Calibration of a pollination model using Approximate Bayesian Computation

joint work with Ullrika Sahlin, Yann Clough and Henrik G. Smith (from Lund University)

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Plan

1. Introduction
2. Pollination model
3. Data
4. Statistical model
5. Approximate Bayesian Computation
6. Results
7. Conclusion and discussion

Introduction

Introduction

- In 2000, the United Nations launches the *Millenium Ecosystem Assessment*
- Initial goal: “identify the impacts of ecosystem changes on human well-being, and actions needed to enhance the conservation and sustainable use of those systems”
- It popularized the term ecosystem services
  - the benefits humans obtain from ecosystems
  - e.g. : crop pollination, oxygen production by plants, carbon sequestration, ...
- One of the final recommendation was to assess these ecosystem services and include them in public policy decision making

Introduction

- One possible answer: develop models for ecosystem services, and use these models to evaluate the impact of changes (climate change, agricultural practices, management interventions, ...) on the ecosystems
- These models are often complex, and rarely calibrated on experimental data (rely on expert judgment, literature data, ...)

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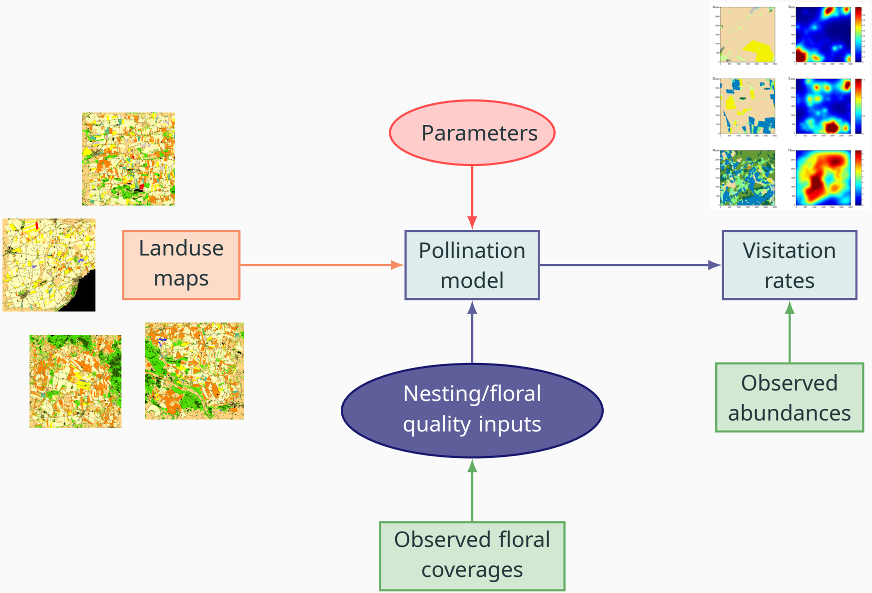
Objectives

- Build a mechanistic model for pollination at the landscape level
- Propose a methodology to calibrate the model on experimental data

Pollination model

General structure



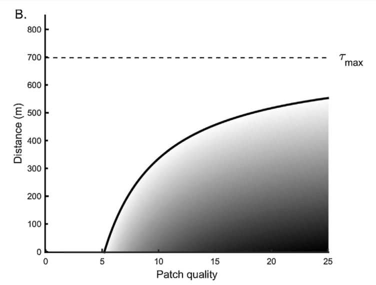
Central-place foragers model (Olsson et Bolin, 2014, Olsson et al. 2015)

Based on a fitness isocline curve

$$\tau_f = \tau_0 \left(1 - \frac{f_0}{f}\right),$$

with:

- f_0 minimum floral value that will be visited by bees
- τ_0 the maximum distance travelled by a bee



(Olsson et al. 2015)

CPF model - suitability of a nest

- For a bee nesting in patch i and floral patch j at distance d_{ij} :

$$\Delta_{ij} = \tau_0 \left(1 - \frac{f_0}{f_j}\right) - d_{ij},$$

$\rightarrow$ distance spare flying for patch j compared to what they were willing to fly for a patch of that quality.

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- Suitability or fitness of a nest in patch i:

$$s_i = \sum_j \Delta_{ij} \mathbf{1}_{\Delta_{ij} > 0},$$

$\rightarrow$ distance a bee will spare flying when its nest is surrounded by patches of good quality.

CPF model - optimization of foraging

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- A bee in a nest with high suitability exploit less patches further away compared to a bee in a nest with low suitability
- We define a new maximum distance the bee is prepared to travel from patch i:

$$\tau_i = \frac{\tau_0}{1 + \exp((\sqrt{s_i} - a)/b)}. \tag{1}$$

$\rightarrow$ patch-specific maximum distance.

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  $\rightarrow$ patch-specific maximum distance.
- We define:

$$\Delta^*_{ij} = \tau_i \left(1 - \frac{f_0}{f_j}\right) - d_{ij}, \tag{2}$$

  $\rightarrow$ contribution from patch j to fitness of the bees in patch i.

- The number of foraging bees from nest in patch *i* to floral resources in patch *j* is

$$r_{i \to j} = q_i \frac{\Delta_{ij}^*}{\sum_{j=1}^{J} \Delta_{ij}^*}.$$

where q_i is habitat quality (usually, the number of bees nesting there).

CPF model - visitation rate

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  where q_i is habitat quality (usually, the number of bees nesting there).

- The (instantaneous) total number of bees visiting patch i is:

$$\nu_i = \sum_{j=1}^{J} r_{j \to i},$$

Data

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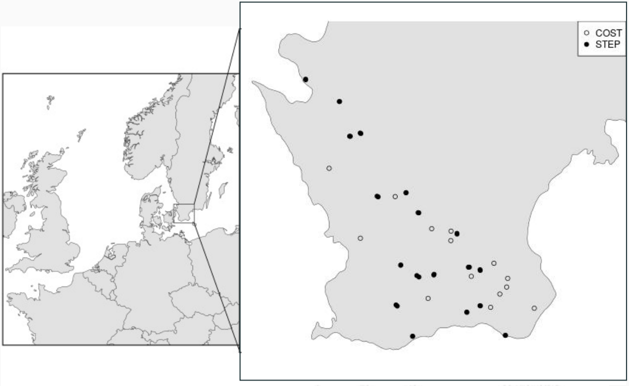
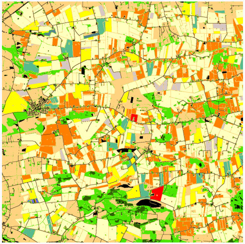


- Two studies on pollinator abundances in southern Sweden
- Data collected in four different years, several times a year (covering 3 different periods of bumblebees life cycle)
- Number of bees flying or foraging in a given transect for a given period of time was recorded

Model inputs

For each sampling site *i*, each year *j* and each period *k*:

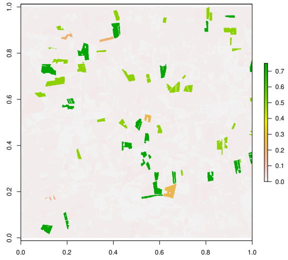
A landscape map



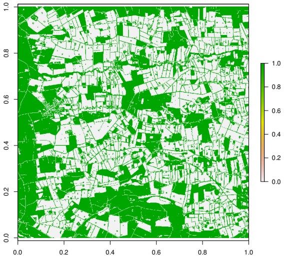
$\Downarrow$

denoted by $\mathcal{M}_{ijk}$

A floral quality map



A nesting map



informed by expert judgement or literature data

Statistical model

Statistical model - Bayesian formulation

- y_{ijk}: observed nb of bees on site i, year j and period k.

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- **Likelihood**
$$\begin{cases} y_{ijk} \mid \lambda_{ijk}, \theta & \sim \mathcal{P}(c_i \cdot \lambda_{ijk}) \\ \log \lambda_{ijk} & = \log \nu_i(\theta, \mathcal{M}_{jk}) + \beta_k + \varepsilon_{ijk} \\ \varepsilon_{ijk} & \sim \mathcal{N}(0, \sigma^2). \end{cases}$$
  - c_i a known scaling parameter,
  - λ_{ijk} the real intensity of the visitation rates,
  - β_k a period-specific parameter
- Complete vector of parameters $\psi = (\tau_0, f_0, a, b, \beta_1, \ldots, \beta_K, \sigma^2)$

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- Complete vector of parameters $\psi = (\tau_0, f_0, a, b, \beta_1, \ldots, \beta_K, \sigma^2)$
- **Priors**
$$\tau_0 \sim \mathcal{LN}_{[0,1000]}(\log(1000), 1) \quad f_0 \sim \mathcal{LN}(\log(0.1), 1)$$
$$a \sim \mathcal{U}([100, 1000]) \quad b \sim \mathcal{U}([100, 1000])$$
$$\beta_k \sim \mathcal{N}(\mu_k, \sigma_k^2), \quad k = 1, \ldots, K$$
$$\sigma^2 \sim \mathcal{IG}(\xi, \eta)$$

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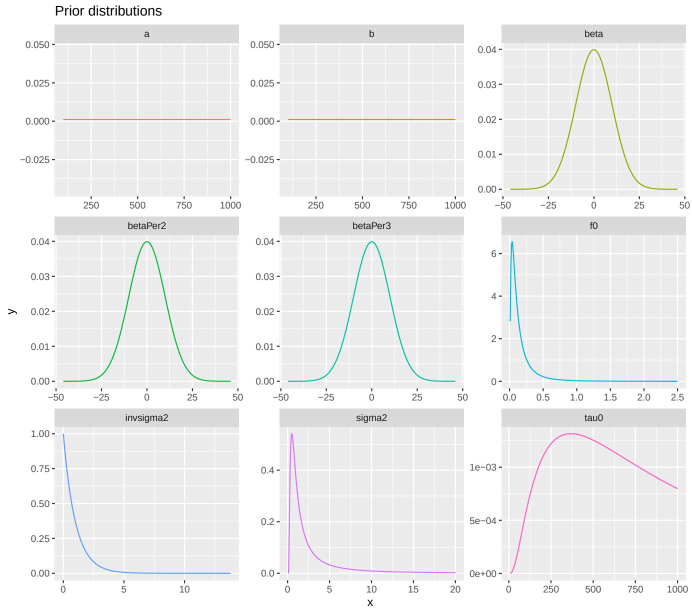

Approximate Bayesian Computation

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- how to handle this situation?
  - change the likelihood so that it becomes tractable? $\rightarrow$ can introduce biais, do not reflect the “true” process as we think it is generated
  - use an estimation method which can deal with untractable likelihood $\rightarrow$ approximate Bayesian computation (ABC)

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ABC rejection sampling

Input: a threshold ε and a distance d on the set of observations

For $m = 1, \dots, M$:

1. draw a sample $\psi^{(m)}$ from the prior distribution
2. generate a set of observations $y^{(m)}$ using $p(y \mid \psi)$
3. if $d(y_{obs}, y^{(m)}) \leq \varepsilon$, keep $\psi^{(m)}$
4. **Output**: a sample of size M_ε with all the accepted sets of parameters $\psi^{(m)}$

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- The accepted values follow the ABC posterior distribution $\pi_\varepsilon(\psi \mid y)$:

$$\pi_{ABC}(\psi \mid y_{obs}) \propto \int f(y|\psi)p(\psi)\mathbb{1}_{A_\varepsilon}(y)dy,$$

where $A_\varepsilon = \{y; d(y_{obs}, y) < \varepsilon\}$.

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  - replace crude rejection by kernel smoothing $\to$ each sample is used, with a weight $w_m = K(d(y_{obs}, y^{(m)}))$

ABC - choice of summary statistics

- Choice of the summary statistics is crucial: a poor specification leads to a poor approximation of the posterior, but choosing **sufficient statistics** has no impact, i.e. $\pi(\psi \mid s_{obs}) = \pi(\psi \mid y_{obs})$.

Objective

find low-dimensional summary and highly informative summary statistics

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- A statistic is said to be sufficient w.r.t. a parameter θ if it carries all the necessary knowledge to perform inference for θ. More precisely, s is sufficient iff:

$$\mathbb{P}(y \in A \mid s(y) \in B, \theta) = \mathbb{P}(y \in A \mid s(y) \in B)$$

  ex. : for a Gaussian i.i.d. sample $\mathcal{N}(\mu, \sigma^2)$, the sample mean is sufficient for μ.

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Methods based on regression adjustment

- **First idea:** build a relationship between the parameter values and the summary statistics values, e.g. via regression techniques.

$$\psi_i^{(m)} = m_i(s(y^{(m)}) - s_{obs}) + \sigma(s(y^{(m)}))\varepsilon_{im}, \quad i = 1, \dots, p$$

  Then, samples from $\pi_{ABC}(\psi \mid s_{obs})$ are obtained via:

$$\psi_i^{*(m)} = \hat{m}_i(s_{obs}) + \left(\psi_i^{(m)} - \hat{m}(s(y^{(m)}))\right) \frac{\hat{\sigma}(s_{obs})}{\hat{\sigma}(s(y^{(m)}))}$$

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- Estimation via weighted least squares, where weights are given by a chosen smoothing kernel K
- Local regression around $s_{obs} \to$ lowers the effect of the distance between $s(y^{(m)})$ and s_{obs}.
- Can be combined with dimension reduction methods to further reduce the summary statistics dimension

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Some methods based on regression adjustment (shortnames for methods compared in this work are marked in blue):

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  2. perform a second LocNLH regression using parameters values samples from p_D, the conditional prior of the parameters given that they fall in D
- penalized regression (Wegmann et al. 2016)
- best subset selection (via criteria such as AIC or BIC for example) (Numes and Balding 2010, Blum et al. 2013)

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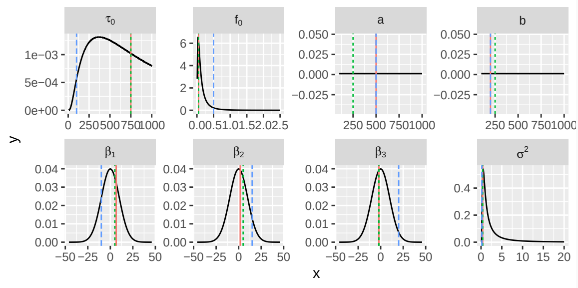
Some examples

- Raynal et al. 2016 suggested the use of random forests combined with quantile regression [qRF]
- Other machine learning algorithms could be used, e.g. gradient boosting methods, also combined with quantile regression [qGBM]

Results

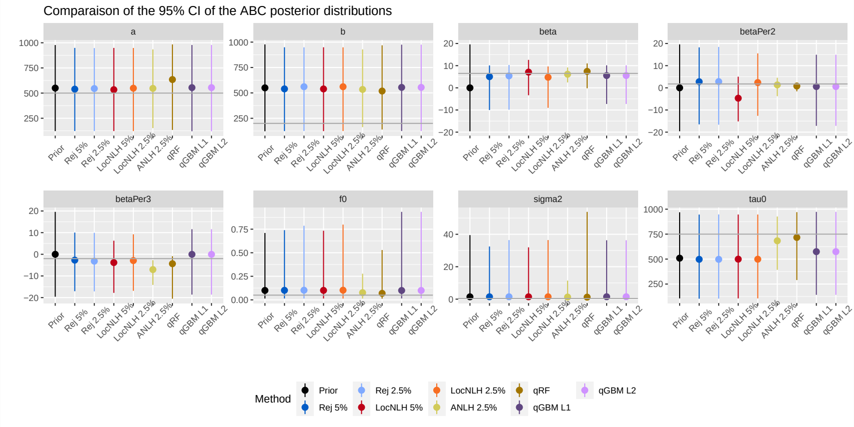
Simulated data

- We simulated three datasets corresponding to different parameter values

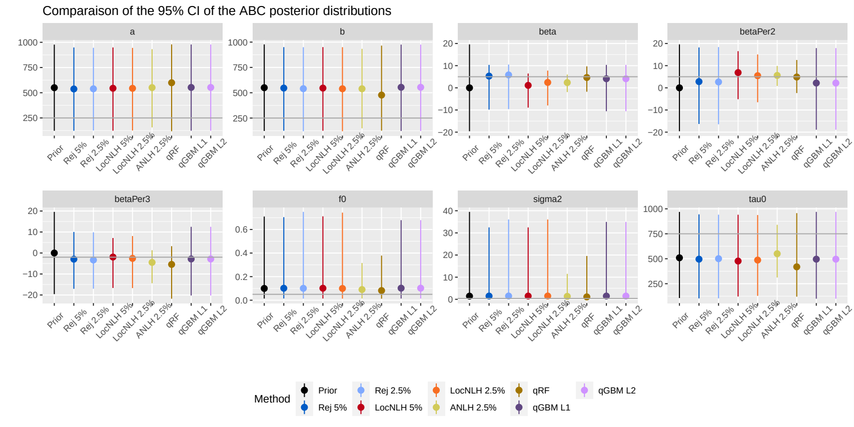


- We generated $M = 100\,000$ parameter samples from the prior and M corresponding simulated datasets
- Summary statistics were defined as : min, max, mean, quantiles of order 25%, 50% (median), 75%, and number of 0 observed per landuse type, and per combination of landuse type and flowering period
  $\rightarrow$ from 790 observations to 112 summary statistics

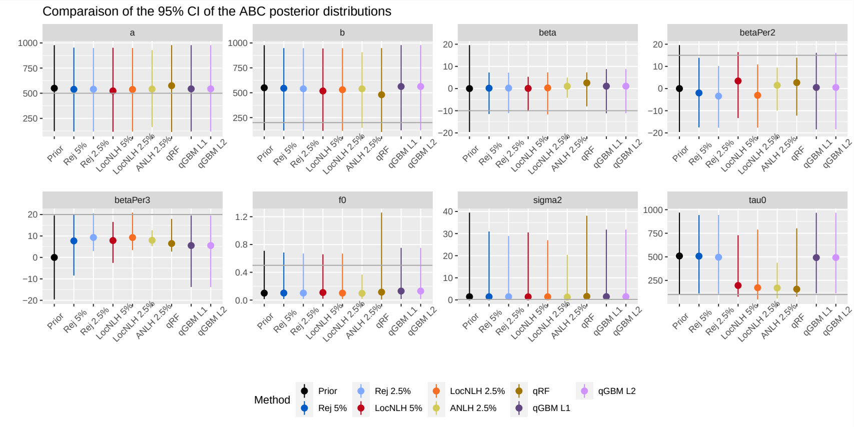
Results (no local linear approaches)



Results (no local linear approaches)



Results (no local linear approaches)

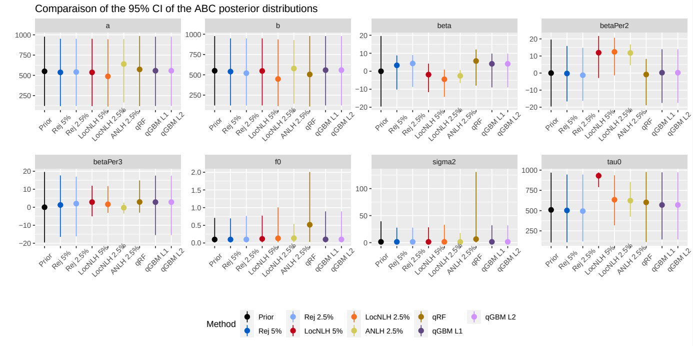


Results – best results

Posterior mean - posterior median for CPF model parameters

Param	True	LocNLH 5%	LocNLH 2.5%	ANLH 5%	qRF
τ_0	750	512 / 499	510 / 498	679 / 684	**691 / 717**
	750	501 / 476	512 / 486	**556 / 550**	454 / 419
	100	249 / 196	230 / 173	**192** / 170	214 / **158**
f_0	0.05	0.167 / 0.101	0.174 / 0.103	**0.095** / 0.076	0.102 / **0.069**
	0.05	0.166 / 0.102	0.164 / 0.100	**0.111** / 0.090	**0.111 / 0.082**
	0.05	0.172 / 0.109	0.167 / 0.101	**0.124 / 0.097**	0.212 / 0.114
a	500	**532 / 534**	539 / 548	545 / 547	592 / 635
	250	**537 / 544**	538 / 543	549 / 551	588 / 599
	500	**526 / 522**	534 / 537	545 / 540	559 / 575
b	200	538 / 537	548 / 559	541 / 534	**526 / 518**
	250	542 / 545	538 / 538	542 / 539	**498 / 477**
	200	519 / 517	526 / 529	533 / 539	**510 / 480**

Real data



Param	MAP	95% CI
τ_0	623	[425 ; 855]
f_0	0.137	[0.021 ; 0.540]
a	640	[166 ; 948]
b	578	[144 ; 928]

Param	MAP	95% CI
β_1	-2.54	[-6.53 ; 0.624]
β_2	11.8	[4.54 ; 16.8]
β_3	-0.270	[-3.54 ; 2.56]
σ^2	1.40	[0.253 ; 17.8]

Conclusion and discussion

Conclusion

- Preliminary results suggest that some parameters are difficult to infer
- Methods based on nonlinear local regression perform better
- Improvements are needed to enhance predictive quality
- Results are conditional on the floral and nesting maps

Perspectives

- Evaluate the performances of the methods on more simulated data (computation of RMSE)
- Another choice for the summary statistics?
- Influence of the origin of the data (STEP study vs COST study)?
- Use the estimated ABC posterior distribution to tune likelihood-free MCMC algorithms (initialization of the chain, choice of the proposal distribution) (e.g. Wegmann 2009)
- Evaluate the influence of the input maps