# Mesoscopic analysis of ecological networks using Hill numbers



*Jump In* Jackson Pollock



OSUG

Marc Ohlmann PhD supervisor : Wilfried Thuiller Aim : assess the diversity of one or several ecological communities that are interacting through an ecological network

 $\alpha$ -diversity : *richness of a community* 

β-diversity : "the extent of change in community composition, or degree of community differentiation, in relation to a complex-gradient of environment, or a pattern of environments" Whittaker 1960

Classic diversity index omit interactions



Conclusion

Measuring diversity of a community ?

A diversity of metrics

Shannon entropy

Simpson index

Unifying framework : True diversity (Hill numbers, 1973)

set of organisms colours are species



Suppose that (E) is composed of N individuals, belonging to S distinct species, with relative abundances :  $(p_1, ..., p_s)$ 

$$M_{q-1} = \sqrt[q-1]{\sum_{i=1}^{S} p_i p_i^{q-1}} \quad \text{Gen}$$

$$D_q = 1/M_{q-1} = (\sum_{i=1}^{S} p_i^q)^{1/(1-q)}$$

$$D_1 = exp(-\sum_{i=1}^{S} p_i log(p_i))$$

Generalized mean of order q-1

$$D_q = 1/M_{q-1} = (\sum_{i=1}^S p_i^q)^{1/(1-q)}$$
 Hill number of order q



order q

$$D_q = 1/M_{q-1} = (\sum_{i=1}^{S} p_i^q)^{1/(1-q)}$$
 Hill number of

$$D_0 = S$$
 Species richness

$$D_1 = exp(\sum_{i=1}^S p_i log(p_i))$$
 Shannon entropy (exp)  
 $D_2 = rac{1}{\sum_{i=1}^S p_i^2}$  Inverse of Simpson inc

Inverse of Simpson index



order q

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### Examples and simulations

## Conclusion

Comparing two communities

β-diversity concept : "The extent of change in community composition, or degree of community differentiation, in relation to a complexgradient of environment, or a pattern of environments' Whittaker, 1960

Meta-community



community 2

community 1

## Examples and simulations

### Conclusion

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β-diversity concept :

"The extent of change in community composition, or degree of community differentiation, in relation to a complexgradient of environment, or a pattern of environments' Whittaker, 1960



$$\beta(q) = \begin{cases} \frac{\gamma(q)}{(\omega(\alpha_1(q))^{1-q} + (1-\omega)(\alpha_2(q))^{1-q})^{\frac{1}{1-q}}} & q \neq 1\\ \frac{\gamma(q)}{exp(\omega \log(\alpha_1(q)) + (1-\omega)\log(\alpha_2(q)))} & q = 1 \end{cases}$$
$$\omega = \frac{n_1}{n_1 + n}$$
$$1 \leq \beta(q) \leq 2\\ \beta(q) \leftarrow \beta(q) - 1\\ 0 \leq \beta(q) \leq 1 \end{cases}$$

ratio of generalised means in presence of two classifications (species and space) cf Tuomisto, 2010, Ecography





Interaction network

Introduction	Definitions	Examples and simulations	Conclusion
G a network ,A its adjacency matrix, V(G) set of nodes, $ V(G)  = n$ E(G) set of edges, $ E(G)  = LQ set of classes(e.g. species, functional groups)$		roups)	

# Definitions

G a network , A its adjacency matrix, V(G) set of nodes, |V(G)| = nE(G) set of edges, |E(G)| = LQ set of classes(e.g. species, functional groups)

#### Connectance

$$\begin{array}{l} \mathbf{Q}{=}\{q\}\\ \pi=Pr(i\rightarrow j|i,j\in q)\\ C=\hat{\pi}=L/S^2 \end{array}$$

Connectance does not take into account species identity

Not a measure of diversity, since there is only one group



# Definitions

G a network , A its adjacency matrix, V(G) set of nodes, |V(G)| = nE(G) set of edges, |E(G)| = LQ set of classes(e.g. species, functional groups)

#### Connectance

$$\begin{array}{l} \mathbf{Q} = \{q\} \\ \pi = Pr(i \rightarrow j | i, j \in q) \\ C = \hat{\pi} = L/S^2 \end{array}$$

Not a measure of diversity, since there is only one group

#### Link number

$$Q = \{S_1, ..., S_n\} \\ \pi_{i,j} = Pr(i \to j | , i \in S_i, j \in S_j) \\ \hat{\pi}_{i,j} = A_{i,j} \in \{0, 1\} \\ L = \sum_{1 \le i,j \le n} A_{i,j}$$

# Measure of $\alpha$ -diversity















#### Conclusion

G a network ,A its adjacency matrix, V(G) set of nodes, |V(G)| = nE(G) set of edges, |E(G)| = L $Q = \{1, ..., |Q|\}$   $1 \le |Q| \le n$ 

$$\alpha_k = \frac{\sum_{i=1}^n \mathbf{1} \mathbf{I}(i \in k)}{n}$$
$$l_{k,l} = \frac{\sum_{i=1}^n \mathbf{1} \mathbf{I}(i \to j, i \in k, j \in l)}{L}$$
$$\pi_{k,l} = \Pr(i \to j | i \in k, j \in l)$$
$$\hat{\pi}_{k,l} = \frac{\sum_{i=1}^n \mathbf{1} \mathbf{I}(i \to j, i \in k, j \in l)}{\sum_{i=1}^n \mathbf{1} \mathbf{I}(i \in k) \sum_{j=1}^n \mathbf{1} \mathbf{I}(j \in l)}$$

$$D_{q,\alpha} = \left(\sum_{k=1}^{|Q|} \alpha_k^q\right)^{\frac{1}{1-q}} D_{q,l} = \left(\sum_{1 \le k, l \le |Q|} l_{k,l}^q\right)^{\frac{1}{1-q}} D_{q,\pi} = \left(\sum_{1 \le k, l \le |Q|} \left(\frac{\pi_{k,l}}{\sum_{k,l} \pi_{k,l}}\right)^q\right)^{\frac{1}{1-q}}$$

Hill number on group proportion Hill number on links proportion

Hill number on connectance matrix

Conclusion

### The network dissimilarity problem

 $\begin{array}{l} G_1 \ a \ network, A_1 \ its \ adjacency \ matrix \\ V(G_1) \ set \ of \ nodes, |V(G_1)| = n_1 \\ E(G_1) \ set \ of \ edges, |E(G_1)| = L_1 \end{array}$ 

 $\begin{array}{l} G_2 \ a \ network, A_2 \ its \ adjacency \ matrix \\ V(G_2) \ set \ of \ nodes, |V(G_2)| = n_2 \\ E(G_2) \ set \ of \ edges, |E(G_2)| = L_2 \end{array}$ 



## Macroscopic comparison

Q : set of groups of the metaweb **Elton niches= species that have a similar position in the network** Q={q} Compare connectance of the two networks



## Charles Elton (1900-1991)

# Microscopic comparison

$$Q_{1} = \{S_{1}, \dots, S_{n_{1}}\}$$

$$Q_{2} = \{S_{1}, \dots, S_{n_{2}}\}$$

$$Q_{meta} = Q_{1} \cup Q_{2}$$

$$Q_{inter} = Q_{1} \cap Q_{2}$$

What already exists : Poisot, 2012

$$\beta_{WN} = 1 - \frac{A_1 \odot A_2}{(|A_1| + |A_2|)/2}$$

## Microscopic comparison

$$Q_{1} = \{S_{1}, ..., S_{n_{1}}\}$$
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$$Q_{meta} = Q_{1} \cup Q_{2}$$
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What already exists : Poisot, 2012

$$\beta_{WN} = 1 - \frac{A_1 \odot A_2}{(|A_1| + |A_2|)/2}$$

Both species turnover and plasticity of interactions (at a species level) contribute to  $\beta_{_{WN}}$ 

Aim : seperate these two effects

# Microscopic comparison



$$G_{1}^{inter} = induced.subgraph(G_{1}, Q_{inter})$$

$$A_{1}^{inter} : adjacency matrix$$

$$G_{2}^{inter} = induced.subgraph(G_{2}, Q_{inter})$$

$$A_{2}^{inter} : adjacency matrix$$

## **Microscopic comparison**





Mesoscopic comparison?





$$\alpha_{G_{meta},k} = \omega \alpha_{G_2,k} + (1-\omega) \alpha_{G_1,k}$$
$$\omega = \frac{n_1}{n_1 + n_2}$$
$$l_{G_{meta},k} = \omega l_{G_2,k} + (1-\omega) l_{G_1,k}$$
$$\omega = \frac{L_1}{L_1 + L_2}$$

$$\alpha_{G_{meta},k} = \omega \alpha_{G_2,k} + (1-\omega)\alpha_{G_1,k}$$
$$\omega = \frac{n_1}{n_1+n_2}$$
$$l_{G_{meta},k} = \omega l_{G_2,k} + (1-\omega)l_{G_1,k}$$
$$\omega = \frac{L_1}{L_1+L_2}$$



$$\beta_{\alpha}(q) = \frac{D_{\alpha}^{G_{meta}}(q)}{(\omega(D_{\alpha}^{G_{1}}(q))^{1-q} + (1-\omega)(D_{\alpha}^{G_{2}}(q)^{1-q})^{\frac{1}{1-q}}} \beta_{L}(q) = \frac{D_{L}^{G_{meta}}(q)}{(\omega(D_{L}^{G_{1}}(q))^{1-q} + (1-\omega)(D_{L}^{G_{2}}(q)^{1-q})^{\frac{1}{1-q}}}$$

$$\beta_{\alpha}(q) = \frac{D_{\alpha}^{G_{meta}}(q)}{(\omega(D_{\alpha}^{G_{1}}(q))^{1-q} + (1-\omega)(D_{\alpha}^{G_{2}}(q)^{1-q})^{\frac{1}{1-q}})}$$
$$\beta_{L}(q) = \frac{D_{L}^{G_{meta}}(q)}{(\omega(D_{L}^{G_{1}}(q))^{1-q} + (1-\omega)(D_{L}^{G_{2}}(q)^{1-q})^{\frac{1}{1-q}})}$$





G1

G2

$$1 \le \beta_{\alpha,L}(q) \le 2$$
  
$$\beta_{\alpha,L}(q) \leftarrow \beta_{\alpha,L}(q) - 1$$
  
$$0 \le \beta_{\alpha,L}(q) \le 1$$

A particular case : Tim Poisot case (microscopic turnover)

$$Q_1 = \{1, ..., n_1\} Q_2 = \{1, ..., n_2\} \omega = 1/2$$

Easy to show that :

$$\beta_{\alpha}(0) = \beta_{S}$$
$$\beta_{L}(0) = \beta_{WN}$$

A particular case : Tim Poisot case (microscopic turnover)

 $Q_1 = \{1, ..., n_1\}$  $Q_2 = \{1, ..., n_2\}$  $\omega = 1/2$ 

Easy to show that :

$$\beta_{\alpha}(0) = \beta_{S}$$
$$\beta_{L}(0) = \beta_{WN}$$

Do change of link diversity reflect change in connectivity ? Yes, but there is a group size effect too !

Here, we work on :  $\begin{array}{c} {
m G}_1^{inter} \\ {
m G}_2^{inter} \end{array}$ 

$$\begin{split} & \left( \pi_{k,l}^{G_1^{inter}} \right)_{1 \leqslant k,l \leqslant |Q_{inter}|} \\ & \left( \pi_{k,l}^{G_2^{inter}} \right)_{1 \leqslant k,l \leqslant |Q_{inter}|} \\ & \pi_{k,l}^{G_{meta}^{inter}} = \omega \pi_{k,l}^{G_1^{inter}} + (1-\omega) \pi_{k,l}^{G_2^{inter}} \end{split}$$

$$D_{\pi}(q) = \frac{1}{(\omega(D_{\pi}^{G_{1}^{inter}}(q))^{1-q} + (1-\omega)(D_{\pi}^{G_{2}^{inter}}(q)^{1-q})^{\frac{1}{1-q}}}$$

A particular case : Tim Poisot case (microscopic scale)

$$egin{aligned} & \mathbb{Q}_1 = \{1, ..., n_1\} \ & Q_2 = \{1, ..., n_2\} \ & \omega = 1/2 \end{aligned} \qquad eta_\pi(0) = eta_{OS} \end{aligned}$$







Forbidden links between species ?



(A)

Frequency

(B)

Frequency

#### Definitions

# Examples and simulations A trait perspective

#### Forbidden links between species ?



Figure 1. Frequency Distribution of Matching Traits (e.g., Body Size or Phenology) in a Consumer and a Resource Species. Interaction is possible whenever  $x_{\text{consumer}} \ge x_{\text{resource}}$ . (A) Mismatching between both trait means and intraspecific variability prevent interaction, leading to a totally forbidden link. (B) Mismatching occurs between trait means but intraspecific variability allows interaction, leading to a partially forbidden link. The difference between (A) and (B) thus, our ability to infer interactions - may depend on how broadly intraspecific trait variability has been assessed in space and time (Box 1).

These binary relations neglect the intraspecific trait variability compared to interspecific trait variability

Taken from : *The labile limit of forbidden interactions* Gonzalez-Varo Trends in Ecology and Evolution, 2016



 $\mu_{consumer}$ 

Totally forbidden link

Partially forbidden link

Allowed

interaction

Trends in Ecology & Evolution

 $\mu_{resource}$ 



 $Pr(i \to j | i \in A, j \in B) = \pi$ 

 $\Pr(i \rightarrow j | i \in A, j \in B) = 1$ 

 $Pr(i \rightarrow j | i \in C, j \in D) = 0$ 

Conclusion

Conclusion

Can we neglect intra-specific traits variability ?



A mono-trophic view of traits intra/interspecific variability

Taken from :

A multi-trait approach reveals the structure and the relative importance of intra- vs. interspecific variability in plant traits Albert et al., Functionnal Ecology, 2010

Conclusion

Can we neglect intra-specific traits variability ?



A mono-trophic view of traits intra/interspecific variability

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For trophic networks : Interaction is the resultant of a match between a vulnerability trait (prey) and a foraging trait (predator) see : Gravel et al., 2016



Examples and simulations A trait perspective

Conclusion

For trophic networks : Interaction is the resultant of a match between a vulnerability trait (prey) and a foraging trait (predator) see : Gravel et al., 2016



# Examples and simulations A trait perspective



Definitions

# relevant in that case

Introduction



Taken from :

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#### Definitions

# Examples and simulations A trait perspective





Macroscopic scale : no variations Microscopic scale : too much variations Mesoscopic scale : strong pattern

#### Definitions

Examples and simulations Rarefactions curves

## Aim : Test the robustness of mesoscopic metrics to incomplete sampling

Using a model of food web :

the niche model (Williams and Martinez, 2000)

Inferring classes of nodes using Stochastic Block Model ( a model of community detection)



Taken from : Simple rules yield complex food webs, Williams and Martinez, 2000, Nature



Rarefaction curves : how robust are our metrics to incomplete species sampling ?



number of species sampled

A framework using Hill numbers that allows network comparison at different scales, from macroscopic scale to microscopic scale

Ecological networks might evolve at different Elton niches scales, especially if you're interested in microbial/soil ecology

Mesoscopic analysis is more robust to incomplete sampling

Key question : *how to determine Elton niches* ? Using network topology ? Traits ?