# Multi-scale Finite Element Method for incompressible flow in perforated domain

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## Abstract

Simulating the flow in a multi-scale media with many obstacles, such as nuclear reactor cores, is very challenging. Indeed, to capture the finest scales of the flow, one needs to use a very fine mesh, which often leads to intractable simulations due to the lack of computational resources. To overcome this limitation, various multi-scale methods have been developed in the literature to attempt to resolve scales below the coarse mesh scale. In this contribution, we focus on the Multi-scale Finite Element Method (MsFEM).

The MsFEM uses a coarse mesh on which one defines basis functions which are no longer the classical polynomial basis functions of finite elements, but which solve fluid mechanics equations on the elements of the coarse mesh. These functions are themselves numerically approximated on a fine mesh taking into account all the geometric details, which gives the multi-scale aspect of this method.

Based on the work of [1, 2], we develop an enriched non-conforming MsFEM to solve viscous incompressible flow in heterogeneous media. Our MsFEM is in the vein of the classical non-conforming Crouzeix-Raviart finite element method with high-order weighting functions. We perform a rigorous theoretical study of our MsFEM in two and three dimensions at both continuous and discrete levels. At the numerical level, we implement the MsFEM to solve the Stokes and the Oseen problems, in two and three dimensions, in a massively parallel framework in FreeFEM [3].

The perspective of this work is to be able to solve the Navier-Stokes equations in a perforated domain at high Reynolds number using MsFEM basis functions. Furthermore, to complete the study of our MsFEM, we are investigating on an a posteriori error estimate for MsFEM.

**Keywords**: Multi-scale Finite Element Method, Incompressible Flow, Stokes equations, Oseen equations, Parallel computing.

## References

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