Time-Consistent Supervisory Accounting: the Cases of Insurance and Banking

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Modeling and managing financial risks, Paris, January 10, 2011

[1]) Partial support from AERF/CKER, The Actuarial Foundation is gratefully acknowledged.

Aims of Talk

A adapt theory of multiperiod risk measures (of "net"positions) to the specifics of situation:

- liquid assets,
- illiquid obligations,
- a typical insurance framework;

B explore application to illiquid assets, as in banking under duress conditions.

Tool Box

$$T = 2,$$

$$(\Omega, \mathcal{F}_1, \mathcal{F}_2, \mathbb{P})$$

a numéraire / benchmark / "risk-free" investment tool
$$r = (1, r_1, r_2), r_{12} = r_2/r_1.$$

$\mathcal{M}$ set of neutral pricing measures

N_1 date 0, zero cost portfolios,

N_2 date 1, zero cost portfolios,

Supervisory Risk Assessment

(change sign w.r.t. risk measures).

regard an adapted process (X_1, X_2) of (cumulative) amounts of date 1 and date 2 money.

Expressions $\Psi_0(X_1, X_2)$, $\Psi_1(X_2)$ with many properties like:

- measurability,
- cash invariance:
  $\Psi_0(X_1 + a \cdot r_1, X_2 + a \cdot r_2) = \Psi_0(X_1, X_2) + a$,
- time consistency:
  $\Psi_0(X_1, X_2) = \Psi_0(X_1, \Psi_1(X_2) \cdot r_{12})$

Acceptability

acceptability:

$$\begin{array}{rrcl} \text{at date } 0: & \Psi_0(X_1, X_2) & \geq & 0 \\ \text{at date } 1: & \Psi_1(X_2) & \geq & 0. \end{array}$$

notice

$$\begin{array}{cl} (X_1 - \Psi_0(X_1, X_2)r_1, X_2 - \Psi_0(X_1, X_2)r_2) & \text{is } 0\text{-acceptable} \\ (X_2 - \Psi_1(X_2) \cdot r_{12}) & \text{is } 1\text{-acceptable.} \end{array}$$

Links between market and risk assessment

A. Liaisons dangereuses: case of

$$\Psi_0(X_1, X_2) < 0$$

but

$$\Psi_0(X_1 + D_1, X_2 + D_1 \cdot r_{12} + D_2) \geq 0 \text{ for some } D_1 \in N_1, D_2 \in N_2.$$

B. Market consistency (hedgeable components, CFK): for functionals Φ_0, Φ_1:

$$\Phi_0(X_1 + D_1, X_2 + D_1 \cdot r_{12} + D_2) = \Phi_0(X_1, X_2)$$

for all $D_1 \in N_1, D_2 \in N_2$.
Similarly for Φ_1.

Two-sided Supervisory Accounting for Insurance

right side:

an exogenous, illiquid obligation cash-flow process:

$$Z = (0, Z_1, Z_2)$$

with "value" process (with $\mathbb{Q} \in \mathcal{M}$):

$$\overline{Z}_0 = \mathbb{E}_{\mathbb{Q}}\left[Z_1/r_1 + Z_2/r_2\right], \overline{Z}_1 = \mathbb{E}_{\mathbb{Q}}\left[Z_2/r_{12}\middle|\mathcal{F}_1\right], \ \overline{Z}_2 = 0$$

left side:

"self-financed" asset process, combined with the obligations, provides a current asset value process C:

$$C_0,\ C_1 = C_0 r_1 + D_1 - Z_1,\ C_2 = C_1 r_{12} + D_2 - Z_2,$$

$$D_1 \in N_1,\ D_2 \in N_2,\ \Delta C = D - Z \text{ (as flows)}$$

$$D \text{ "trading risk exposure" process, } C \simeq (C_0, D_1, D_2).$$

Acceptability of a Business Plan

given: $(C_0, D_1, D_2), Z$:

0-acceptability via the risk assessment condition:

$$\Psi_0(C - \overline{Z}) \geq 0$$

$\Psi_0(C - \overline{Z}) =: F_0(C, Z)$ is called the free capital

since $(C - F_0(C, Z) \cdot r, Z)$ is also acceptable.

Remark: Industry speaks of solvency rather than

acceptability, and defines it in a "bottom-up" way.

Cornerstone Concepts: Supervisory Provision and Optimal Replicating Portfolio

Supervisory cover of Z by C: $\Psi_0(C - \overline{Z}) \geq 0$

Cheapest cover:

- in absolute:
$$SP_0(Z) := \inf \left\{ C_0 \middle| (C_0, D_1, D_2), \Psi_0(C - \overline{Z}) \geq 0, \right\}$$
called supervisory provision.
- given D_1, D_2, $(C - F_0(C, Z) \cdot r, Z)$ is acceptable:
$$C_0 - F_0((C_0, D_1, D_2), Z) \geq SP_0(Z).$$
- given C_0, find special trading risk exposures D_1^*, D_2^* with
$$C_0 = F_0((C_0, D_1^*, D_2^*), Z) + SP_0(Z)$$
(seen as maximization of free capital).
- Some extra condition on Ψ_0, Ψ_1 will ensure existence of such optimal trading-risk exposure together with the hereditary property $C_1^* = F_1(C^*, Z) + SP_1(Z)$

Acceptability of a Solvable Company via Rebalancing

Remark: for an ORP (C_0, D_1^*, D_2^*), acceptability reduces to $C_0 \geq SP_0(Z)$.

Define "solvability" of (C_0, Z) by: $C_0 \geq SP_0(Z)$.

take an ORP C^* with $C_0^* = C_0$
(given C_0, do your best with (D_1^*, D_2^*)).

apply remark.

Transfer of the Obligations of an Insolvable Company

Case of $C_0 < SP_0 = SP_0(Z)$

- Goal: Find investors accepting to provide resources $A = A(Z)$ as "Eigen"-capital to cover Z by a minimal company with ORP and $C_0^* = SP_0$,
- A is the "regulated" price of $(C_2^*)^+$,
- means: provide investors with "Fremd"-capital $SP_0 - A$, taken, if possible, from old C_0,
- Rule of thumb: $\underbrace{SP_0 - A - \overline{Z}}_{\substack{\text{additional "Fremd"-capital over } \overline{Z} \\ = \text{ costs of the "Eigen"-capital } A}} = \underbrace{q}_{\text{cost-of-capital ratio}} \cdot A$

  $A = (1 + q) \cdot (SP_0 - \overline{Z})$.
- Supervisory tool: "Fremd"-capital $SP_0(Z) - A$ defined as "technical provision" $TP_0 = TP_0(Z)$:

$$TP_0 := \frac{q}{1+q} \cdot SP_0 + \frac{1}{1+q} \cdot \overline{Z}.$$

Finally: if $C_0 \geq TP_0$ the transfer will be possible (via ORP!).

Rephrasing of Acceptability of $((C_0, D_1, D_2), Z)$

$$\overline{Z}_0 \leq TP_0 \leq SP_0$$

- The "Fremd"-capital is $TP_0(Z) =: \overline{Z}_0 + RM_0(Z)$.
- The "Eigen"-capital is
  $AC_0 := C_0 - TP_0(Z) =: C_0 - \overline{Z}_0 - RM_0(Z)$.
- Benchmark for "Eigen"-capital, given D:

$$SCR_0(D, Z) := \inf \left\{ C_0 - TP_0(Z) \middle| \Delta C = D - Z, (C, Z) \text{ acceptable} \right\}$$

  (given (D_1, D_2)), be reasonable in terms of C_0).
- Condition for acceptability becomes

$$SCR_0(D, Z) \leq AC_0 \quad \text{(available/ "Eigen"-capital)}$$

Summary:

given Z, the financial market (by $\mathcal{M}$ or (N_1, N_2)) and (Ψ_0, Ψ_1) determine $SP_0(Z), SP_1(Z)$.

$SP_0(Z), SP_1(Z)$ determine:

- ORP as D_1^*, D_2^*,
- TP via $(C_2^*)^+$, RM,
- AC, to be compared with $SCR(D, Z)$,
  thus stating acceptability in terms of "Eigen"-capital,
- but SPs do not show up in balance sheet!

Supervisory Accounting for a Bank

Modeling a bank:

- owns a loan portfolio with (exogeneously given) inflow-process

$$V = (0, V_1, V_2) \geq 0,$$

- owes deposits with outflow-process

$$U = (0, U_1, U_2),$$

Successive liquid asset values:

$$\begin{aligned} C_1 &= C_0 \cdot r_1 + D_1 + V_1 - U_1 \\ C_2 &= C_1 \cdot r_{12} + D_2 + V_2 - U_2 \end{aligned}$$

With $C = (C_0, C_1, C_2)$

(C, V, U) is the bank's business plan.

Acceptability of a Bank

The value processes $\overline{V}$ and $\overline{U}$ of the respective future flows are defined in analogy to $\overline{Z}$.

The business plan (C, V, U) is acceptable if

$$\Psi_0\left(C_1 + \overline{V}_1 - \overline{U}_1, C_2\right) \geq 0$$

Supervisory Credit

Owning the loan process V and assuming $U = 0$ the bank is allowed to borrow SC_0 at r where

$$SC_0(V) := \sup \Big\{ -C_0 \Big| \text{ there exists } (D_1, D_2) \text{ s.t.} \\ \Psi_0\Big(C_0 r_1 + D_1 + V_1 + \overline{V}_1, \\ (C_0 r_1 + D_1 + V_1) r_{12} + D_2 + V_2\Big) \geq 0 \Big\}$$

One shows that

$$\begin{aligned} SC_0 &\geq 0 \quad \text{and} \\ SM_0 := \overline{V}_0 - SC_0 &\geq 0. \end{aligned}$$

Transfer of a loan process

A loan process V can be sold to another bank for an amount

$$A := A(V) \in (SC_0, \overline{V}_0).$$

Investors are ready to build a (new) bank owning V and borrowing SC_0, for a price A between SC_0 and $\overline{V}_0$:

- money borrowed by the new bank SC_0 (to be reimbursed at T),
- contributed "Eigen"-capital: $A - SC_0$.

There is a gain (on average) of $\overline{V}_0 - A$, hence the regulator picks a cost-of-capital ratio q_0:

$$\overline{V}_0 - A = q_0 \left(A - SC_0\right).$$

This defines A as

$$A = SC_0 + \frac{1}{1+q_0} SM_0, \qquad SM_0 = \overline{V}_0 - SC_0.$$

The old bank has now the liquid asset value $C_0 + A$ to guarantee the deposits U: We are back in a kind of insurance problem.

References

[1] P. Artzner and K.-T. Eisele.
Supervisory accounting: mathematics for provision – and solvency capital – requirements.
to appear in ASTIN Bulletin,
(see: http://www-irma.u-strasbg.fr/∼eisele/), 2010.

[2] K.-T. Eisele and P. Artzner.
Time-consistent supervisory accounting.
Technical report, Paper presented at Risk Day 2010, ETH Zurich
(see: http://www-irma.u-strasbg.fr/∼eisele/), 2010.