Iceberg Orders
Optimal Hiding of Limit Orders

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Role of Iceberg Orders

What is the Iceberg Order?

- Limit order with the freedom to hide a fraction of the order
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Importance

- By exposing their presence, limit order traders give away information
- Thus, generating market impact $\rightarrow$ high transaction costs
- Solution: Control exposure by showing only a fraction, $\Delta$
Role of Iceberg Orders

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Trader’s Question

- What is the optimal display size $\Delta^*$?
- How much transaction costs can be saved?
Section 1

Outline
1. **Model**
   - The Set-up
   - Assumptions
   - Some Analytical Insights

2. **Model Calibration**
   - Modeling Market Impact
   - Optimal Display Strategies
   - Iceberg vs Limit Order

3. **Conclusion**
Section 2

Model
Model Set-Up

At $t_0 = 0$ trader places his Iceberg $(N, \Delta)$ at the best bid price $B_0$, observing initial depth $D_{bid}$ (and imbalance $I_0$).

The Optimal Display Size

$$\Delta^* = \arg\min_{0 \leq \Delta \leq N} W(\Delta). \quad (1)$$
Model Set-Up

At \( t_0 = 0 \) trader places his Iceberg \((N, \Delta)\) at the best bid price \( B_0 \), observing initial depth \( D_{bid} \) (and imbalance \( l_0 \)).

The Optimal Display Size

\[
\Delta^* = \arg \min_{0 \leq \Delta \leq N} W(\Delta) .
\]  

- Expected relative execution price \( W(\Delta) := E[P \mid \Delta]; P = \tilde{P} - NB_0; \).
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The Optimal Display Size

$$\Delta^* = \arg \min_{0 \leq \Delta \leq N} W(\Delta)$$

- Expected relative execution price $W(\Delta) := E[P \mid \Delta]$; $P = \tilde{P} - NB_0$;
- Absolute Execution Price $\tilde{P} = V_T B_0 + (N - V_T)A_T$
- Execution volume $V_T$; Terminal Best Ask $A_T$
Assumptions

■ (Sequential) 2-period model ("Aggregated flow volumes")
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- LBO Priority-Hierarchy: Price, Display and Time
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$$f(s) = (1 - p) \left\{ s = 0 \right\} + \text{"Atom at Zero"}$$

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Assumptions

- (Sequential) 2-period model ("Aggregated flow volumes")
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\[
f(s) = \begin{cases} 
(1 - p) \cdot \{s=0\} + \\
\frac{p}{\alpha} e^{-\frac{s}{\alpha}} \cdot \{s>0\}
\end{cases}
\]

"Atom at Zero"

"Exponential"
Assumptions

- (Sequential) 2-period model ("Aggregated flow volumes")
- LBO Priority-Hierarchy: Price, Display and Time
- (Market and Limit) Order arrival volumes $\hat{y}, y$ and $x$ are distributed according to

$$f(s) = \begin{cases} (1 - p) \{s=0\} + \frac{pe^{-s/\alpha}}{\alpha} \{s>0\} \\ "Atom at Zero" \rightarrow "Exponential" \rightarrow "Evidence" \rightarrow \text{Empirical Distributions} \end{cases}$$

Dell ($T = 30s$)
Subsection 3

Some Analytical Insights
Some Analytical Insights

Proposition (Execution Volume)

If \( p \cdot \alpha, N > 0 \), then

\[
E[V_T \mid \Delta] = \alpha p(1 - \hat{\beta}_r) e^{-\frac{D(1-c)}{\alpha}} \left\{ \left( 1 - e^{-\frac{\Delta}{\alpha}} \right) + (1 - \beta_r)(1 - \gamma_r) \left( e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}} \right) \right\}
\]  

(2)
Some Analytical Insights

**Proposition (Execution Volume)**

*If* $p \cdot \alpha, N > 0$, *then*

$$E[V_T | \Delta] = \alpha p (1 - \hat{\beta}_r) e^{-\frac{D (1-c)}{\alpha}} \left\{ \left( 1 - e^{-\frac{\Delta}{\alpha}} \right) + (1 - \beta_r)(1 - \gamma_r) \left( e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}} \right) \right\}$$

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- $V_T$ and $A_T$ conditionally independent
Some Analytical Insights

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\]

(2)

- \( V_T \) and \( A_T \) conditionally independent
- \( W(\Delta, \beta_r, \beta, A, ...) \overset{!}{=} (N - E[V_T | \Delta]) (E[A_T | \Delta] - B_0) \)
Some Analytical Insights

**Proposition (Execution Volume)**

*If \( p \cdot \alpha, N > 0, \) then*

\[
E[VT | \Delta] = \alpha p (1 - \hat{\beta}_r) e^{-\frac{D(1-c)}{\alpha}} \left\{ \left( 1 - e^{-\frac{\Delta}{\alpha}} \right) + (1 - \beta_r)(1 - \gamma_r) \left( e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}} \right) \right\}
\]

(2)

- \( VT \) and \( AT \) conditionally independent
- \( W(\Delta, \beta_r, \beta, A, ...) \overset{!}{=} (N - E[VT | \Delta])(E[AT | \Delta] - B_0) \)
- \( \alpha, \hat{\beta}_r, ... \) depend on the trader’s \( \Delta \)-choice.
Some Analytical Insights

Hiding and Display Thresholds

Proposition

Assume $A' > 0$. There are bounds $k, m > 0$, such that

$$N \geq k(\hat{\beta}_r, \beta_r, A, p, ...) \implies \Delta^*_W = 0 \quad (3)$$

$$N \leq m(\hat{\beta}_r, \beta_r, A, p, ...) \implies \Delta^*_W = N \quad (4)$$

holds, with $k \geq m$.

- When $N$ sufficiently large, hide them all
Hiding and Display Thresholds

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Assume $A' > 0$. There are bounds $k, m > 0$, such that

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holds, with $k \geq m$.

- When $N$ sufficiently large, hide them all
- When $N$ sufficiently small, show them all
Some Analytical Insights

Results on Optimal Display Sizes

Corollary

Assume $N, \alpha p > 0$.

\[
\lim_{\frac{D}{\alpha} \to \infty} \Delta^* = \begin{cases} 
0 & \text{for } A' \geq 0 \\
N & \text{else}
\end{cases}
\]  

(5)

Corollary (Execution Price vs Execution Volume)

Let $\Delta^*_V$, $\Delta^*_W$ denote the optimal display size w.r.t. transaction price and volume. Assume $A' \geq 0$, then

\[ \Delta^*_W \leq \Delta^*_V. \]  

(6)
Some Analytical Insights

Results on Optimal Display Sizes

**Corollary**

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\lim_{\frac{D}{\alpha} \to \infty} \Delta^* = \begin{cases} 
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\]  \hspace{1cm} (5)

- **Tight vs Shallow stocks (Apple vs Cisco)**

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Let $\Delta^*_V, \Delta^*_W$ denote the optimal display size w.r.t. transaction price and volume. Assume $A' \geq 0$, then

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Some Analytical Insights

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$$
\Delta^*_W \leq \Delta^*_V.
$$

- Tight vs Shallow stocks (Apple vs Cisco)
- Consistent with empirical literature (Bessembinder (2009)).
Some Analytical Insights

Market vs Priority Impact

\[ \frac{d}{d\Delta} W(\Delta) = M_{\text{Market}} - M_{\text{Priority}} \]
Some Analytical Insights

Market vs Priority Impact

\[ \frac{d}{d\Delta} W(\Delta) = M_{Market} - M_{Priority} \] (7)

\[ M_{Priority} = W_\Delta > 0 \quad (\text{lowers Costs } W!) \]
Some Analytical Insights

**Market vs Priority Impact**

\[
\frac{d}{d\Delta} W(\Delta) = M_{Market} - M_{Priority} 
\]

\[ M_{Priority} = W_{\Delta} \overset{!}{=} 0 \quad \text{(lowers Costs W!)} \]

\[ M_{Market} = \hat{\beta}_r(\Delta) W_{\hat{\beta}_r} + \beta_r'(\Delta) W_{\beta_r} + \underbrace{p'(\Delta) W_p + A'(\Delta) W_A}_{\text{Market Order Impact} + \text{Price Impact}} > 0 \]

(7)
Some Analytical Insights

Market vs Priority Impact

\[
\frac{d}{d\Delta} W(\Delta) = M_{\text{Market}} - M_{\text{Priority}}
\]  

\(M_{\text{Priority}} = W_\Delta \overset{!}{=} 0 \) (lowers Costs \(W\!\))

\(M_{\text{Market}} = \hat{\beta}'(\Delta) W_{\hat{\beta}} + \beta'(\Delta) W_{\beta} + p'(\Delta) W_p + A'(\Delta) W_A \geq 0\)

- Market term may **lower or increase** costs \(W\) depending on signs of \(A', \beta'_r, \hat{\beta}_r, p'\) \((A(\Delta) := E[A_T | \Delta])\).
Some Analytical Insights

Absence of Market Penalty?

Corollary (Full Exposure)

Assume $A', \beta', \hat{\beta}_r, -\alpha', -p' < 0$ ("Absence of Market Penalty"). Then

$$\Delta^* = N.$$  (8)
Absence of Market Penalty?

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- Expose Order, when Market does not move against you.
Absence of Market Penalty?

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- Expose Order, when Market does not move against you.
- **Real Markets** move against you! $A', \beta_r', \hat{\beta}_r, -p' \geq 0$. 
Absence of Market Penalty?

Corollary (Full Exposure)

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- Expose Order, when Market does not move against you.
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- Check next slide.
Section 3

Model Calibration
Modeling Market Impact

I-Parametrization of Market Impact

Indeed markets move against you.
I-Parametrization of Market Impact

Indeed markets move against you.

Figure: Imbalance-based Flow - and Price Impact
I-Parametrization of Market Impact

Indeed markets move against you.

\[
T = 10s
\]

\[
I(\Delta) = \frac{D_{\text{bid}} + \Delta - D_{\text{ask}}}{D_{\text{bid}} + \Delta + D_{\text{ask}}}
\]

Figure: Imbalance-based Flow - and Price Impact
I-Parametrization of Market Impact

Indeed markets move against you.

\[ I(\Delta) = \frac{D_{bid} + \Delta - D_{ask}}{D_{bid} + \Delta + D_{ask}} \]

- Harris’ scenario: Presence of *Parasitic* trader’s (1996)

**Figure:** Imbalance-based Flow - and Price Impact
I-Parametrization of Market Impact

Indeed markets move against you.

\[ I(\Delta) = \frac{D_{bid} + \Delta - D_{ask}}{D_{bid} + \Delta + D_{ask}} \]

- Harris’ scenario: Presence of Parasitic trader’s (1996)
- Use this to compute real market optimal display strategies.

**Figure:** Imbalance-based Flow - and Price Impact
Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

**Optimal Display Strategies**

- que = $D_{bid}$
- imbl = $I_0$, size = $N$
- disratio = $\frac{\Delta}{N}$
- $D_{ask} = D_{bid} \frac{1-I_0}{1+I_0}$

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Iceberg Orders
Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

\[\text{que} = D_{\text{bid}}, \quad \text{imbl} = I_0, \quad \text{size} = N\]

\[\text{disratio} = \frac{\Delta}{N}\]

\[D_{\text{ask}} = D_{\text{bid}} \frac{1-I_0}{1+I_0}\]
Calibration Results: The Optimal Display Strategy

- $\text{que} = D_{\text{bid}}$
- $\text{imbl} = I_0, \text{size} = N$
- $\text{disratio} = \frac{\Delta}{N}$
- $(D_{\text{ask}} = D_{\text{bid}} \frac{1-I_0}{1+I_0})$
Calibration Results: The Optimal Display Strategy

- \( \text{que} = D_{\text{bid}}, \)
- \( \text{imbl} = l_0, \) \( \text{size} = N \)
- \( \text{disratio} = \frac{\Delta}{N} \)
- \( (D_{\text{ask}} = D_{\text{bid}} \frac{1-l_0}{1+l_0}) \)
Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

\[
\begin{align*}
\text{que} &= D_{\text{bid}}, \\
\text{imbl} &= l_0, \quad \text{size} = N \\
\text{disratio} &= \frac{\Delta}{N} \\
(D_{\text{ask}} &= D_{\text{bid}} \frac{1-l_0}{1+l_0})
\end{align*}
\]
Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

- \( q \equiv D_{\text{bid}}, \)
- \( \text{imbl} = l_0, \text{size} = N \)
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Calibration Results: The Optimal Display Strategy

- \( \text{que} = D_{\text{bid}} \),
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Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

que = $D_{\text{bid}}$

imbl = $l_0$, size = $N$

disratio = $\frac{\Delta}{N}$

$D_{\text{ask}} = D_{\text{bid}} \frac{1-l_0}{1+l_0}$
Calibration Results: The Optimal Display Strategy

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Optimal Display Strategies

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Calibration Results: The Optimal Display Strategy

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\text{que} = D_{bid}, \quad \text{imbl} = I_0, \quad \text{size} = N \\
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Calibration Results: The Optimal Display Strategy

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Optimal Display Strategies

Calibration Results: The Optimal Display Strategy

- $\text{que} = D_{\text{bid}}$
- $\text{imbl} = l_0$, $\text{size} = N$
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Calibration Results: The Optimal Display Strategy

- \( \text{que} = D_{\text{bid}} \),
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Optimal Display Strategies

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- $\text{disratio} = \frac{\Delta}{N}$
- $D_{\text{ask}} = D_{\text{bid}} \frac{1-l_0}{1+l_0}$
Calibration Results: The Optimal Display Strategy

\[ \text{que} = \frac{D_{\text{bid}}}{N}, \]
\[ \text{imbl} = I_0, \text{ size} = N \]
\[ \text{disratio} = \frac{\Delta}{N} \]
\[ (D_{\text{ask}} = D_{\text{bid}} \frac{1 - I_0}{1 + I_0}) \]
\[ \frac{\Delta^*}{N} \text{ falls in } N \]
Calibration Results: The Optimal Display Strategy

- que $= D_{bid}$,
- imbl $= l_0$, size $= N$
- disratio $= \Delta N$
- \[ D_{ask} = D_{bid} \frac{1 - l_0}{1 + l_0} \]
- $\frac{\Delta^*}{N}$ falls in $N$
- $\frac{\Delta^*}{N}$ increases for large $|l_0|$
Calibration: Benchmark Test (Iceberg vs Limit Order)

- Iceberg always "'beats'" the Limit Order \( \frac{W_{\text{Ice}}}{W_{\text{Limit}}} - 1 < 0 \)
Iceberg vs Limit Order

Calibration: Benchmark Test (Iceberg vs Limit Order)

- Iceberg always "'beats'" the Limit Order \( \left( \frac{W_{\text{Ice}}}{W_{\text{Limit}}} - 1 < 0 \right) \)
- for large order sizes \( (N \gg 0) \)
Iceberg vs Limit Order

Calibration: Benchmark Test (Iceberg vs Limit Order)

- Iceberg always "'beats'" the Limit Order ($\frac{W_{Ice}}{W_{Limit}} - 1 < 0$)
  - for large order sizes ($N \gg 0$)
  - for Imbalance-excess on the same side ($I > 0$)
**Calibration: Benchmark Test (Iceberg vs Limit Order)**

- Iceberg always "'beats'" the Limit Order ($\frac{W_{Ice}}{W_{Limit}} - 1 < 0$)
  - for large order sizes ($N \gg 0$)
  - for Imbalance-excess on the same side ($I > 0$)
- Up to 70% performance gain.
Conclusion

- Simple sequential trade model with minimal assumptions
- Providing $\Delta^*$ (numerical solution)
- Consistent with prior empirical research on hidden order submission strategies (Bessembinder, 2009)
- Main-Feature: Priority-Gain vs Market Impact Loss
- $\Delta^*$: Trade-Off between Market Impact and Priority-Gain
- Markets inherently penalize ”exposure
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  - At best bid: Hide more for ”tight“ stocks (there i’s only marginal priority-gain!)
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- Furthermore
  - At best bid: Hide more for "tight" stocks (there i's only marginal priority-gain!)
  - Optimal display ratio $\frac{\Delta^*}{N}$ decreases with $N$
Conclusion

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- Providing $\Delta^\ast$ (numerical solution)
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- $\Delta^\ast$: Trade-Off between Market Impact and Priority-Gain
- Markets inherently penalize "exposure"
- Furthermore
  - At best bid: Hide more for "tight" stocks (there is only marginal priority-gain!)
  - Optimal display ratio $\frac{\Delta^\ast}{N}$ decreases with $N$
  - Usage of Icebergs can significantly boost performance, particularly for large $N$ and when initial imbalance on the same side ($I_0 \gg 0$)
Thank you!