Modèles structurés multi-niveaux de dynamiques épidémiques

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Motivation and goals

School and/or workplace closures: efficient non-pharmaceutical interventions for mitigating epidemics, *e.g.* COVID-19 and influenza (Mendez-Brito et al., 2021; Luca et al., 2018).

⇒ Models with several levels of mixing: explicitly distinguish different types of contact.

⇒ Aim of the present study:

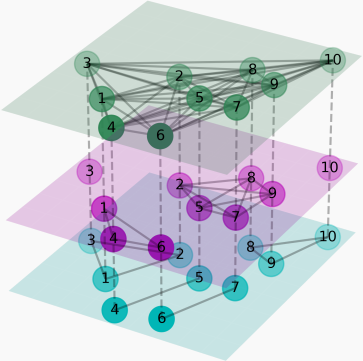
1. Better understanding of the epidemic impact of small contact structures: indicators capturing this impact?
2. Propose reduced models = approximate models that are more prone to mathematical analysis / numerical exploration.

A multilayer model with households and workplaces

Population structure

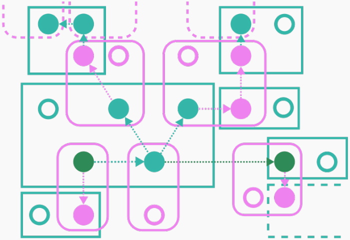
Model introduced by Pellis et al., 2009.

1. **Global** level of mixing $\rightarrow$ homogeneously mixing general population.
2. **Local** level of mixing $\rightarrow$ households and workplaces:
    - Structure size distributions π^H and π^W, maximal size $n_{\max} < \infty$.
    - Each individual is attributed to a household and workplace independently from one another and from other individuals.



Epidemic process

Modified *SIR* model $\rightarrow$ three ways of contamination in a population of size K:



- **General population**: total of S susceptible and I infected individuals $\rightarrow$ infections at rate $\frac{\beta_G}{K} SI$.
- **Intra-household** or **intra-workplace**: s susceptible and i infected members $\rightarrow$ infections at rate $\lambda_X si$ for $X \in \{H, W\}$, respectively.

Duration of infectious periods $\sim Exp(\gamma)$

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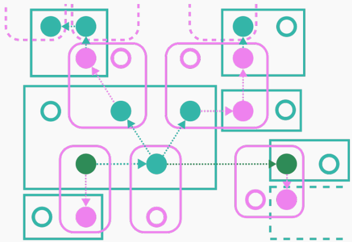


⇒ **Stochastic model of parameters**

$$(K, \underbrace{\pi^H, \pi^W}_{\text{social structure}}, \underbrace{\lambda_h, \lambda_w, \beta_g, \gamma}_{\text{epidemic}}).$$

R_0 and exponential growth rate

Reproduction number R_0: outbreak criterion (threshold $= 1$).

- Intuition: average number of contaminations caused by a 'typical' infected at the beginning of the epidemic.
- Several possible definitions $\rightarrow$ most of them use the fact that some correlations can be neglected at the beginning of the epidemic.
- R_0 introduced by Pellis et al., 2009 $\rightarrow$ associated to the proportions (p_H, p_W, p_G) of infection in each layer.

Exponential growth rate r: characterised by an implicit equation (Pellis et al., 2011) $\rightarrow$ can only be solved numerically.

The epidemiological footprint of contact structures

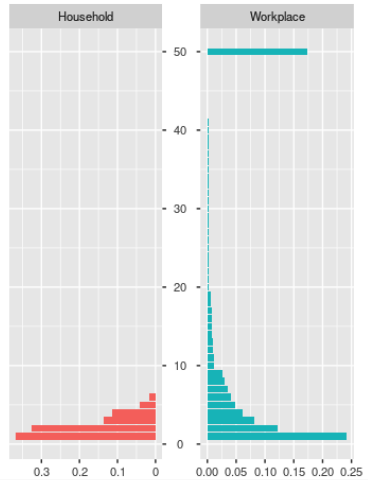
Bansaye, Deslandes, Kubasch and Vergu (2023)

Simulation scenarios

1. Population structure:
    - Large population ($K = 100{,}000$).
    - Fixed household size distribution.
    - Varied workplace size distributions (sizes 1 to 50).

   $\rightsquigarrow$ Teleworking.

2. Epidemic parameters: scenarios differ in terms of
    - Epidemic intensity (R_0, r).
    - Proportions of infection between layers (p_G, p_H, p_W).



The influence of the variance of workplace size distribution of fixed average on key features of the epidemic

We are going to focus on a setting where the size of the population and the number of workplaces are fixed, considering that the latter is given by logistic constraints.

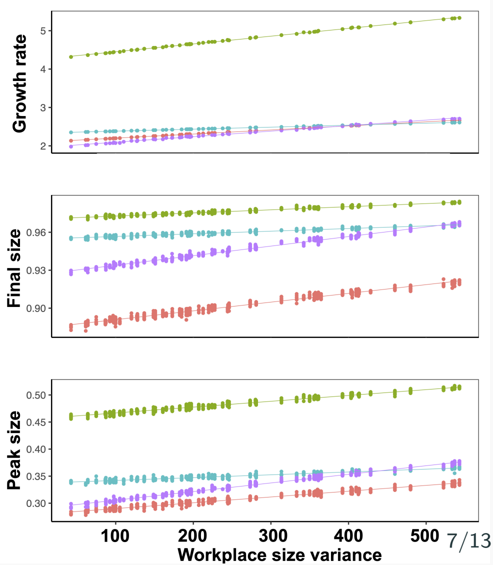
$\Rightarrow$ This implies that the average workplace size is fixed.

We are going to focus on the influence of the workplace size variance on the exponential growth rate r, the peak size and the final size of the epidemic.

The influence of the variance of workplace size distribution of fixed average on key features of the epidemic

For each epidemic scenario $\rightarrow$ simulate epidemics for a variety of workplace distributions, of fixed mean and different variances.

$\Rightarrow$ Linear dependence on the variance: good proxy for the impact of π^W.



Approximation using a uniformly mixing SIR model

Proposed approach: approximate our model by a classical *SIR* model:

$$\begin{cases} S' = -\beta S I \\ I' = \beta S I - \gamma I \\ R' = \gamma I. \end{cases}$$

$\Rightarrow$ How to fit the parameters?

- Removal rate γ usually known (epidemiological expertise).
- Two natural candidates for calibrating β: $\beta = \gamma R_0$ or $\beta = r + \gamma$.

Approximation using a uniformly mixing SIR model

Proposed approach: approximate our model by a classical *SIR* model:

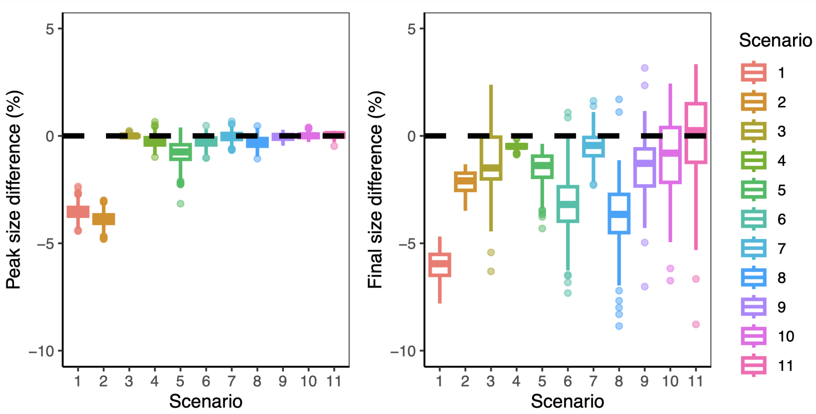
$$\begin{cases} S' = -(r+\gamma)SI \\ I' = (r+\gamma)SI - \gamma I \\ R' = \gamma I. \end{cases}$$

$\Rightarrow$ How to fit the parameters?

- Removal rate γ usually known (epidemiological expertise).
- Two natural candidates for calibrating β: $\beta = \gamma R_0$ or $\beta = r + \gamma$.

The growth rate allows to capture the epidemic footprint of social structures

Comparison of simulation outputs and reduced model predictions
⇒ satisfying results on key features of the epidemic:



Large population approximation

Kubasch (2023)

General idea

Large graph limit: well understood for epidemics on configuration models (Volz, 2008; Decreusefond et al., 2012) $\rightarrow$ no small closed structures.

Reduced models suggested in similar settings (House and Keeling, 2008; Volz et al., 2011) $\rightarrow$ epidemic at the level of structures characterised by a type x:

$$x = (n, s, i)$$

- n: structure size
- s: number of susceptibles
- i: number of infected

Problem: infected individuals correlate the epidemic states of their household and workplace.

General idea

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$$x = (n, s, \tau)$$

- n: structure size
- s: number of susceptibles
- τ: remaining infectious periods ($\tau \in \mathbb{R}^n$)

Solution: keep track of each infected's remaining infectious period (similar in spirit to Ball et al., 2014).

General idea

$\Rightarrow$ Measure-valued Markov process ζ^K describing the epidemic process in a population of size $K \to$ model reduction in two steps:

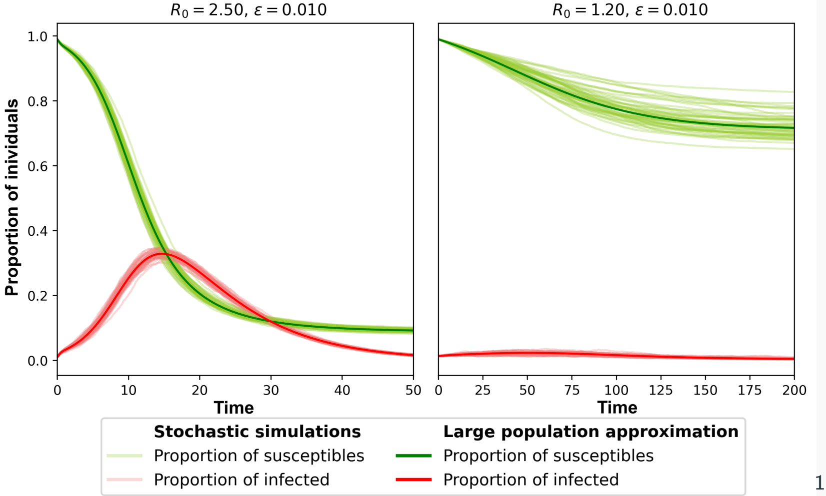
1. Tightness-identification-uniqueness strategy inspired by Tran, 2006 $\rightsquigarrow$ convergence in distribution when $K \to \infty$ to the unique **deterministic solution** η of :

$$\langle \eta_T^X, f_T \rangle = \langle \eta_0^X, f_0 \rangle + \int_0^T \langle \eta_t^X, \mathcal{A} f_t \rangle dt + \lambda_X \int_0^T \langle \eta_t^X, \mathbf{si}(f_t^{\mathcal{I}} - f_t) \rangle dt$$
$$+ \lambda_{\overline{X}} \int_0^T \frac{\langle \eta_t^{\overline{X}}, \mathbf{si} \rangle}{\langle \eta_t^{\overline{X}}, \mathbf{s} \rangle} \langle \eta_t^X, \mathbf{s}(f_t^{\mathcal{I}} - f_t) \rangle dt + \beta_G \int_0^T \frac{\langle \eta_t^H, \mathbf{i} \rangle}{\langle \eta_0^H, \mathbf{n} \rangle} \langle \eta_t^X, \mathbf{s}(f_t^{\mathcal{I}} - f_t) \rangle dt.$$

2. Extract from η a **finite-dimensional closed dynamical system**.

Simulations

Examples: Stochastic simulations (SSA) in a population of size $K = 10000$ compared to the reduced model.



Conclusion and perspectives

Key messages

- Proxy for the epidemic impact of the workplace size distribution: size distribution variance, exponential growth rate.
- Two reduced models:
  - Homogeneous mixing *SIR* model calibrated using the growth rate $\rightarrow$ key characteristics of the epidemic;
  - Large population approximation $\rightarrow$ asymptotically exact trajectories.
- Perspective: Closer study of the exponential growth phase?

Thank you for your attention!

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