

## Segmentation sur arbre: quelques applications

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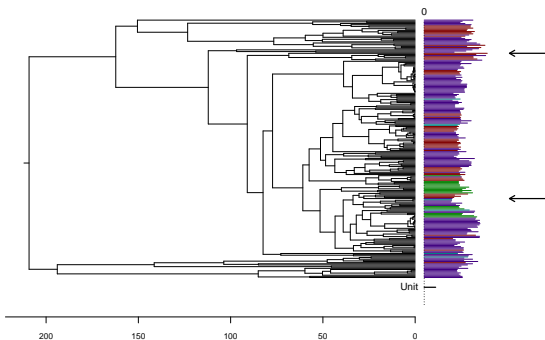
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Journées MMB  
12 Mars 2020



# Introduction



*Dermochelys Coriacea*



*Homopus Areolatus*

Turtles phylogenetic tree with habitats.  
(Jaffe et al., 2011).

- How can we explain the diversity, while accounting for the phylogenetic correlations ?
- Modelling: a shifted stochastic process on the phylogeny.

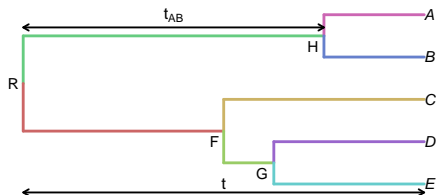
- 1 Stochastic Processes on Trees
- 2 Statistical Inference
- 3 Application in Ecology
- 4 Application in metagenomics

- 1 Stochastic Processes on Trees
  - Principle of the Modeling
  - Shifts
  - Two Mathematical Formulations

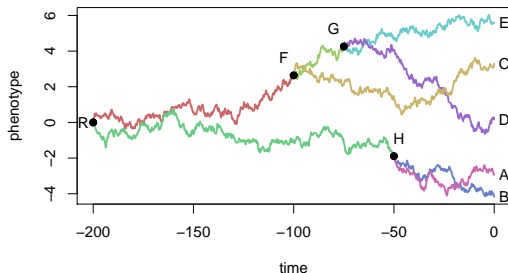
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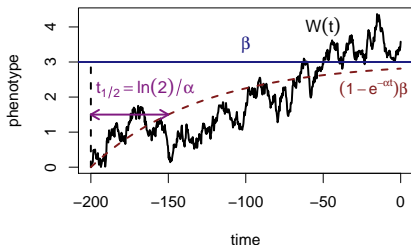
The tree is known.  
Only *tip* values are observed



Brownian Motion:

$$\text{Var}[A | R] = \sigma^2 t$$

$$\text{Cov}[A; B | R] = \sigma^2 t_{AB}$$



$$dW(t) = \alpha[\beta(t) - W(t)]dt + \sigma dB(t)$$

Deterministic part:

- $\beta(t)$ : primary optimum, mechanistically defined.
- $\ln(2)/\alpha$ : phylogenetic half live.

Stochastic part:

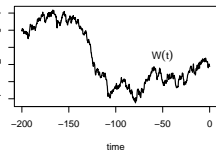
- $W(t)$ : actual optimum (trait value).
- $\sigma dB(t)$  Brownian fluctuations.

# BM vs OU

Equation

Stationary State

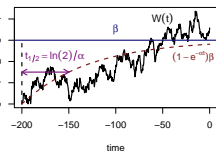
Variance



$$dW(t) = \sigma dB(t)$$

None.

$$\sigma_{ij} = \sigma^2 t_{ij}$$

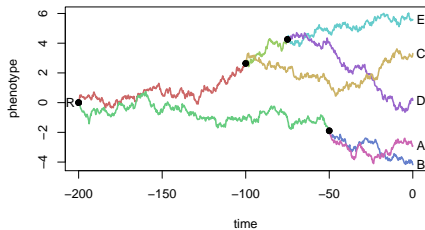
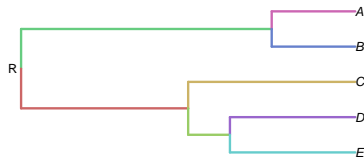


$$dW(t) = \sigma dB(t) + \alpha[\beta(t) - W(t)]dt$$

$$\begin{cases} \mu = \beta_0 \\ \gamma^2 = \frac{\sigma^2}{2\alpha} \end{cases}$$

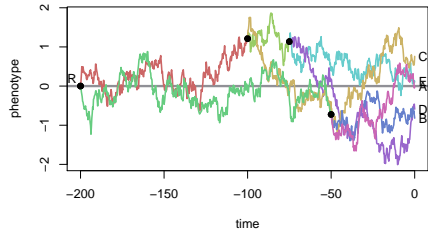
$$\sigma_{ij} = \gamma^2 e^{-\alpha(t_i+t_j)} \times (e^{2\alpha t_{ij}} - 1)$$

# Shifts



BM Shifts in the **mean**:

$$m_{\text{child}} = m_{\text{parent}} + \delta$$

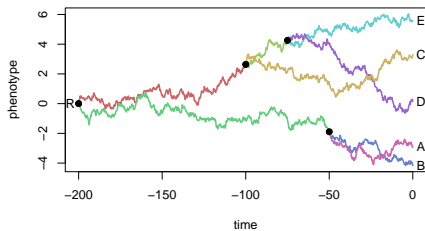
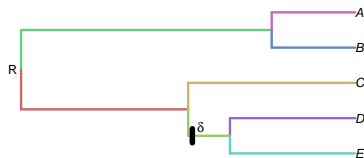


OU Shifts in the **optimal value**:

$$\beta_{\text{child}} = \beta_{\text{parent}} + \delta$$

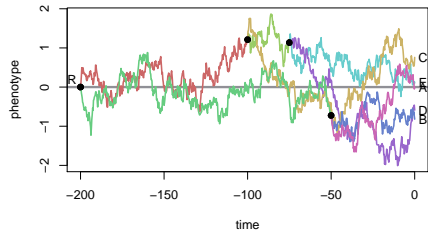


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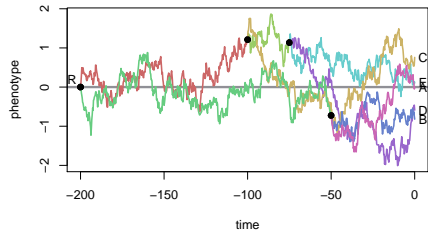
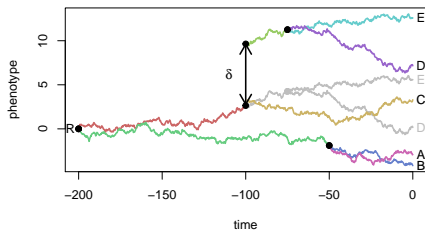
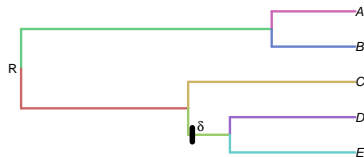
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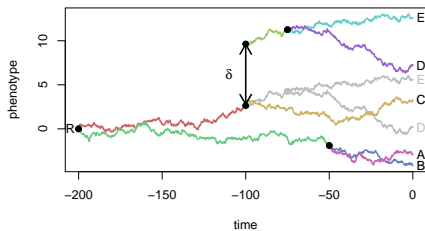
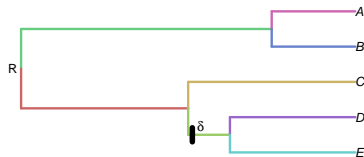
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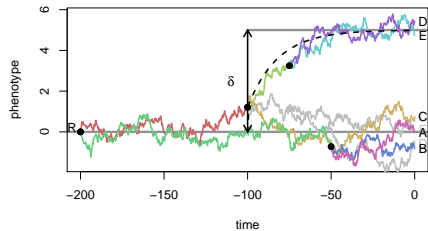
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# Shifts



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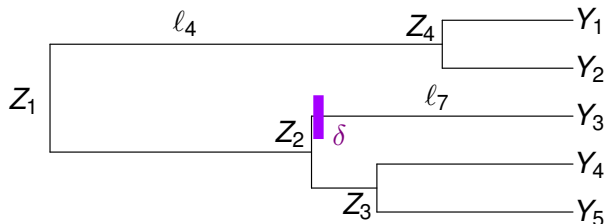
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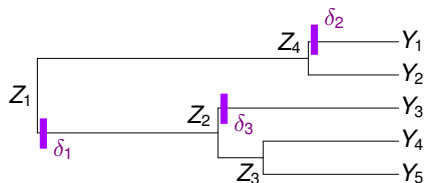
# Incomplete Data Model



$$\begin{aligned} \text{BM } Z_4|Z_1 &\sim \mathcal{N}\left(Z_1, \sigma^2 l_4\right) \\ Y_3|Z_2 &\sim \mathcal{N}\left(Z_2 + \delta, \sigma^2 l_7\right) \end{aligned}$$

$$\text{OU } Y_3|Z_2 \sim \mathcal{N}\left(Z_2 e^{-\alpha l_7} + (1 - e^{-\alpha l_7})(\beta Z_2 + \delta), \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha l_7})\right)$$

# Linear Regression Model

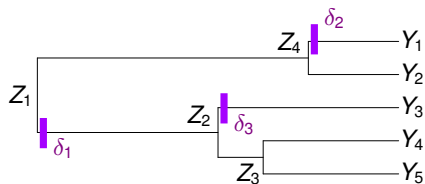


$$\Delta = \begin{pmatrix} \mu \\ \delta_1 \\ 0 \\ 0 \\ \delta_2 \\ 0 \\ \delta_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad T\Delta = \begin{pmatrix} \mu + \delta_2 \\ \mu \\ \mu + \delta_1 + \delta_3 \\ \mu + \delta_1 \\ \mu + \delta_1 \end{pmatrix}$$

$$T = \begin{matrix} & Z_1 & Z_2 & Z_3 & Z_4 & Y_1 & Y_2 & Y_3 & Y_4 & Y_5 \\ \begin{matrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$BM: \quad Y = T\Delta^{BM} + E^{BM}$$

# Linear Regression Model



$$\Delta = \begin{pmatrix} \lambda \\ \delta_1 \\ 0 \\ 0 \\ \delta_2 \\ 0 \\ \delta_3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$TW(\alpha)\Delta = \begin{pmatrix} \lambda + w_5\delta_2 \\ \lambda \\ \lambda + w_2\delta_1 + w_7\delta_3 \\ \lambda + w_2\delta_1 \\ \lambda + w_2\delta_1 \end{pmatrix}$$

$$W(\alpha) = \text{Diag}(1 - e^{-\alpha(h-t_{pa(i)})}, 1 \leq i \leq m+n)$$

$$\lambda = \mu e^{-\alpha h} + \beta_0(1 - e^{-\alpha h})$$

$$BM: Y = T\Delta^{BM} + E^{BM}$$

$$OU: Y = TW(\alpha)\Delta^{OU} + E^{OU}$$

## Expectations

$$\mathbb{E}[Y | X_1 = \mu] = T \underbrace{W(\alpha)\Delta^{OU}}_{\Delta^{BM}}$$

**Remark:**  $\mu^{BM} = \lambda^{OU} = \mu e^{-\alpha h} + \beta_0(1 - e^{-\alpha h})$

## Variance

$$\text{Cov}[Y_i; Y_j | X_1 = \mu] = \sigma^2 \times \underbrace{\frac{1}{2\alpha} e^{-2\alpha h} (e^{2\alpha t_{ij}} - 1)}_{t'_{ij}}$$

OU  $\iff$  BM on a re-scaled tree with  $t' = \frac{1}{2\alpha} e^{-2\alpha h} (e^{2\alpha t} - 1)$

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### Remarks:

- This only works for an *ultrametric* tree.
- The laws of the internal nodes is changed.
- This is *not* the following standard time transformation:

### Lemma (Brownian Solution for the OU)

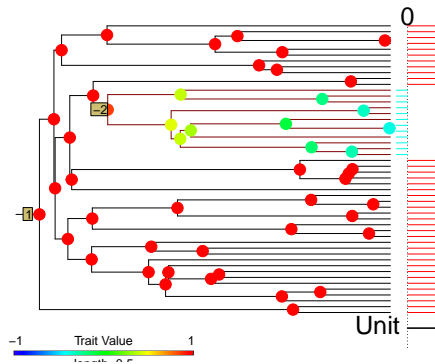
The stochastic process defined by:

$$X_t = X_0 e^{-\alpha t} + \beta(1 - e^{-\alpha t}) + \frac{\sigma}{\sqrt{2\alpha}} e^{-\alpha t} B_{e^{2\alpha t} - 1}$$

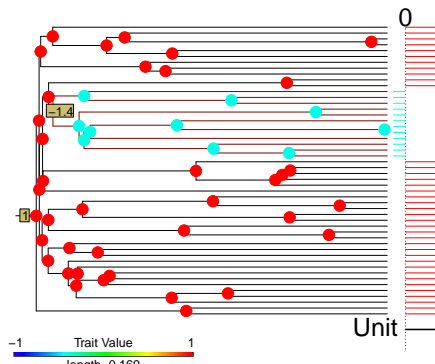
is an OU, solution of the EDS  $dX_t = \alpha(\beta - X_t) + \sigma dB_t$ .



OU  $\iff$  BM on a re-scaled tree with  $t' = \frac{1}{2\alpha} e^{-2\alpha h} (e^{2\alpha t} - 1)$



OU:  $\lambda = \beta_0 = \mu = 1$  and  $t_{1/2} = 0.5$



Equivalent BM on a re-scaled tree

1 Stochastic Processes on Trees

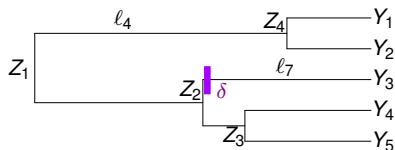
2 Statistical Inference

- EM Algorithm
- Model Selection

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# EM Algorithm: number of shifts K fixed



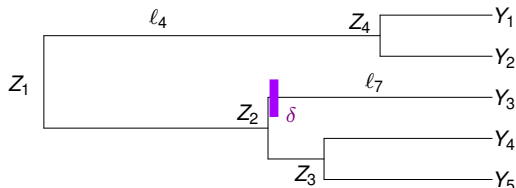
$$\left. \begin{aligned} Z_4 | Z_1 &\sim \mathcal{N}\left(\lambda_1 Z_1 + (1 - \lambda_1)\beta, \frac{\sigma^2}{2\alpha}(1 - \lambda_1^2)\right) \\ Y_3 | Z_2 &\sim \mathcal{N}\left(\lambda_7 Z_2 + (1 - \lambda_7)(\beta + \delta), \frac{\sigma^2}{2\alpha}(1 - \lambda_7^2)\right) \\ \lambda_i &= e^{-\alpha \ell_i} \end{aligned} \right\}$$

$$p_\theta(Z, Y) = p_\theta(Z_1) \prod_{1 < j \leq m} p_\theta(Z_j | Z_{\text{parent}(j)}) \prod_{1 \leq i \leq n} p_\theta(Y_i | Z_{\text{parent}(i)})$$

**EM Algorithm**  $\log p_\theta(Y) = \mathbb{E}_\theta[\log p_\theta(Z, Y) | Y] - \mathbb{E}_\theta[\log p_\theta(Z) | Y]$

**E step** Given  $\theta^h$ , compute  $p_{\theta^h}(Z | Y)$

**M step**  $\theta^{h+1} = \text{argmax}_\theta \mathbb{E}_{\theta^h}[\log p_\theta(Z, Y) | Y]$



Compute the following quantities:

$$\mathbb{E}^{(h)}[Z_j | Y], \text{Var}^{(h)}[Z_j | Y], \text{Cov}^{(h)}[Z_j, Z_{\text{parent}(j)} | Y]$$

- Using Gaussian properties. Need to invert matrices: complexity in  $O(n^3)$ .
- Using Gaussian properties **and** the tree structure: "Upward-Downward" algorithm. Complexity in  $O(n)$ .

Maximize:

$$\mathbb{E} [\log p_{\theta}(X) \mid Y] = - \sum_{j=2}^{m+n} C_j(\alpha, \text{shifts}) + \mathcal{F}^{(h)}(\mu, \gamma^2, \sigma^2, \alpha)$$

- $\mu, \gamma^2, \sigma^2$ : simple maximization
- Discrete location of  $K$  shifts
  - ↳ Exact and fast for the BM
  - ↳ Hill-climbing heuristics for the OU
- $\alpha$ : numerical maximization and/or on a grid
  - ↳ Generalized EM

# Starting point and choice of $K$

## Starting point

**Shifts:** Fast estimate based on Lasso regression (see next section).

**Selection strength  $\alpha$ :** Initialization using couples of tips and robust estimate of  $\alpha$ .

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## Choosing $K$

**Assumption  $\alpha$  fixed**

$$Y = TW(\alpha)\Delta + \gamma E \quad , \quad E \sim \mathcal{N}(0, V(\alpha))$$

**Models**

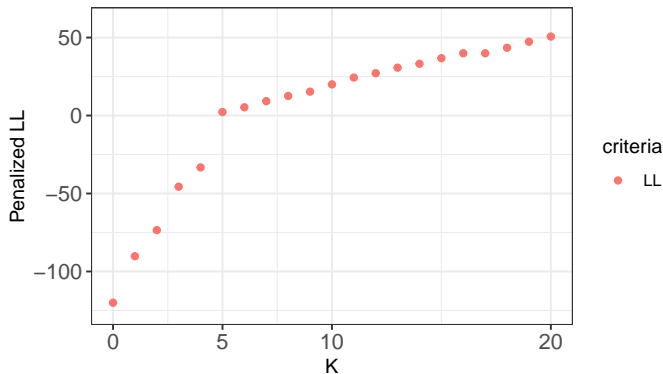
$\eta \in \bigcup_{K=0}^{p-1} \mathcal{S}_K^{PI}$ : Identifiable parcimonious allocations of shifts

**EM Estimators**

$$\hat{Y}_K = \underset{\eta \in \mathcal{S}_K^{PI}}{\operatorname{argmin}} \left\| Y - \hat{Y}_\eta \right\|_V^2$$

# Model Selection: Penalized Likelihood

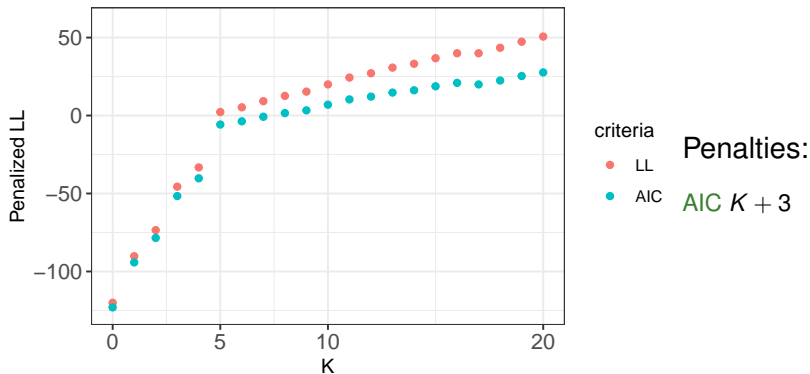
Idea  $\hat{K} = \operatorname{argmax}_{0 \leq K \leq p-1} \left\{ \frac{n}{2} \log \left( \frac{1}{n} \|Y - \hat{Y}_K\|_V^2 \right) - \frac{1}{2} \operatorname{pen}'(K) \right\}$





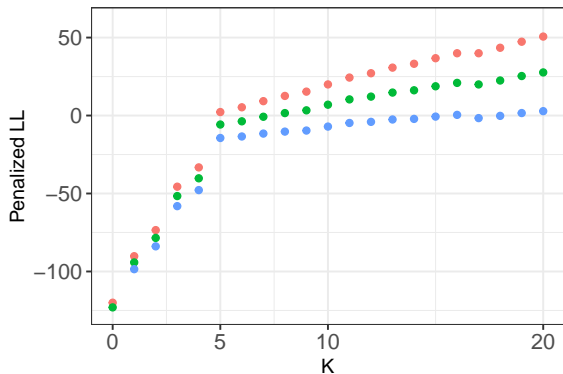
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criteria

● LL

● AIC

● BIC

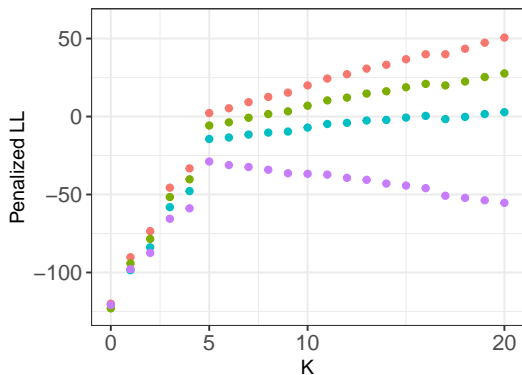
Penalties:

AIC  $K + 3$

BIC  $\frac{1}{2}(K + 3) \log(n)$

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criteria

- LL
- AIC
- BIC
- LINselect

Penalties:

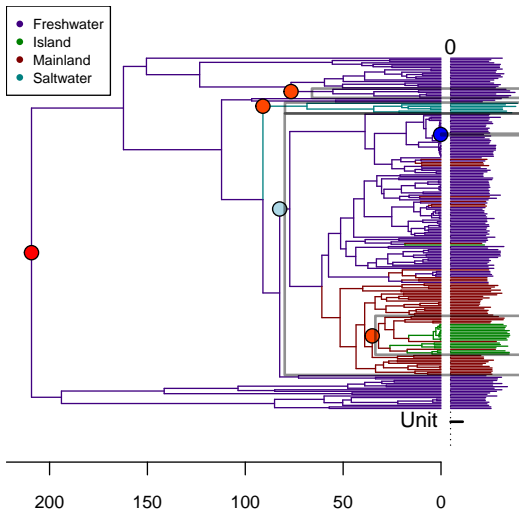
AIC  $K + 3$

BIC  $\frac{1}{2}(K + 3) \log(n)$

LINselect  $\operatorname{pen}(n, K, |S_K^{PI}|)$

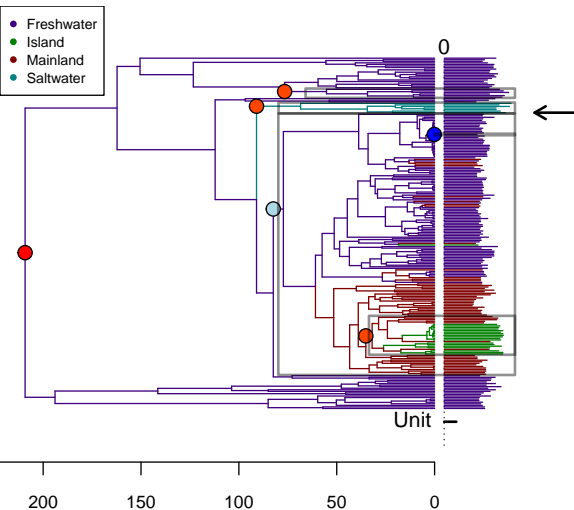
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# Turtles Dataset



Colors: habitats.  
Boxes: selected EM regimes.

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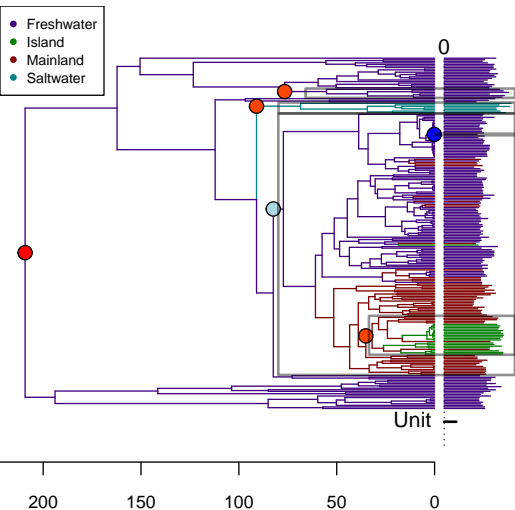


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*Chelonia mydas*

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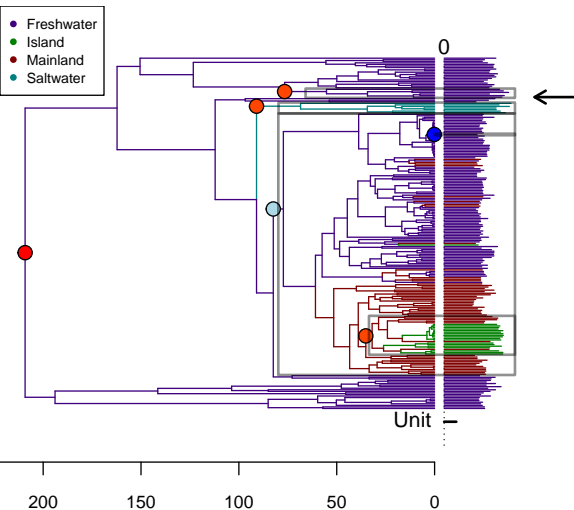


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*Geochelone nigra abingdo*

# Turtles Dataset



Chitra indica

Colors: habitats.  
Boxes: selected EM regimes.



## Univariate framework

P. Bastide, M. Mariadassou, S. Robin (2016), Detection of adaptive shifts on phylogenies by using shifted stochastic processes on a tree. *JRSS-B*. doi:10.1111/rssb.12206

## Extension to multivariate framework

P. Bastide, C. Ané, S. Robin, M. Mariadassou (2018), Inference of Adaptive Shifts for Multivariate Correlated Traits. *Syst. Biol.*. doi:10.1093/sysbio/syy005

Package `PhylogeneticEM`: available on GitHub, on the CRAN.

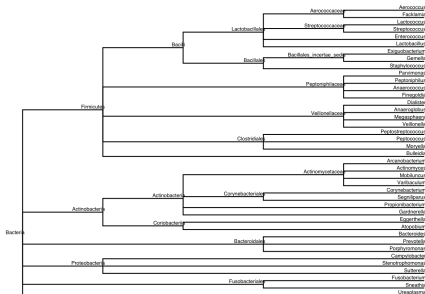
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  - Differential abundance testing
  - Mathematical model
  - Inference

A species  $\times$  sample count table

	Taxa	A1	A2	A3	B1	B2	B3
1	Lactobacillus	2318	1388	1361	2256	88	1770
2	Prevotella	0	1	1	0	525	7
3	Megasphaera	0	1	0	0	402	0
4	Sneathia	0	0	0	0	302	0
5	Atopobium	0	1	0	0	84	0
6	Streptococcus	0	0	3	0	0	0
7	Dialister	0	1	0	0	152	4
8	Anaerococcus	0	1	3	2	0	9
9	Peptoniphilus	0	1	0	0	7	2
10	Eggerthella	0	0	0	0	2	0

## Taxonomic / phylogenetic tree

	Phylum	Class	Order	Family	Genus
1	Actinobacteria	Actinobacteria	Actinomycetales	Actinomycetaceae	Actinobaculum
2	Actinobacteria	Actinobacteria	Actinomycetales	Actinomycetaceae	Actinomyces
3	Actinobacteria	Actinobacteria	Actinomycetales	Actinomycetaceae	Arcanobacterium
4	Actinobacteria	Actinobacteria	Actinomycetales	Actinomycetaceae	Mobiluncus
5	Actinobacteria	Actinobacteria	Actinomycetales	Actinomycetaceae	Varibaculum
6	Actinobacteria	Actinobacteria	Bifidobacteriales	Bifidobacteriaceae	Bifidobacterium
7	Actinobacteria	Actinobacteria	Bifidobacteriales	Bifidobacteriaceae	Gardnerella

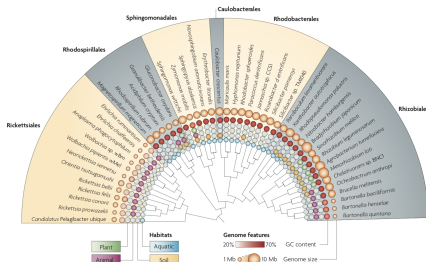


# Differential abundance analysis

- For each taxa  $i$  (in  $\{1, \dots, n\}$ ), test
  - $H_{0i}$ : Abundances are **equal** in groups  $A$  and  $B$
  - $H_{1i}$ : Abundances are **not equal** in groups  $A$  and  $B$
- **Hundred** of univariate tests and p-values
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- **Hundred** of univariate tests and p-values
- Need for a multiple testing correction procedure



- Taxa / group associations may show a **phylogenetic signal**
- Similar taxa  $\Rightarrow$  similar levels of association
- Can we leverage the tree when correcting the tests?

## Standard assumptions on $p$ -values

- Under  $H_{0i}$ ,  $p_i \sim \mathcal{U}(0, 1)$
- Under  $H_{1i}$ ,  $p_i \preceq \mathcal{U}(0, 1)$

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## Standard assumptions on $z$ -scores

- Under  $H_{0i}$ ,  $z_i = \Phi^{-1}(p_i) \sim \mathcal{N}(0, 1)$
- Under  $H_{1i}$ ,  $z_i = \Phi^{-1}(p_i) \sim \mathcal{N}(m_i, 1)$  with  $m_i < 0$



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## Tractable assumptions on $z$ -scores vector

- $Z = (z_1, \dots, z_n) \sim \mathcal{N}(M, V(\alpha))$  where
  - $M = (m_1, \dots, m_n) \in \mathbb{R}_-^n$
  - $V(\alpha)$  is the variance matrix of an OU on a tree.

## Standard assumptions on $p$ -values

- Under  $H_{0i}$ ,  $p_i \sim \mathcal{U}(0, 1)$
- Under  $H_{1i}$ ,  $p_i \preceq \mathcal{U}(0, 1)$

## Standard assumptions on $z$ -scores

- Under  $H_{0i}$ ,  $z_i = \Phi^{-1}(p_i) \sim \mathcal{N}(0, 1)$
- Under  $H_{1i}$ ,  $z_i = \Phi^{-1}(p_i) \sim \mathcal{N}(m_i, 1)$  with  $m_i < 0$

## Tractable assumptions on $z$ -scores vector

- $Z = (z_1, \dots, z_n) \sim \mathcal{N}(M, V(\alpha))$  where
  - $M = (m_1, \dots, m_n) \in \mathbb{R}_-^n$
  - $V(\alpha)$  is the variance matrix of an OU on a tree.

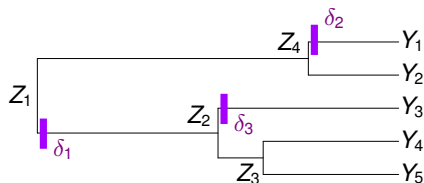
Assume that  $z$ -scores evolve as an OU on the tree with a **sign constraint** on the mean.

# Linear regression model

Tree-structure enforced by decomposition  $M = TW(\alpha)\Delta$ .

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$$\Delta = \begin{pmatrix} 0 \\ \delta_1 \\ 0 \\ 0 \\ \delta_2 \\ 0 \\ \delta_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad TW(\alpha)\Delta = \begin{pmatrix} w_5\delta_2 \\ 0 \\ w_2\delta_1 + w_7\delta_3 \\ w_2\delta_1 \\ w_2\delta_1 \\ 0 \end{pmatrix}$$

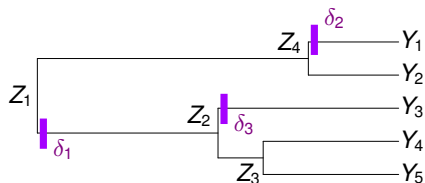
$$T = \begin{matrix} & Z_1 & Z_2 & Z_3 & Z_4 & Y_1 & Y_2 & Y_3 & Y_4 & Y_5 \\ \begin{matrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$OU: \quad Z = TW(\alpha)\Delta + E$$

$$W(\alpha) = \text{Diag}(1 - e^{-\alpha(h-t_{pa(i)})}, 1 \leq i \leq m+n)$$

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$$W(\alpha) = \text{Diag}(1 - e^{-\alpha(h - t_{pa(i)})}, 1 \leq i \leq m + n)$$

**Goal:** Find  $\{i : m_i < 0\}$

# Estimating $M$

The MLE of  $\Delta$  (and in turn  $M$ ) is solution to

$$\underset{\Delta \text{ s.t. } TW(\alpha)\Delta \leq 0}{\operatorname{argmax}} \|Z - TW(\alpha)\Delta\|_{2, V(\alpha)^{-1}}^2$$

Equivalent to<sup>1</sup>:

$$\underset{\Delta \text{ s.t. } C\Delta \leq 0}{\operatorname{argmax}} \|Y - X\Delta\|_2^2$$

---

<sup>1</sup>with  $C$ ,  $Y$  and  $X$  some simple transforms of  $Z$  and  $TW(\alpha)$ ,  $V(\alpha)$

<sup>2</sup>Using a variant of the LASSO shooting algorithm

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Add a  $\ell_1$ -penalty to sparsify the solution and solve<sup>2</sup>

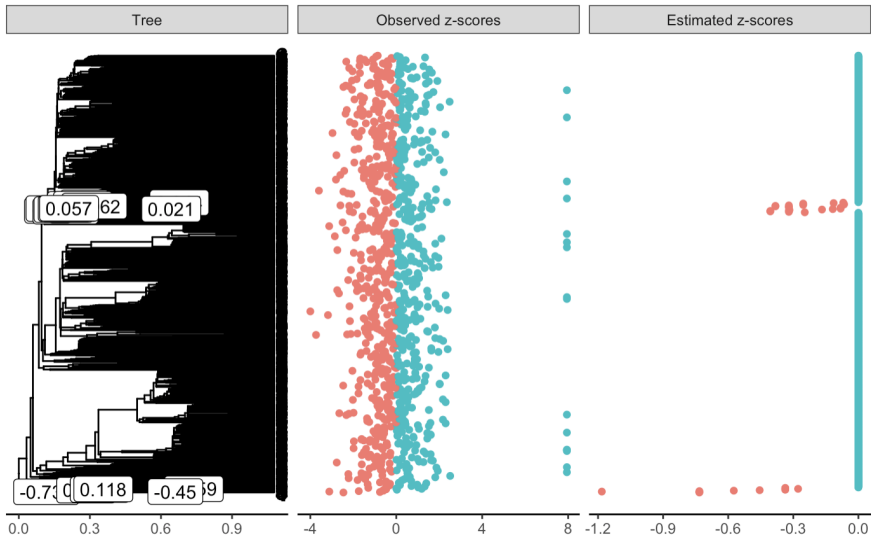
$$\hat{\Delta} = \underset{\Delta \text{ s.t. } C\Delta \leq 0}{\operatorname{argmax}} \|Y - X\Delta\|_2^2 + \lambda \|\Delta\|_1 \quad (1)$$

using penalized likelihood for selection of  $\alpha$  and  $\lambda$

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<sup>1</sup>with  $C$ ,  $Y$  and  $X$  some simple transforms of  $Z$  and  $TW(\alpha)$ ,  $V(\alpha)$

<sup>2</sup>Using a variant of the LASSO shooting algorithm





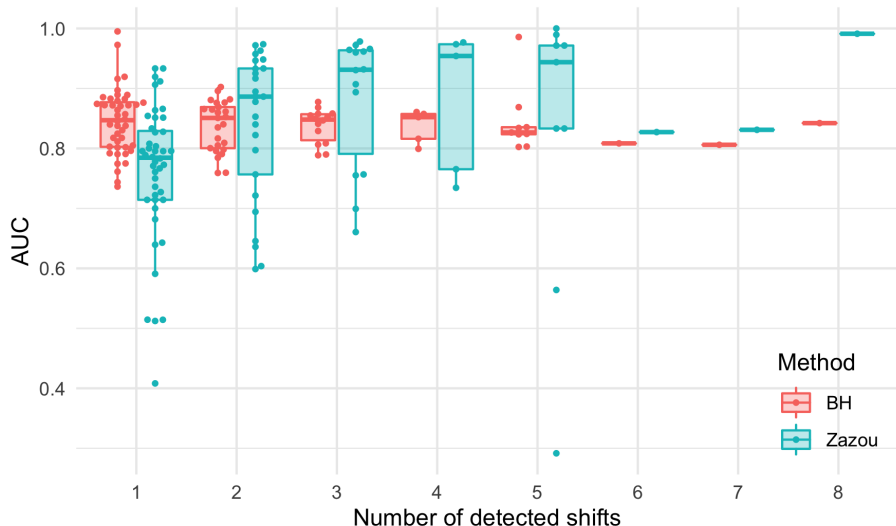
# Testing null components of $M$

## Decision rule

- $\hat{m}_i \neq 0 \Rightarrow$  reject  $H_{0i}$
- $\hat{m}_i = 0 \Rightarrow$  do not reject  $H_{0i}$

# Comparison with non-hierarchical procedures

AUC on simulated data (higher is better)



- `zazou` R package: under active development on GitHub.

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$\hat{M} = TW(\alpha)\hat{\Delta}$  is bias

- Unbias  $\hat{M}$  (using desparsified lasso)
- Build consistent confidence intervals for  $m_j$

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