward markets.

Our analysis predicts that there is a negative relationship between the development of forward markets and firms' incentives to merge with vertically-related segments. Other things equal, we expect the prevalence of vertical integration to be higher in industries that are more subject to uncertainty and where other risk management mechanisms are less readily available.

This should also be true in industries where goods are non storable or where storage costs are higher. Since, when goods are durable, production timing can be adjusted so as to manage risk, forward markets and vertical integration should be less important than in industries that have non-durable goods. This is undoubtedly a reason why risk management and vertical integration are central to the ongoing reforms in the electricity industry.

In addition, be there developed forward markets or not, we expect vertical integration to be more widespread in industries that are subject to greater risk aversion, in particular through greater regulatory pressure, higher bankruptcy costs and higher corporate taxes.

We have highlighted risk management aspects of vertical integration by ruling out issues related to imperfect competition. Such concerns would undoubtedly bring about additional features. For example, Allaz (1992) has pointed out that forward trading may reduce producers' market power, which would likely reduce the surplus generated by the upstream segment. In contrast, vertical integration sometimes extends market power (Bolton and Whinston (1993), Chemla (2003), Rey and Tirole (2006)), which may well have an upward effect on both spot and retail prices in the presence of powerful producers. Our analysis could then serve as a benchmark to quantify the effects of imperfect competition.

Another avenue for future research may be to examine the decision of whether or not trading activities and operating activities should be under the same corporate roof. Such analysis would likely combine issues related to trading and arbitrage with transactions costs, e.g. financial constraints, as in Gromb and Vayanos (2002, 2010), with questions such as those discussed in this paper.

A Appendix: Proofs

A.1 Proof of Proposition 3.1

Maximizing the mean-variance criteria over α_k yields the following first order condition for agent $k \in \mathcal{R}$:

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\lambda_k \text{Cov}[(p^* - w^*)D, (p^* - w^*)\alpha_k^* D + \Pi_k^g],$$

By convexity, the first-order condition is sufficient and gives:

$$\alpha_k^* = \frac{\mathbb{E}[(p^* - w^*)D]}{2\lambda_k \text{Var}[(p^* - w^*)D]} - \frac{\text{Cov}[(p^* - w^*)D, \Pi_k^g]}{\text{Var}[(p^* - w^*)D]}.$$

The coupling constraint $\sum_{k \in \mathcal{R}} \alpha_k^* = 1$ gives the condition on p^* :

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, \Pi_I^g],$$

and allows us to derive (3.3).

A.2 Proof of Proposition 3.2

Assume that some retailers are risk-neutral, so that the aggregate risk aversion coefficient $\Lambda_{\mathcal{R}}$ also tends to zero. Suppose equation (3.3) has two non-negative solutions $p_- \leq p_+$. The Taylor expansion of these roots around $\Lambda_{\mathcal{R}} = 0$ reads:

$$\begin{aligned} p_- &\simeq p^0 + \frac{2\Lambda_{\mathcal{R}}}{\mathbb{E}[D]\text{Var}[D]} \left(\text{Var}[D]\text{Cov}[wD, wD - \Pi_I^g] - \text{Cov}^2[D, wD - \frac{1}{2}\Pi_I^g] \right. \\ &\quad \left. + 2\text{Cov}^2[D, wD - \frac{1}{2}\Pi_I^g - p^0 D] \right) \\ p_+ &\simeq \frac{\mathbb{E}[D]}{2\Lambda_{\mathcal{R}}\text{Var}[D]}. \end{aligned}$$

Hence, as $\Lambda_{\mathcal{R}}$ tends to 0, p_- converges to p^0 , while p_+ tends to infinity. Thus, p_- is the only possible economically relevant root, and the equilibrium is unique.

A.3 Proof of Proposition 3.3

Let p_{NI}^* be the smallest solution to (3.3) in the absence of vertical integration, and p_I^* be this solution when at least one producer is integrated with a retailer. Suppose that $\mathcal{R} \cap \mathcal{P} = \emptyset$, i.e. there is no integrated producer who have both generation and retail activities. Equation (3.3) then boils down to:

$$0 = \mathbb{E}[(p_{NI}^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p_{NI}^* - w^*)D]. \quad (\text{A.1})$$

Hence, it is also greater than the expected spot price.

When there are integrated agents, i.e. $\mathcal{R} \cap \mathcal{P} \neq \emptyset$, the equation that determines the retail price can be written

$$0 = \mathbb{E}[(p_I^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p_I^* - w^*)D, (p_I^* - w^*)D + \Pi_I^g].$$

Suppose that one retailer, say i , chooses to vertically integrate with one producer. Then, $\Lambda_{\mathcal{R}}$ is unchanged and $\Pi_I^g = \Pi_i^g$.

Since Π^r and Π_I^g are negatively correlated,

$$0 \geq \mathbb{E}[(p_I^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p_I^* - w^*)D]. \quad (\text{A.2})$$

Inequality (A.2), combined with (A.1), implies that p_I^* is lower than p_{NI}^* .

A.4 Proof of Proposition 4.1

If $k \in \mathcal{R}$, the first order condition to profit maximization can be written:

$$0 = M \begin{bmatrix} f_k^* \\ \alpha_k^* \end{bmatrix} - \begin{bmatrix} \frac{\mathbb{E}[w^* - q^*]}{2\lambda_k} - \text{Cov}[P^*, \Pi_k^g] \\ \frac{\mathbb{E}[(p^* - w^*)D]}{2\lambda_k} - \text{Cov}[(p^* - w^*)D, \Pi_k^g] \end{bmatrix},$$

where M is the variance-covariance matrix of vector $[w^*, (p^* - w^*)D]$. By inverting the system, we obtain f_k^* and α_k^* as functions of p^* and q^* . If $k \notin \mathcal{R}$, Π_k does not depend on α_k and the first order condition satisfies:

$$0 = \mathbb{E}[w^* - q^*] - 2\lambda_k \text{Cov}[w^* - q^*, (w^* - q^*)f_k^* + \Pi_k^g].$$

The market-clearing constraint (2.4) can be written:

$$\begin{aligned} 0 &= \sum_{k \in \mathcal{K}} f_k^* \\ &= \frac{\text{Var}[(p^* - w^*)D]}{\Delta} \left(\frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\ &\quad - \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\Delta} \left(\frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\ &\quad + \frac{\mathbb{E}[w^* - q^*]}{2\text{Var}[w^*]} \left(\frac{1}{\Lambda} - \frac{1}{\Lambda_{\mathcal{R}}} \right) - \frac{\text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g]}{\text{Var}[w^*]}, \end{aligned}$$

where Δ is the determinant of M . This leads to:

$$\begin{aligned} 0 &= \Delta \left(\frac{\mathbb{E}[w^* - q^*]}{2\Lambda} - \text{Cov}[w^*, \sum_{k \in \mathcal{K}} \Pi_k^g] \right) \\ &\quad - \text{Cov}[w^*, (p^* - w^*)D] \text{Var}[w^*] \left(\frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\ &\quad + \text{Cov}^2[w^*, (p^* - w^*)D] \left(\frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right). \end{aligned} \quad (\text{A.3})$$

The load satisfaction constraint (2.1) reads:

$$\begin{aligned}
1 &= \sum_{k \in \mathcal{R}} \alpha_k^* \\
&= -\frac{\text{Cov}[w^*, (p^* - w^*)D]}{\Delta} \left(\frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\
&\quad + \frac{\text{Var}[w^*]}{\Delta} \left(\frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right),
\end{aligned}$$

which yields, using (A.3):

$$0 = \frac{\mathbb{E}[w^* - q^*]}{2\Lambda} - \text{Cov}[w^*, (p^* - w^*)D] + \sum_{k \in \mathcal{K}} \Pi_k^g.$$

As $\sum_{k \in \mathcal{K}} \Pi_k^g = w^*D - C(D)$, we obtain equation (4.3). Using this result to simplify (A.3), we derive the desired (4.4). Finally, from these two equations, we can re-arrange for f_k^* and α_k^* to obtain (4.1) and (4.2).

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