Modeling the evolution of ecological interaction networks

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    - Project present species niche into the future.
    - Do not consider species interactions.
- We want a model capable of capturing these two phenomena.

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Spotted knapweed (*Centaurea maculosa*) is an invasive species in North America introduced from Europe in the 1890s.


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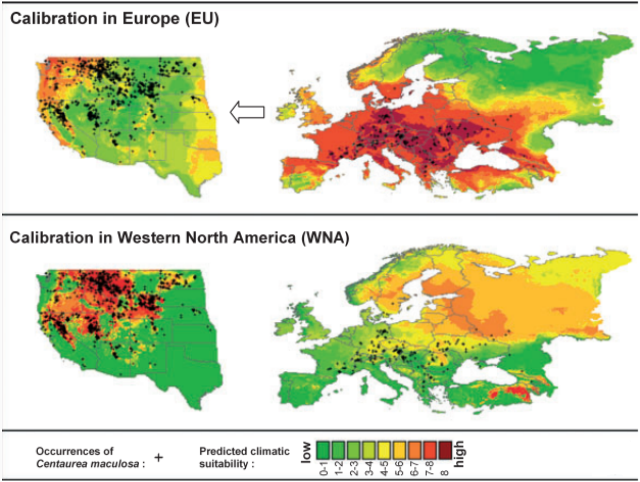

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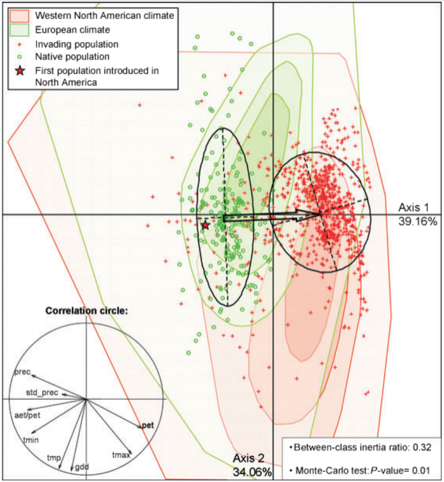

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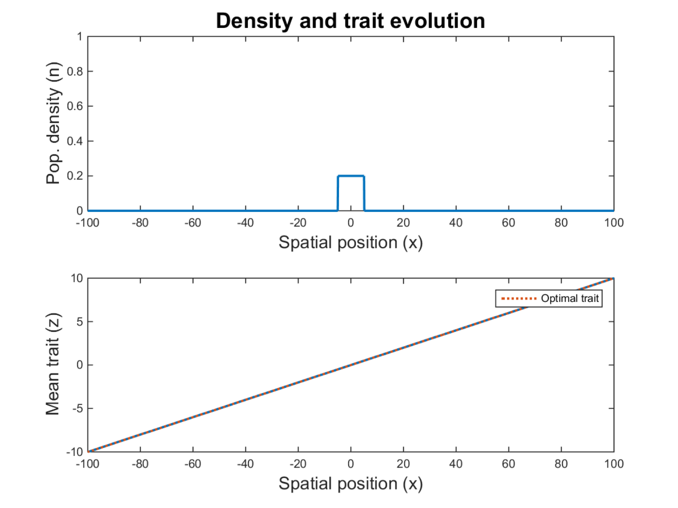

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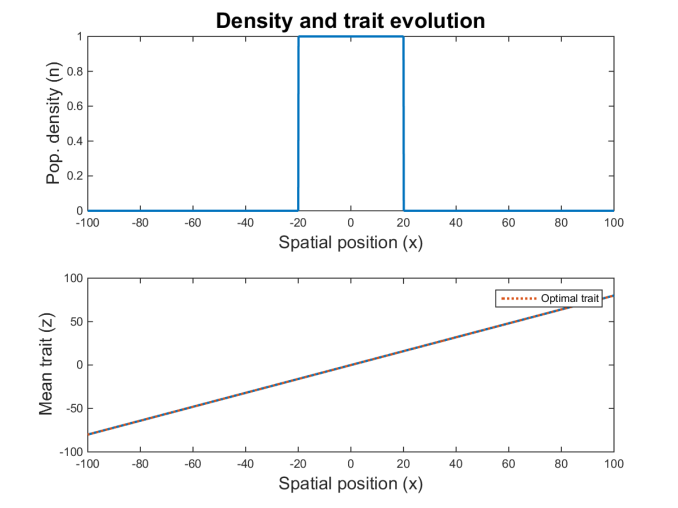

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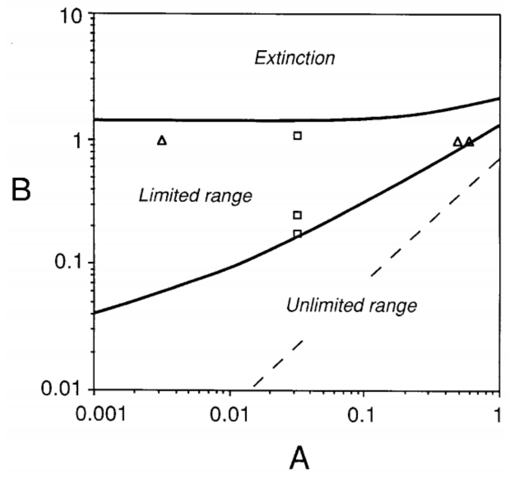

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Where $i = 1$ denotes the prey population:

$$r_1(x,t) = r_1^{max} - \frac{1}{2V_1^s}\left(\bar{Z}_1(x,t) - \theta(x)\right)^2 + I_1(x,t)$$

and

$$I_1(x,t) = -\alpha_1 r_1^{max}\frac{N_1(x,t)}{K} - \beta_{12}N_2(x,t).$$

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And $i = 2$ denotes the predator population:

$$r_2(x,t) = l_2(x,t) - \frac{1}{2V_2^s}\left(\bar{Z}_2(x,t) - \theta(x)\right)^2 - d$$

and

$$l_2(x,t) = \beta_{21} N_1(x,t) - \alpha_2 N_2(x,t),$$

we choose $\beta_{21} = c\beta_{12}$.

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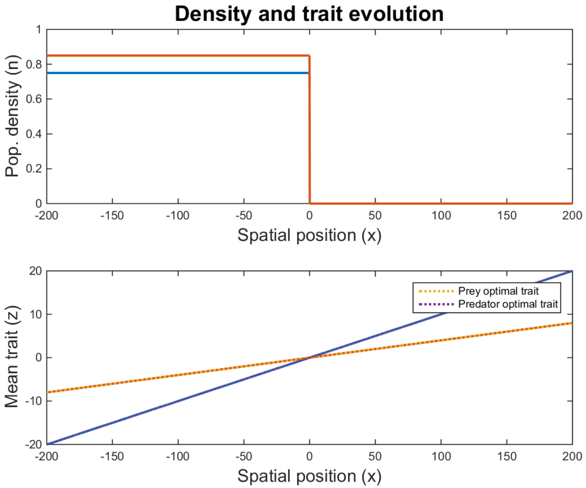
It is a generalization of Kirkpatrick and Barton's equations.

Solution example



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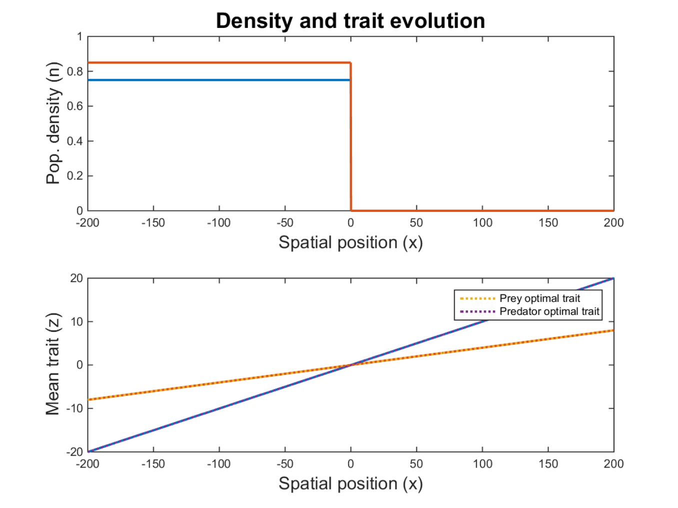
We want to study propagation speeds these states.

Propagation dynamics



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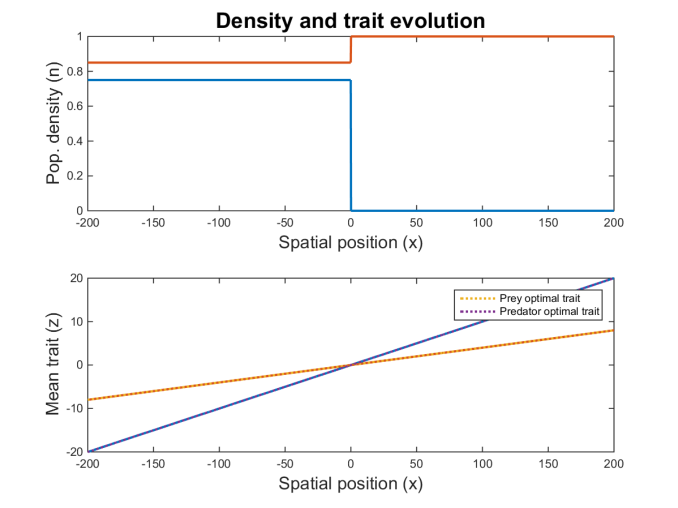

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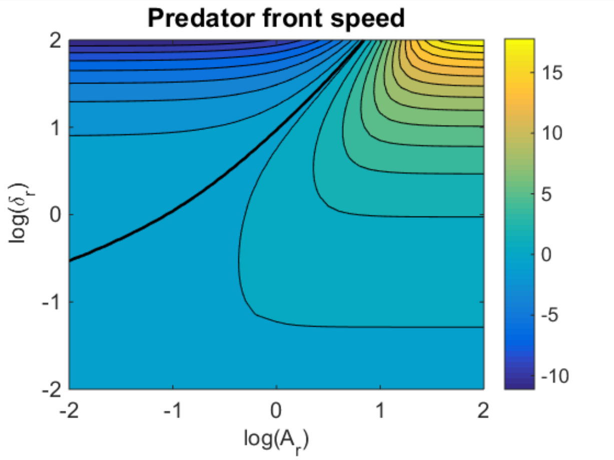
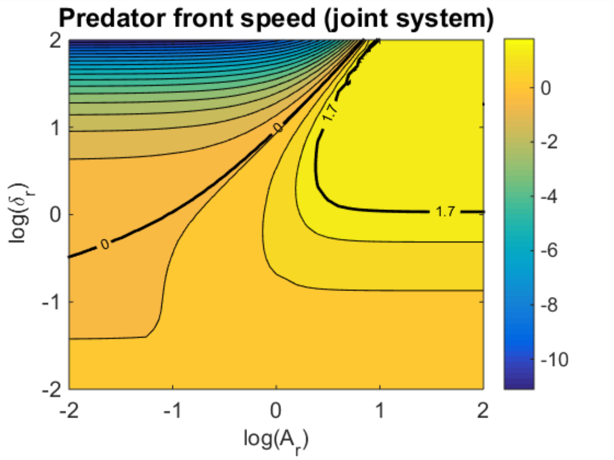


Predator invasion front. The zero speed line is thickened.

$(A_r = A_2/A_1,\ \delta_r = \sigma_2^2/\sigma_1^2)$

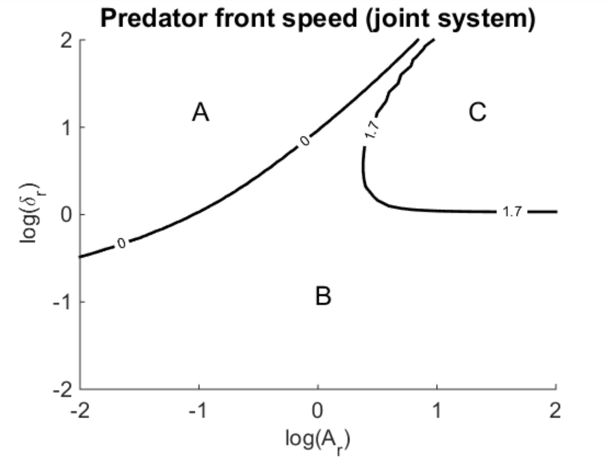
Results



Transition between extinction and coexistence.

$(A_r = A_2/A_1,\, \delta_r = \sigma_2^2/\sigma_1^2)$

Results



Three phases can be identified.

$(A_r = A_2/A_1,\ \delta_r = \sigma_2^2/\sigma_1^2)$

Conclusions

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- Advancing predator fronts can be of one out of two types, implying different geographical distributions.
- The studied parameters do not seem to allow predators to control the prey's geographical range.

Further questions

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$$\theta(x, t) = b_1 x + b_2 t.$$
- How can we generalize the model to other kinds of interaction?

Questions?

Thank you!