# *From Asian Options to Commodities*

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# Asian Options and Stochastic Time Changes

G - Yor ( *Notes aux Comptes Rendus* 1992, *Mathem Finance1993*)

1. In order to find the "exact" price of an Asian option in the Black-Scholes – Merton setting
   - use the property that a geometric Brownian motion is a *time-changed* squared-Bessel process

     $$S(t) = BESQ\,(X(t))$$

     where the time change X(t) is completely specified by the parameters of the geometric Brownian motion (S(t))

   - Use also two times Girsanov theorem and obtain the Laplace transform of the Asian option price with respect to the time- to- maturity

2. Introduce the idea of a stochastic time change to represent stochastic volatility

3. This powerful approach was used in CGMYSV (*Math Fin*, 2003), where the undesirable property of independence of increments in the Lévy process CGMY is broken by the introduction of a stochastic volatility, itself written as a time change. Hence, the log- price process Z is written as

$$Z(t) = CGMY\ (\ A(t))$$

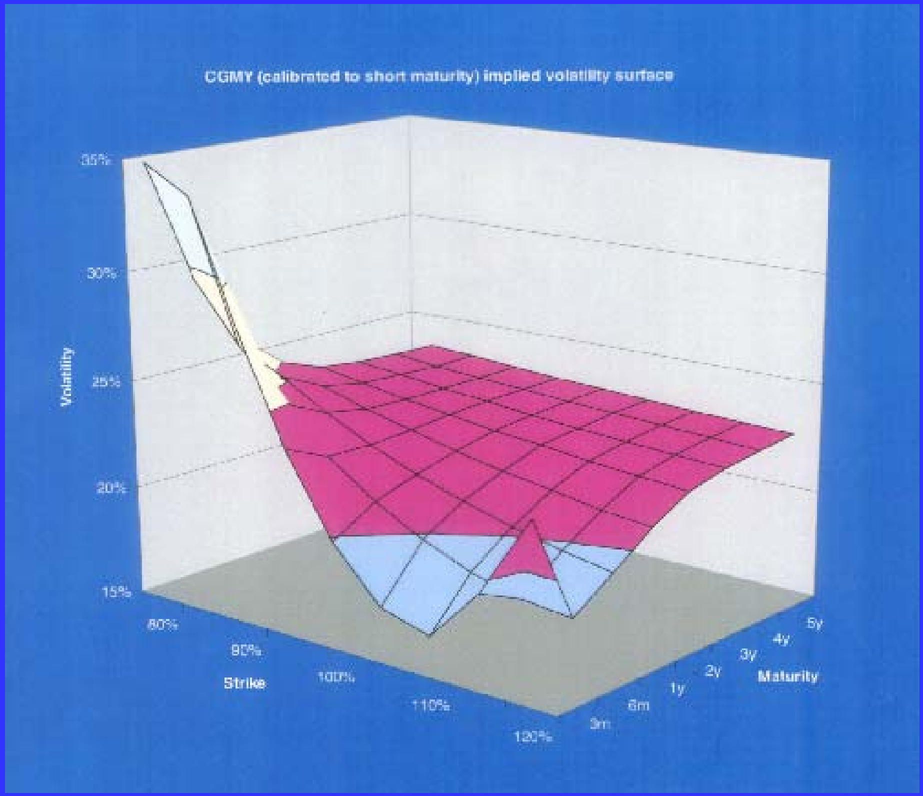
where A was chosen to be the integral of a square- root process D.

→Since D(t) is positive, A(t) - its integral from 0 to t - is increasing almost surely, the only general condition a time change needs to satisfy

The mean- reversion property of D creates volatility clustering, as observed by many authors ( Bates, Fouque, Barndorff- Nielsen)

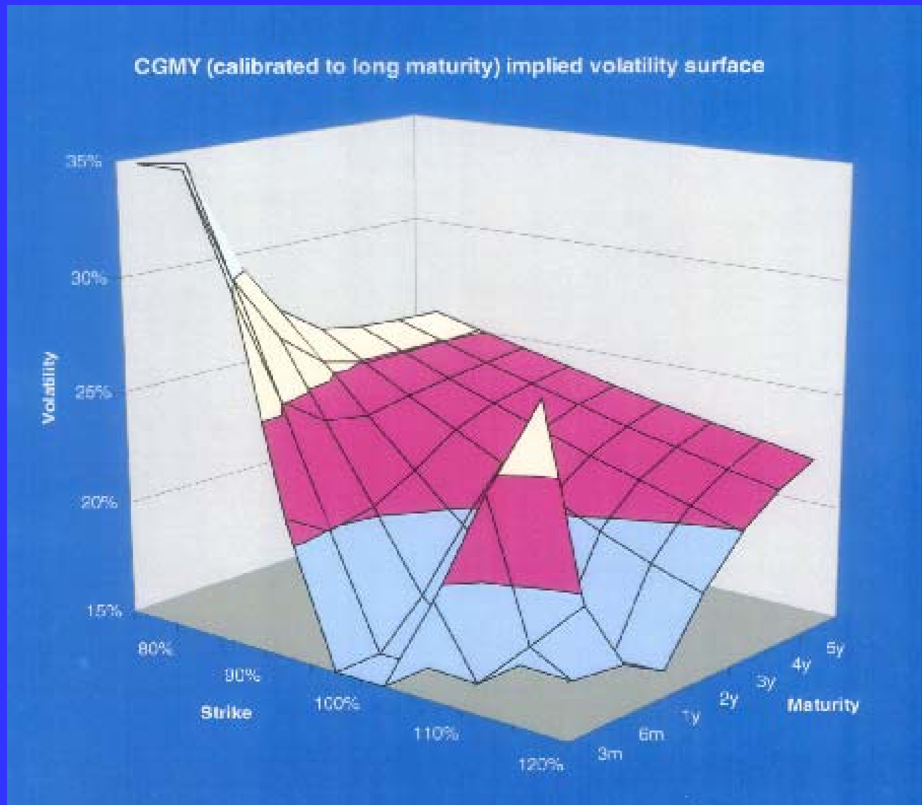
→The process Z now is able to calibrate the volatility surface while CGMY was able to calibrate well only short (or long) maturities. The number of parameters in Z is seven ( in fact 6 because of the drift condition under Q)

# CGMY (2002) volatility surface : short calibration




- Very flat at long maturity

# CGMY volatility surface : long calibration




- Much better at long maturity
- Short maturity bad

$\longrightarrow$ Eydeland- G (RISK, 1995) invert the Laplace transform of these quantities.

For instance, in the case of the Asian call option, the Laplace transform

$$\varphi(h) = \int_0^{+\infty} e^{-h\tau} C(\tau) d\tau$$

is first changed into

$$\varphi_1(h) = \varphi(h) e^{\alpha h}$$

where the factor $\alpha$ is chosen to make $\varphi$ regular in the domain of complex numbers with a positive real part

$\longrightarrow$ Because $\varphi_1$ is *holomorphic* in the $h \geq 0$, its inverse Laplace transform maybe expressed in terms of its inverse Fourier transform

$\longrightarrow$ For computational efficiency, Eydeland -G use the Fast Fourier transform and derive the inversion of the function $\varphi_1$ (or equivalently, $\varphi$)

$\longrightarrow$ Carr - Madan (1999) use the same multiplier in their paper on the Fourier transform in option pricing

# Asian or Average Rate Options in Oil, Gold and Agriculturals

An Asian is an option whose pay-off is based on an average of close or open prices, rather than on a single price at maturity

Pay-out for the call holder = max $(0, A(T) – k)$ where $A(T)$ is an average over a period $(0, T)$, calculated daily or weekly, etc…

Asian options are frequently used

- in oil markets, Asian options represent the vast majority
- in emerging markets (to avoid price manipulations at date T)
- in gold: Asian options are as liquid on the London Bullion Exchange as the plain- vanilla ones
- In all other commodity markets : electricity, natural gas, shipping

# Commodity Research Bureau Index – 1990 to 2010


Chart created with NeoTicker EOD © 1998-2009 TickQuest Inc.

# Shipping and Freight as Part of the Commodities World

→ Two types of freight

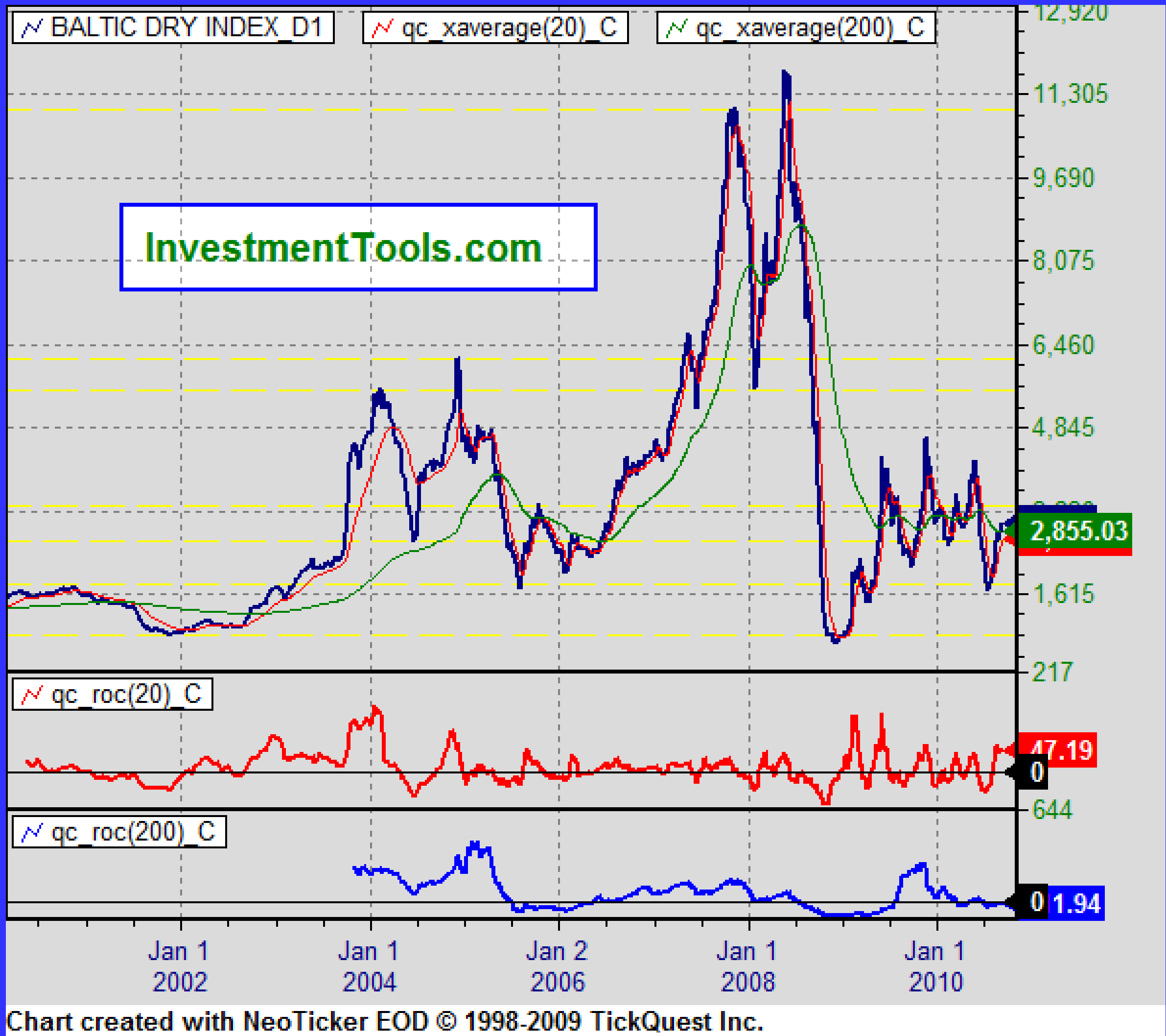
- Dry bulk: Capesize, Panamax, Handymax
- Tankers: Suez-Aframax

→ Major actors in the spot and forward markets: Cargill, Louis Dreyfus, Total, Shell, Deutsche Bank, Morgan Stanley

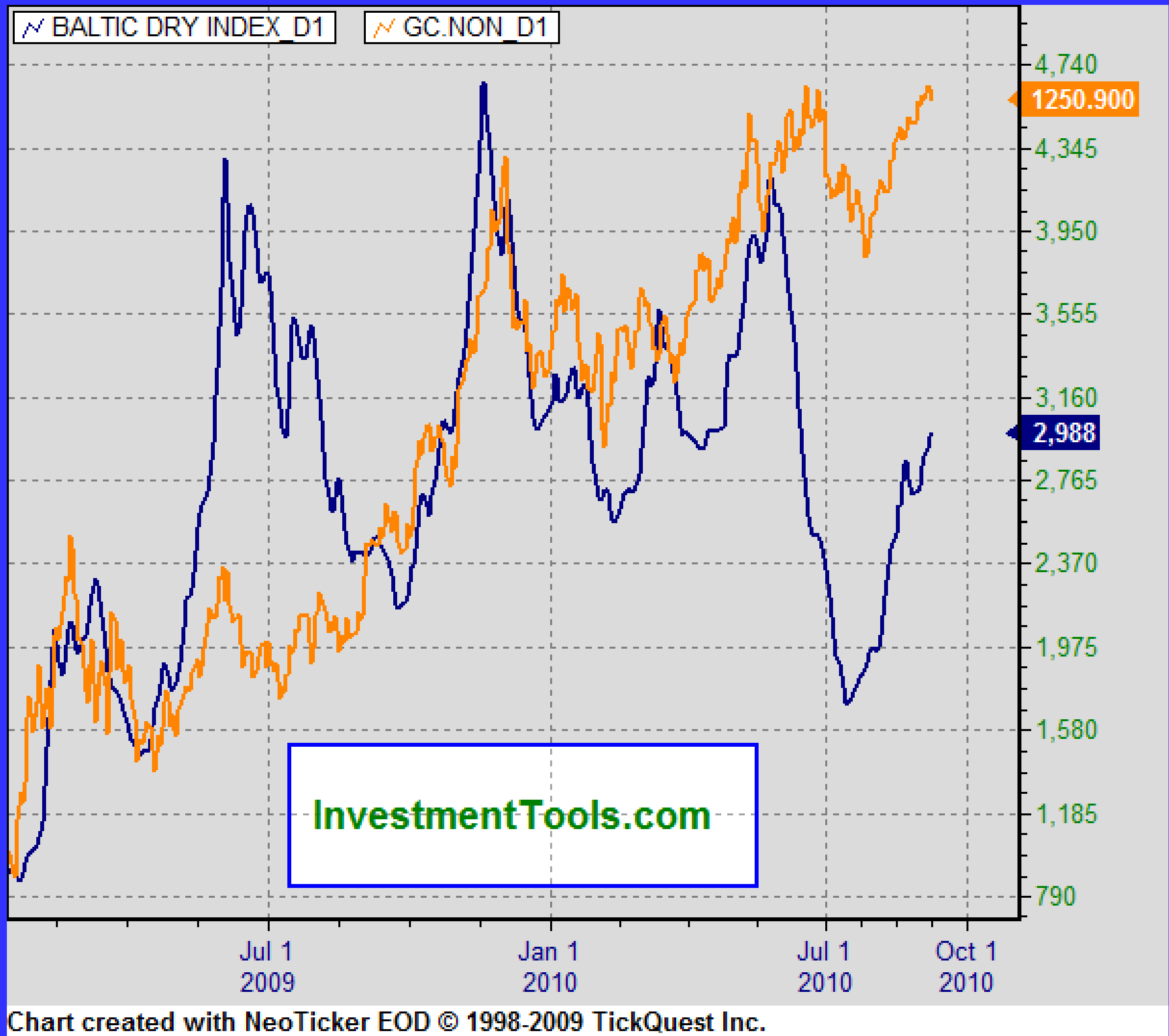
→ Trading activity

- Baltic Exchange (London), used to offer Futures contracts, now only forwards
- Imarex (Oslo), provides daily quotes on maritime shipping
- LCH-Clearnet (London)
- Nymex (New York), very active for dry bulk

# The Baltic Dry Index (BDI) – 2000 to 2010
# Prices dipped from 12,000 in May 08 to 700 in Dec 08!



# BDI and Gold – 2008 to 2010



# Theory of Storage
## Keynes (1936), Kaldor (1939), Working (1949), Brennan (1958)

Three fundamentals results:

→ The convenience yield accounts for the benefit that accrues to the holder of the physical commodity but not to the holder of the futures contract. It is represented as an implicit dividend

→ The volatility of the commodity spot price is high when inventory is low

→ The volatility of Futures contracts decreases with the maturity: "Samuelson effect“

Moreover, forward curves used to be viewed as being mostly in backwardation, the so- called “normal backwardation”, due both to the convenience yield and an assumption of mean- reversion in prices

# Spot-Forward Relationship for a Storable Commodity

Under no arbitrage

$$f^{T}(t) = S(t)\left[\,1 + \underbrace{r(T-t)}_{\text{cos t of financing}} + \underbrace{c(T-t)}_{\text{cos t of storage}} - \underbrace{y_1(T-t)}_{\text{implicit dividend}}\,\right]$$

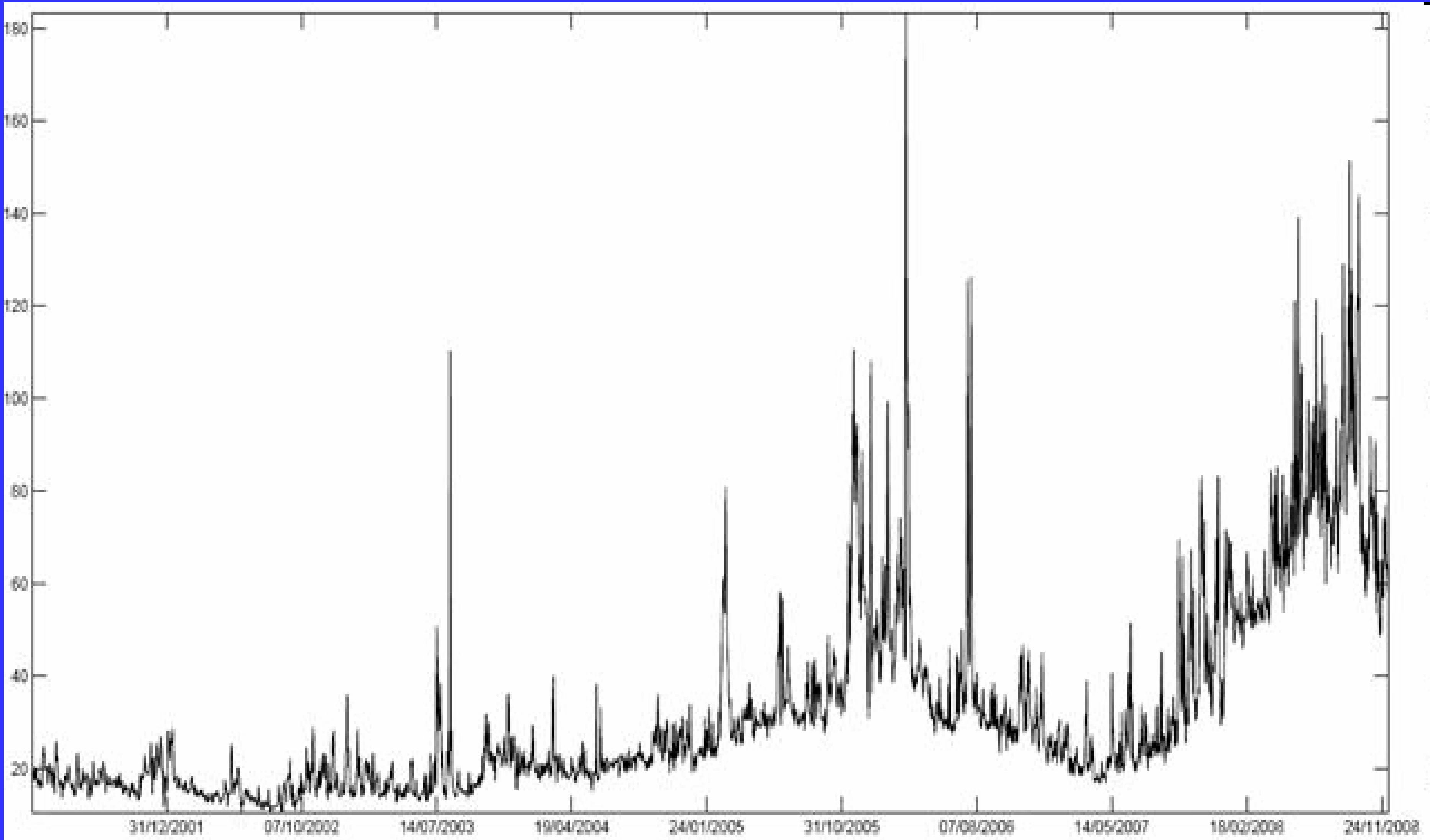
If we define a convenience yield net of cost of storage

$$f^{T}(t) = S(t)\left[1 + (r - y)(T - t)\right]$$

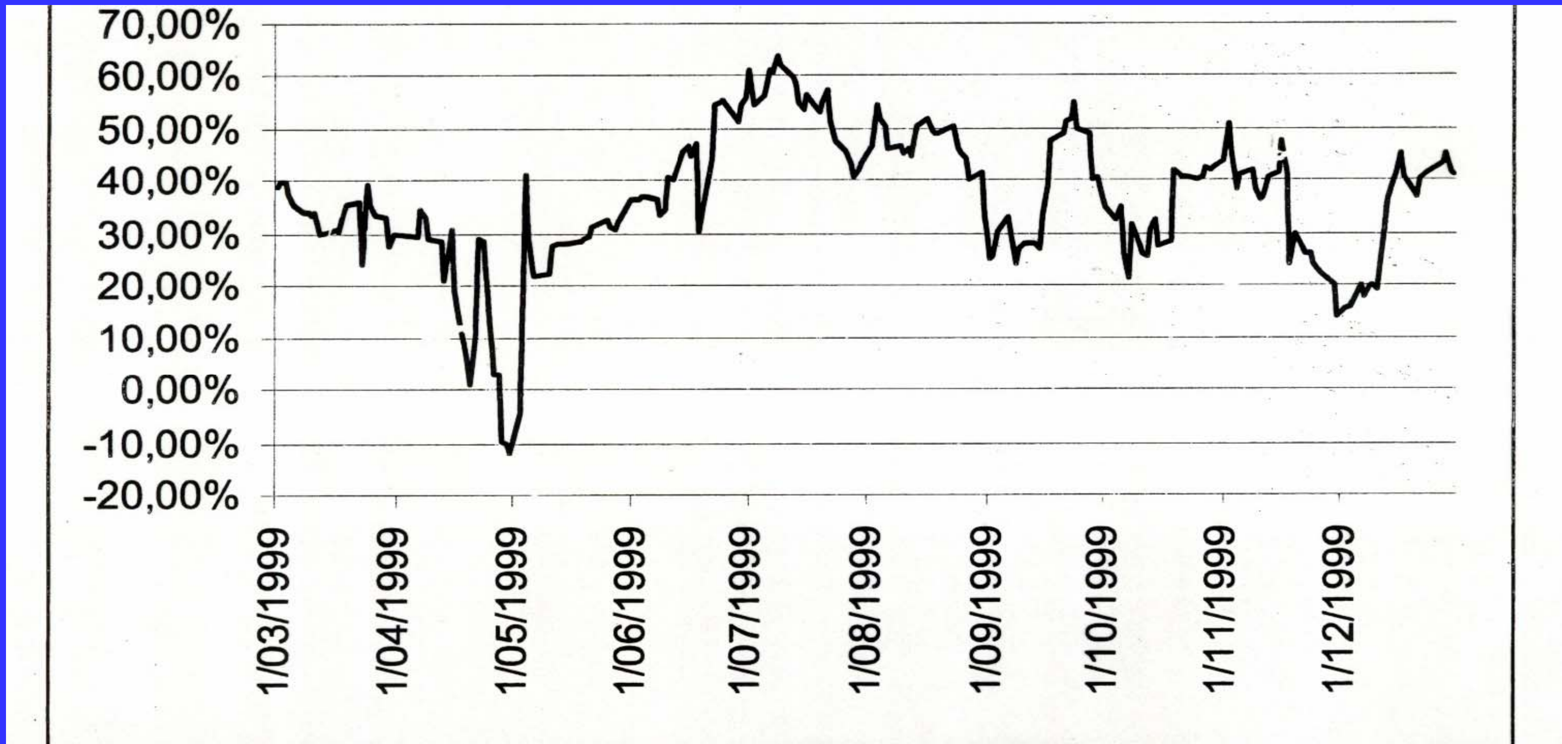
Or in continuous time, at a fixed date t for a given maturity T

$$f^{T}(t) = S(t)e^{(r-y)(T-t)}$$

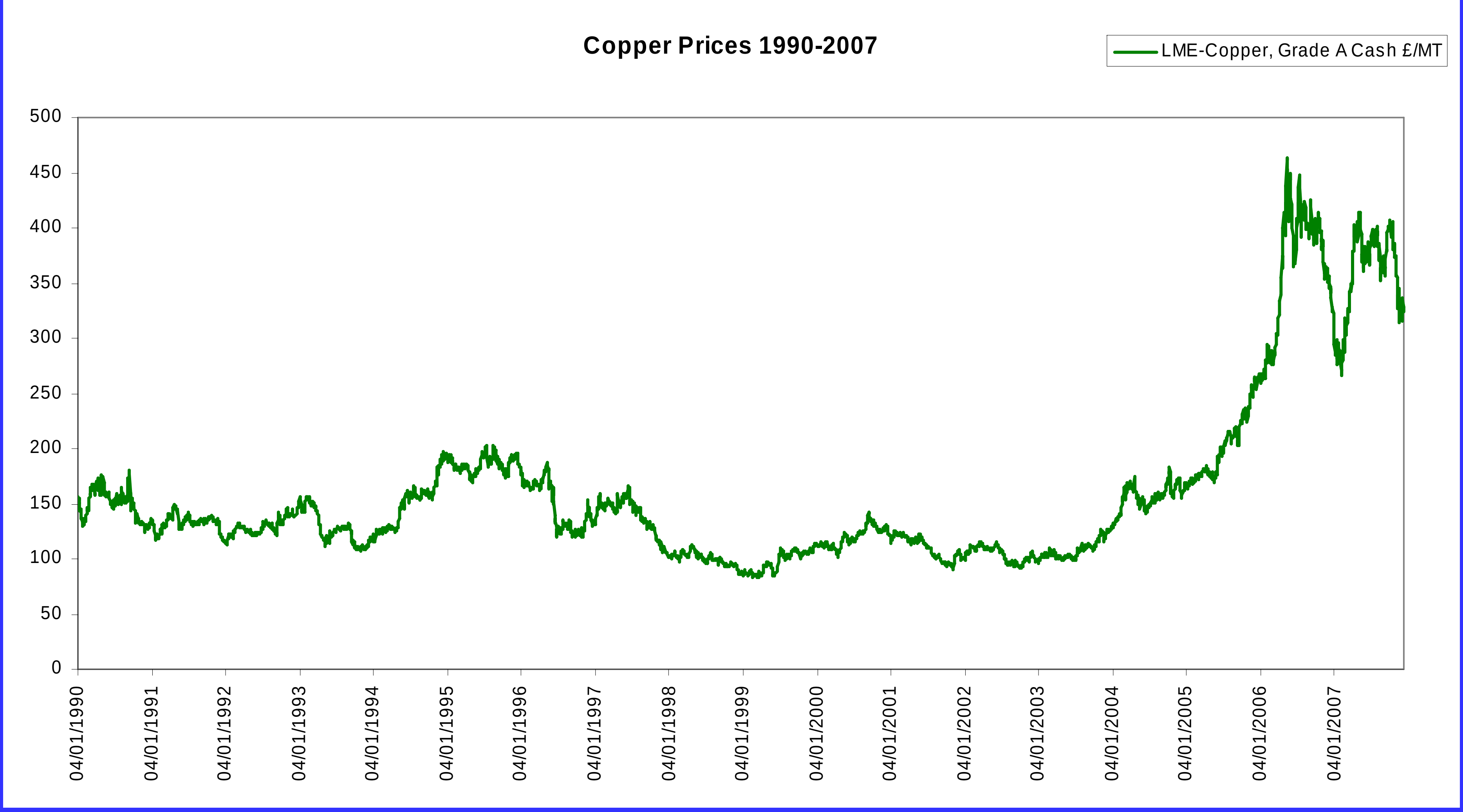
# An Example of Electricity Price Trajectory



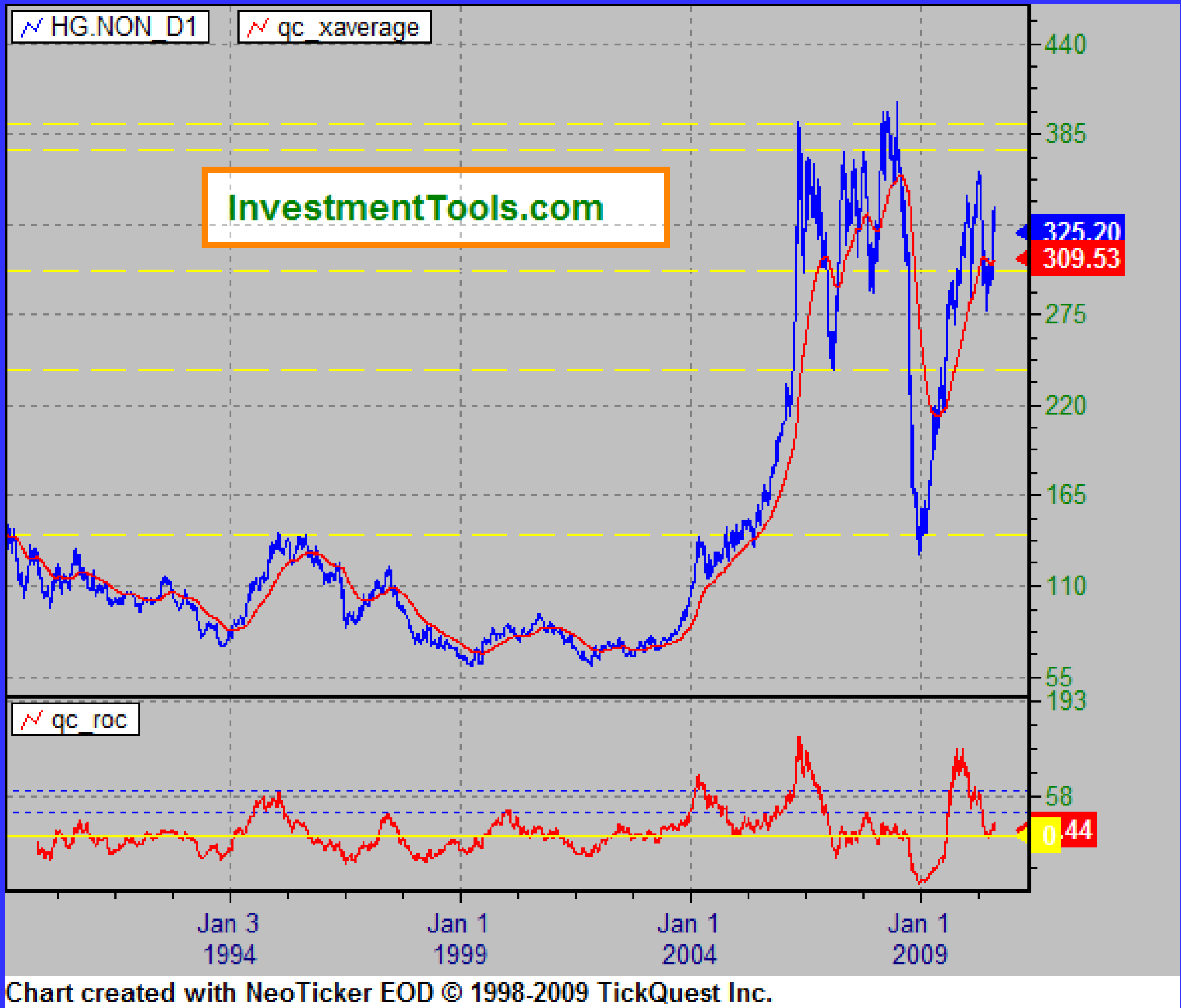
# Correlation Spot- Prompt month (Nordpool):
# The standard convenience yield does not apply to electricity
# (Eydeland- G, *RISK*, 1998)



# LME Copper Prices, in the Sterling Numéraire

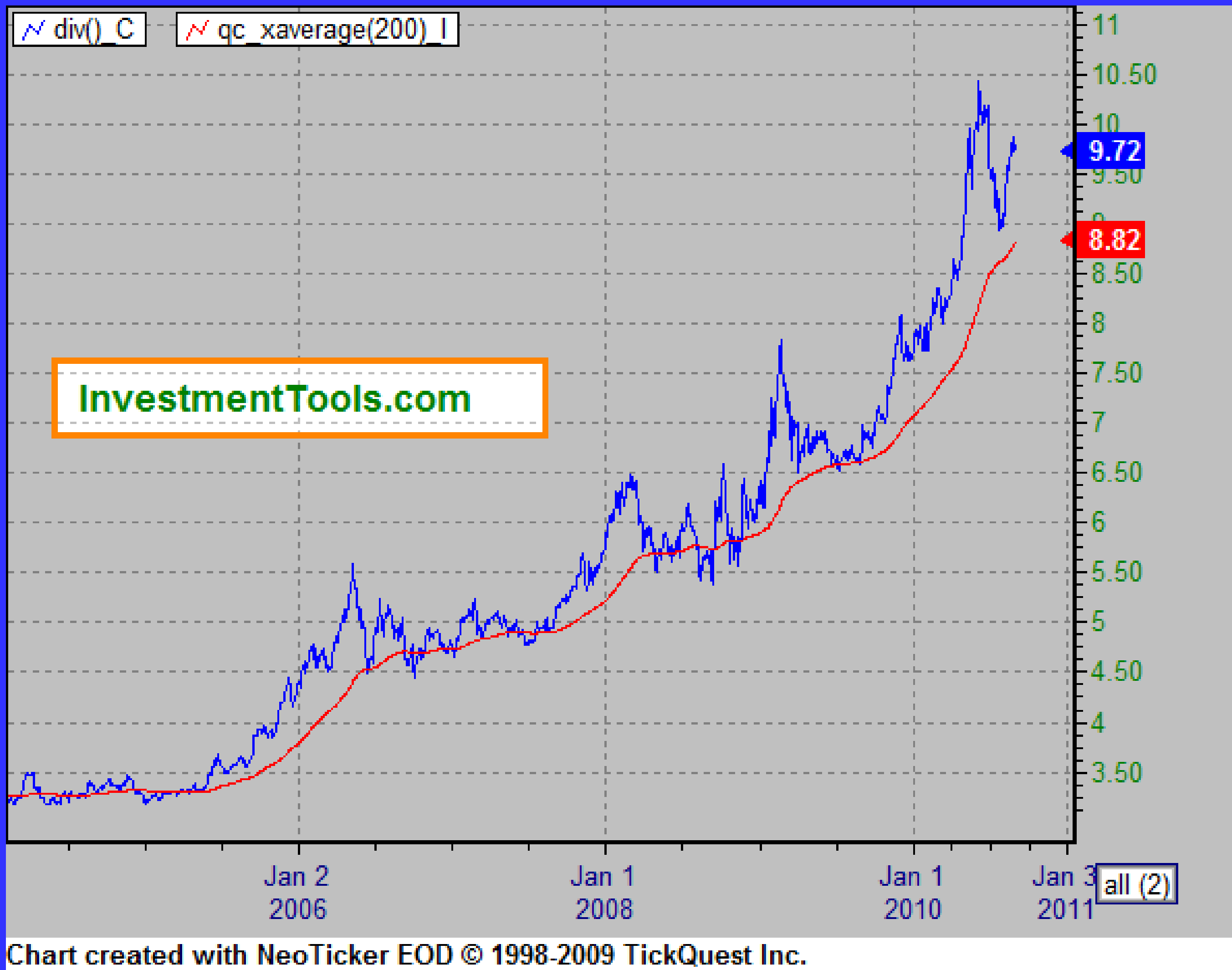


# Copper Prices – 1989 to 2010



# Gold Prices - 2002 to 2010

# Gold Prices in the Euro Numéraire



# Number of Ounces of Gold that can buy the Average US House


Chart created with NeoTicker EOD © 1998-2009 TickQuest Inc.

# Crude Oil (Petroleum)

→ **It is a** complex mixture of various hydrocarbons found in the upper layers of the earth's crust

The commercial drilling of oil began in Titusville, Pennsylvania, in 1859

→ There are over 200 grades of crude oil around the world, differing by the *sulfur* content and *gravity*. The highest quality crudes are those with a low sulfur content and a high specific gravity (API gravity)

→ On this basis, the US WTI and Malaysia's Tapis are the best quality crude oils. The heavier, sour crudes from the United Emirates and Mexico are of a poorer quality and consequently trade at a discount to WTI

# Crude Oil

**Matthew Simmons, *Twilight in the Desert***

**→ "Sooner or later, the worldwide use of oil must peak because oil, like the other two fossils - coal and natural gas - is non renewable"**

**→ Over the past 30 years, daily oil consumption has risen by approximately 33 million barrels, Asia accounting for more than half of this growth in demand**

**→ Current consumption levels suggest that the world's oil supply should last until around 2045 (without including tar sands)**

**→ The world's largest producers are Saudi Arabia (13% of world production), Russia (12%), the United States (7%), Iran (6%) and China (5%)**

**→ The Gulf of Mexico region provides about 29% of the US oil**

# The oil market as a World Market

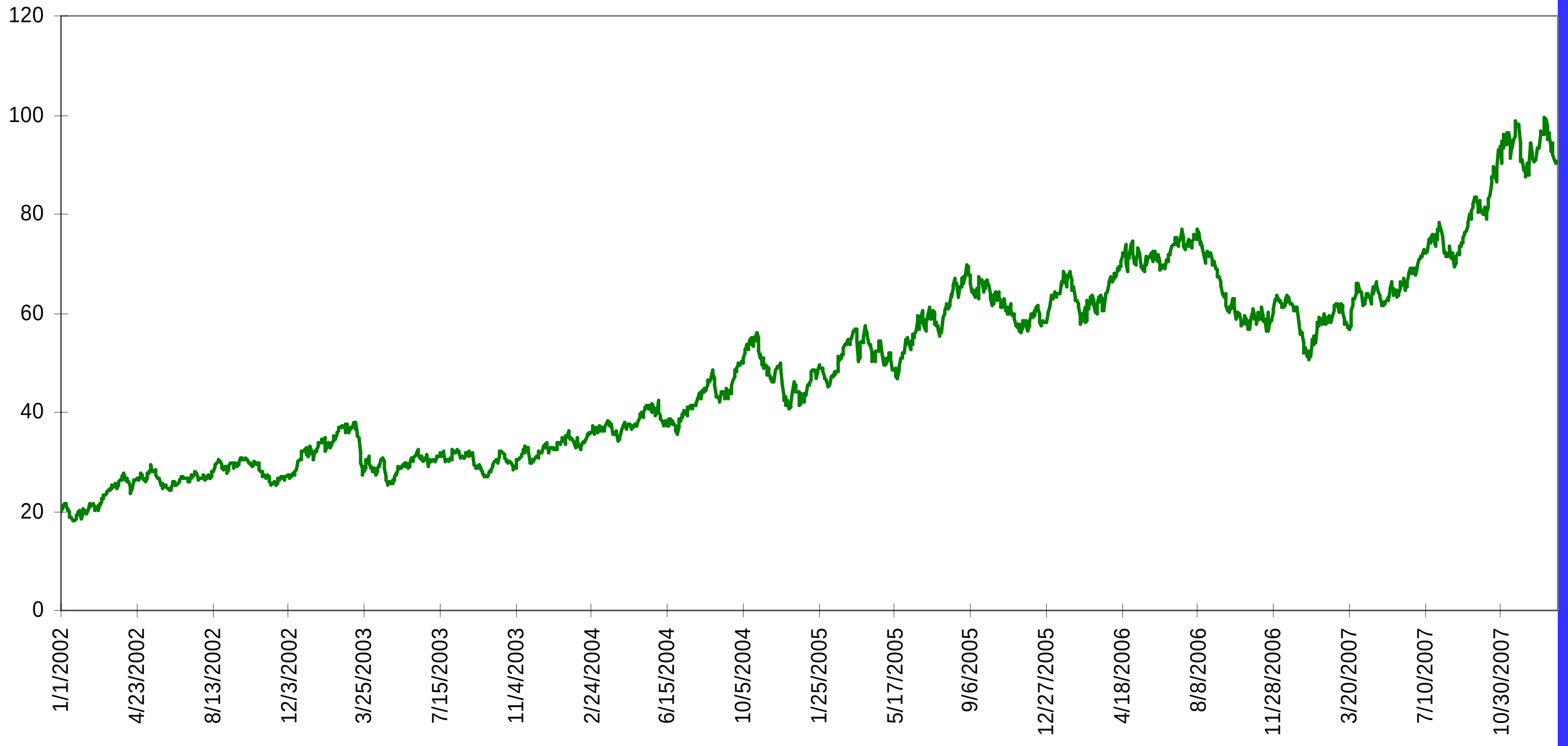
→ Seasonality is not significant since tankers are rerouted to satisfy a surge of demand in a given region

→ The representation of the spot price as a diffusion is mostly acceptable

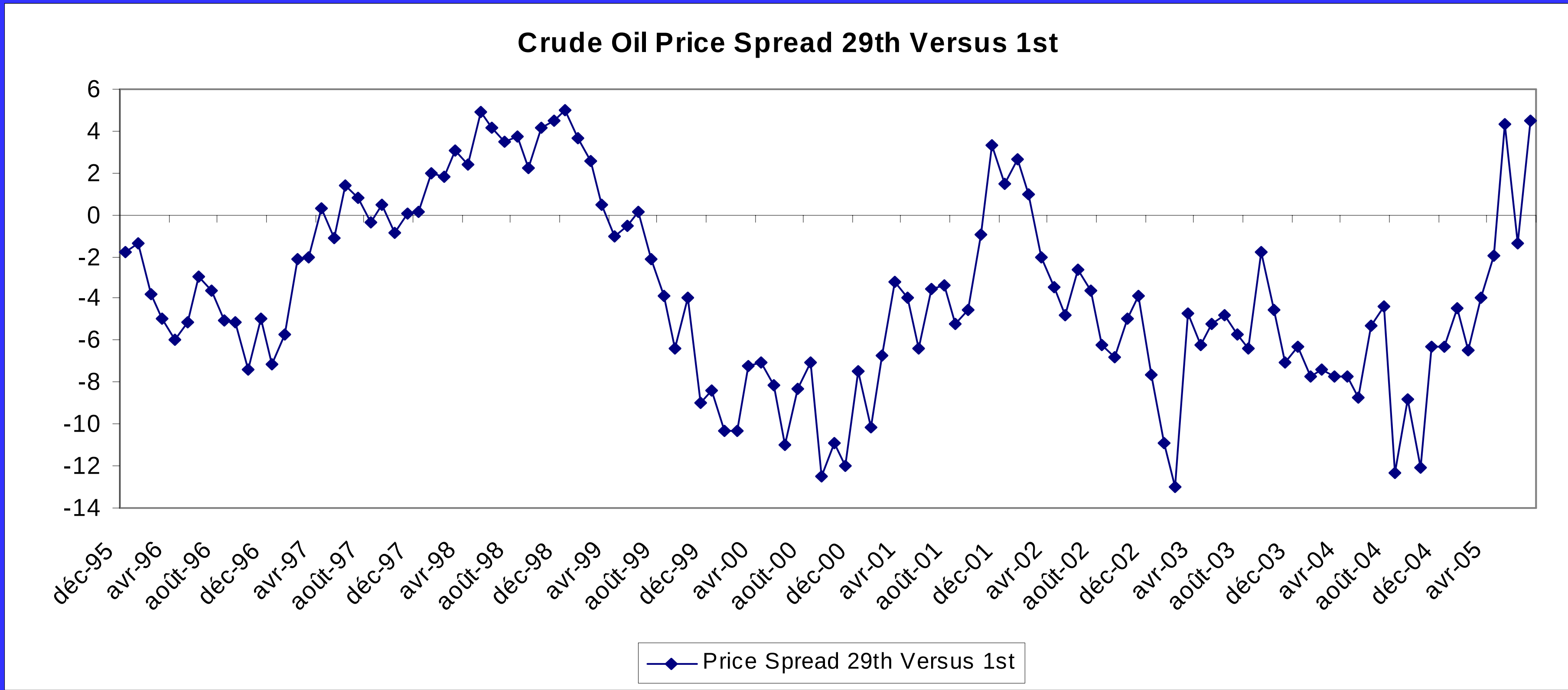
→ It is in the context of this crucial commodity that Brennan and Schwartz (1985), Gibson and Schwartz (1990) remarkably introduced in the valuation of derivative contracts the economic concept of *convenience yield*

→ Gabillon (1991) shows the role of the convenience yield in explaining the role of oil forward curves, as well as the most frequent backwardation observed in oil forward curves up to 1990

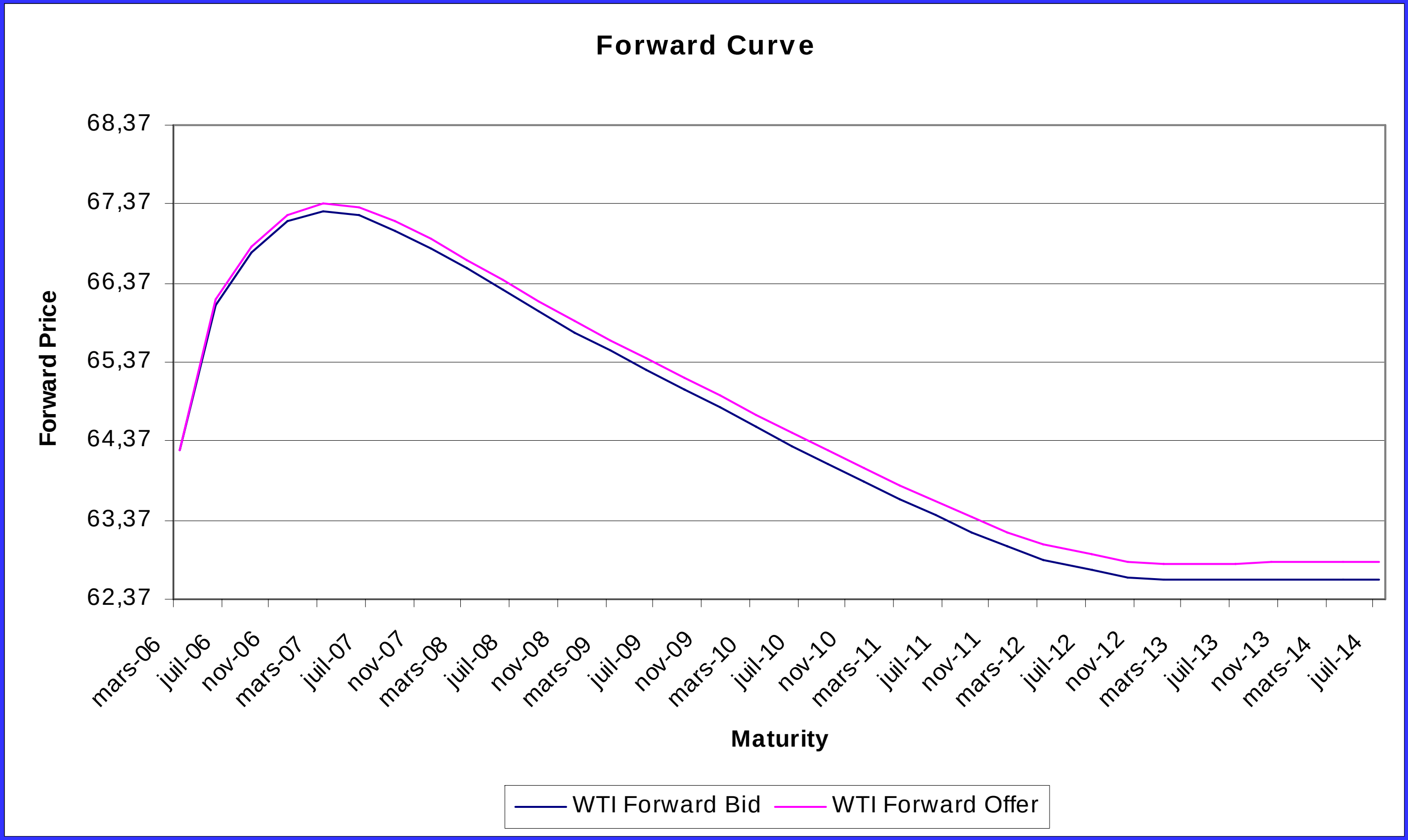
# WTI Oil Prices Jan 2002 - Oct 2007



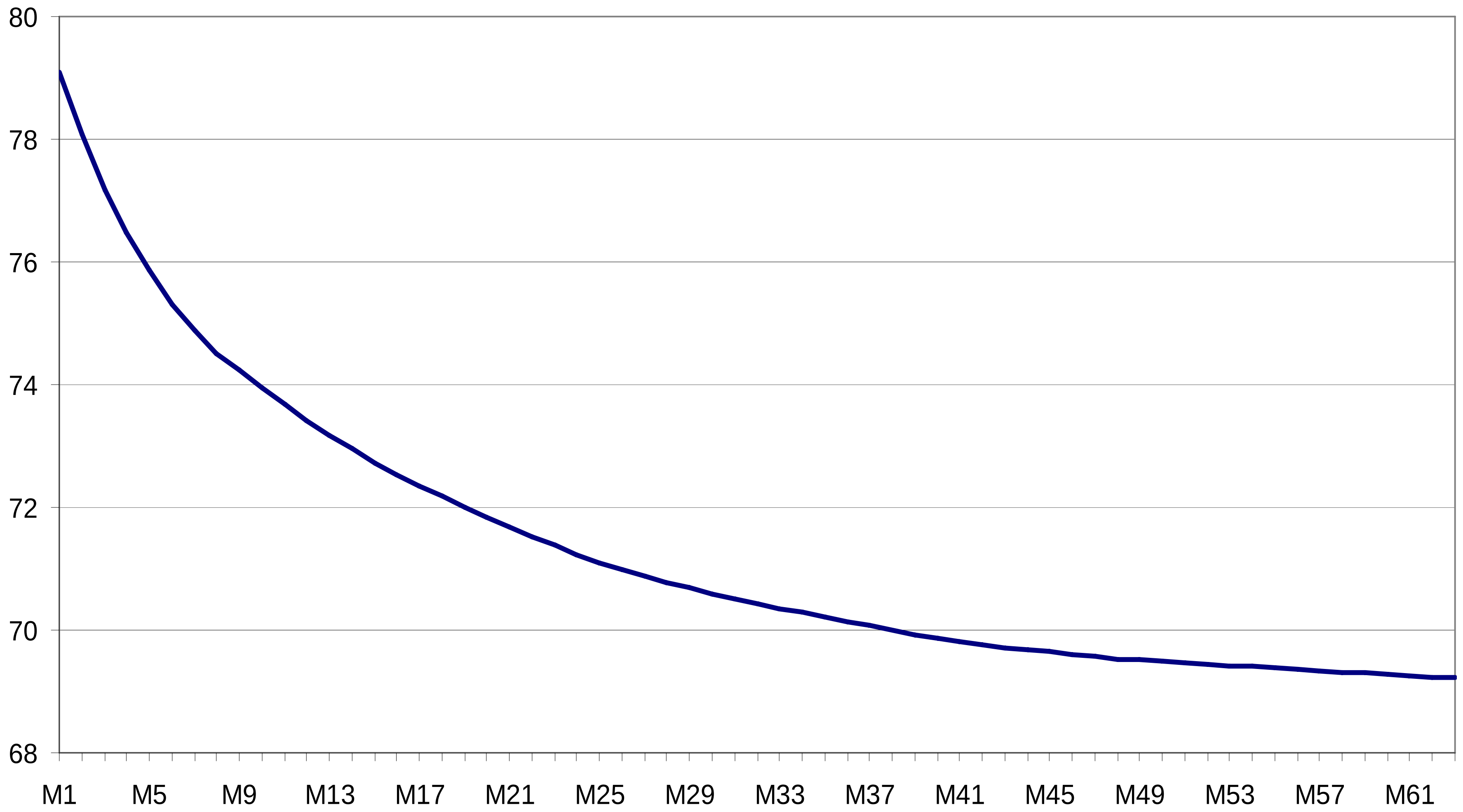
# Spread of the Oil Forward Curve - Dec 1995 / Dec 2005

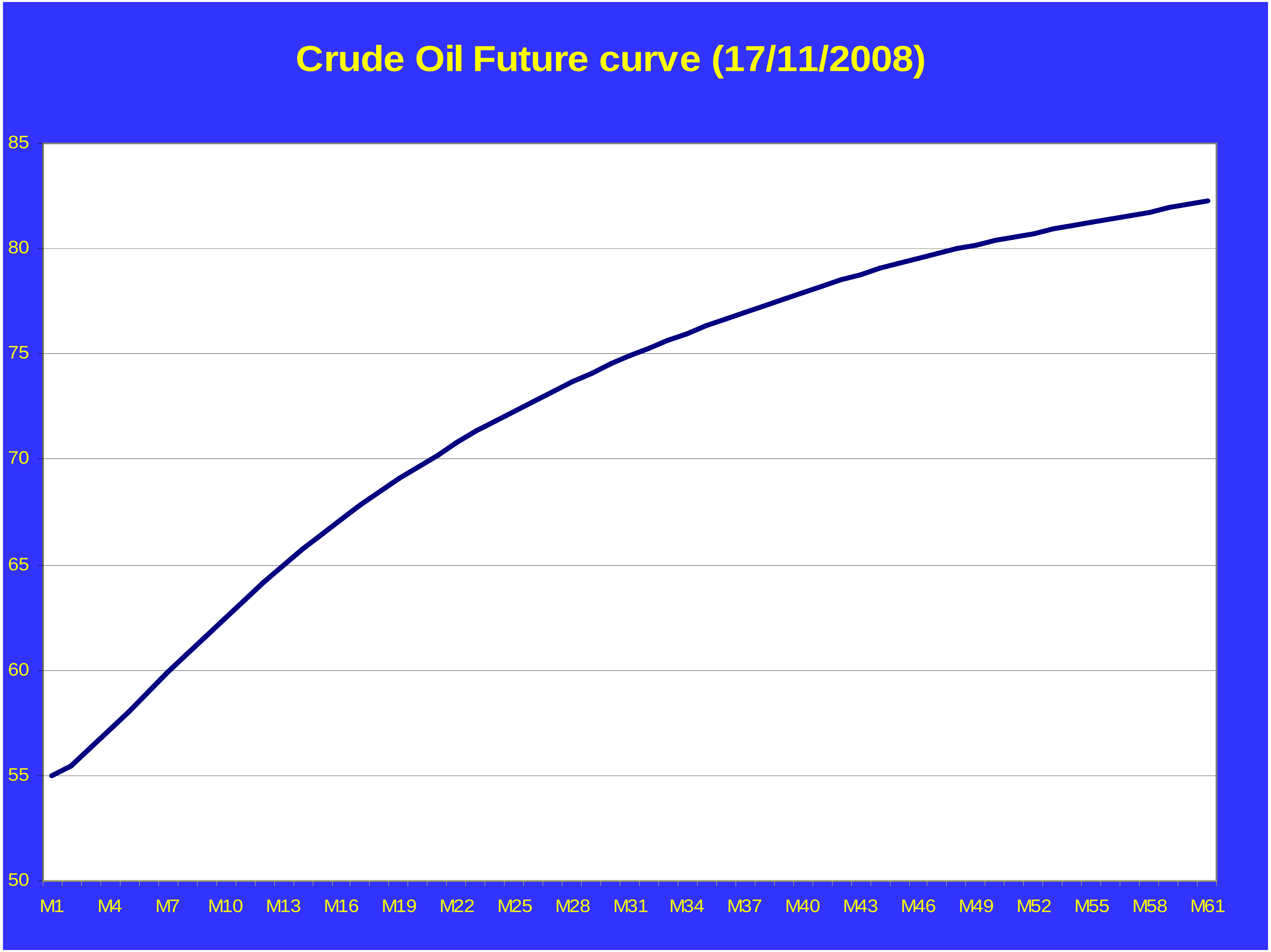


# Oil Forward Curve - March 2006 (Bid and Ask)

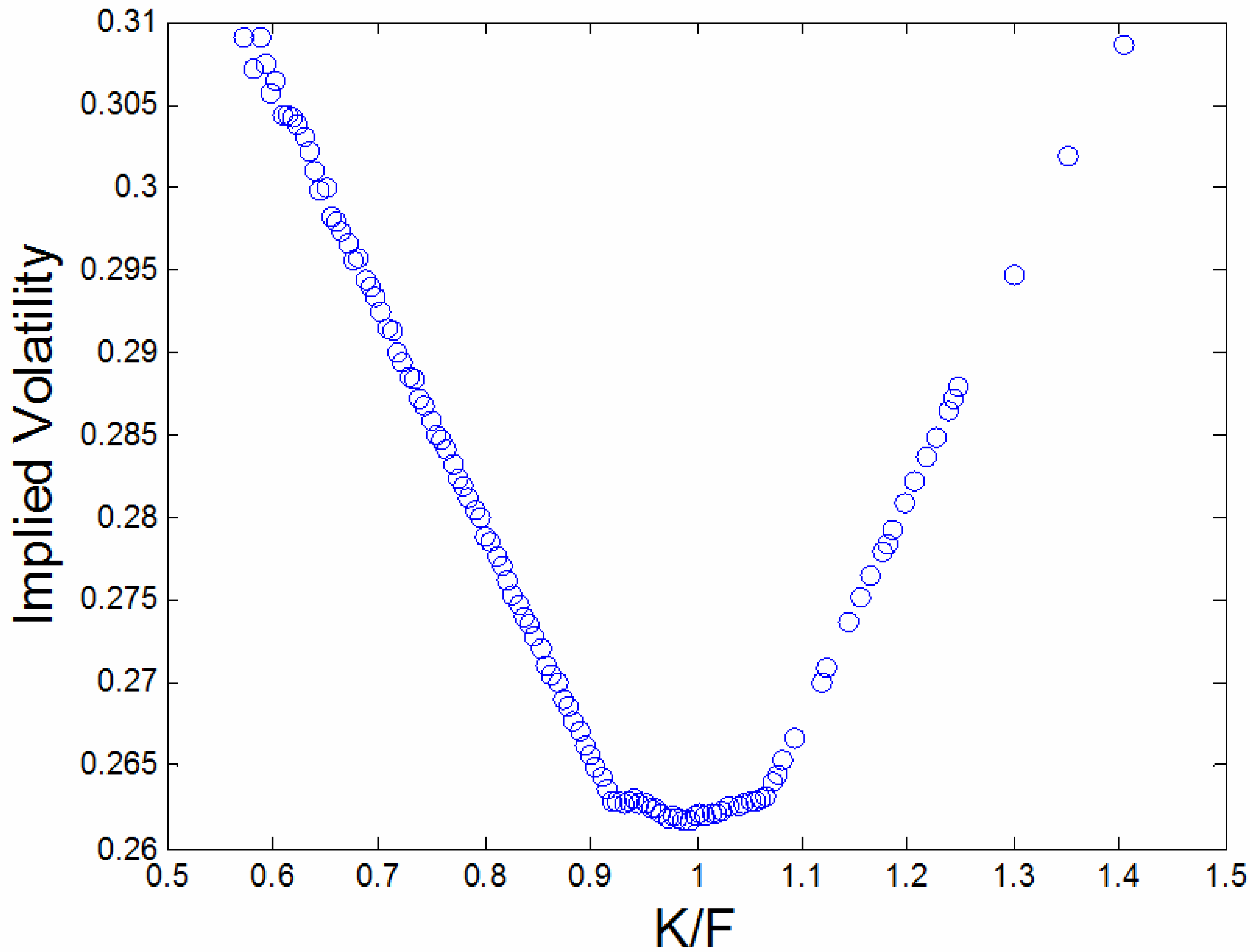


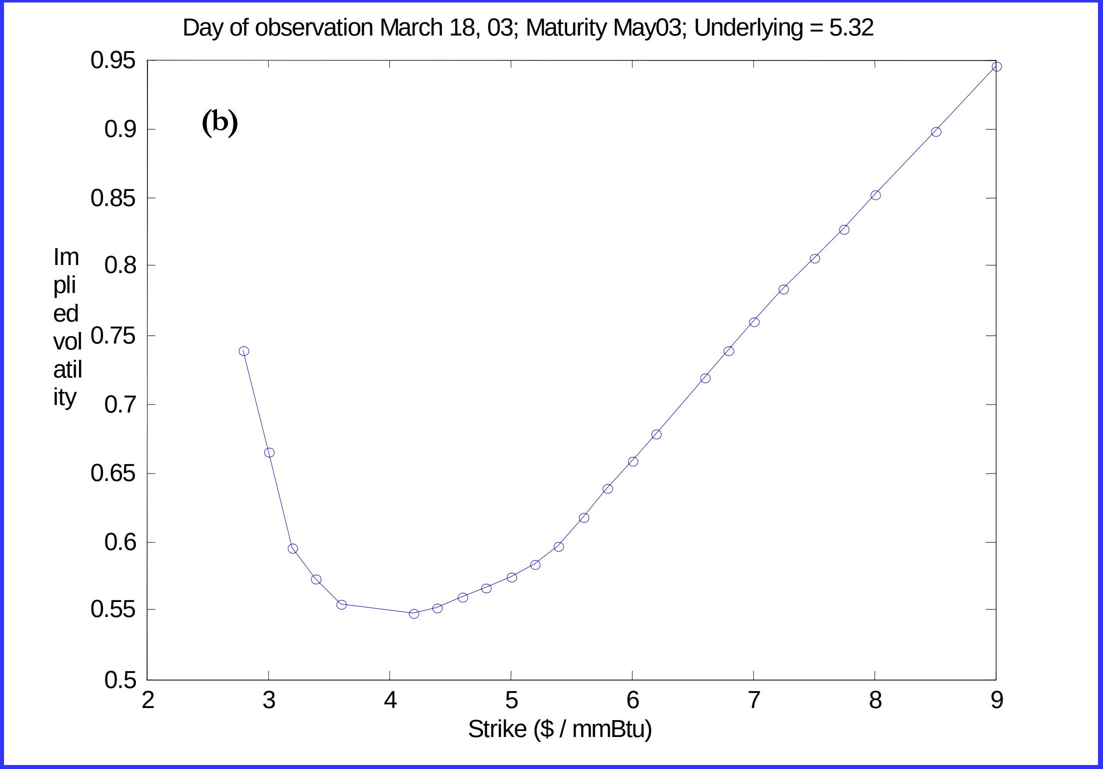
# Back to Backwardation in September 2007




Crude Oil Future curve (17/11/2008)
85
80
75
70
65
60
55
50
M1
M4
M7
M10
M13
M16
M19
M22
M25
M28
M31
M34
M37
M40
M43
M46
M49
M52
M55
M58
M61

# Feb 22, 08 Oil 6 Months implied Volatility

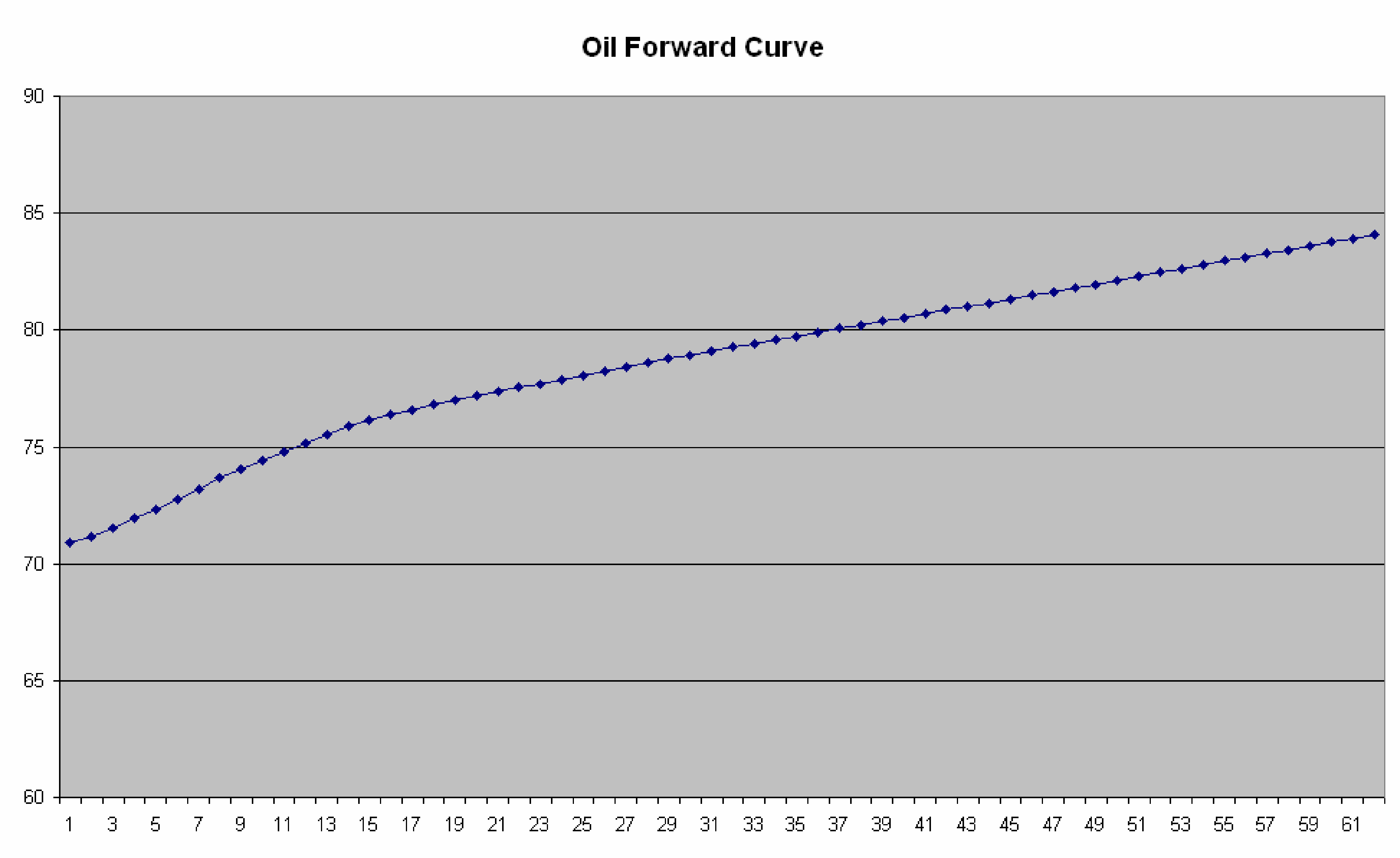




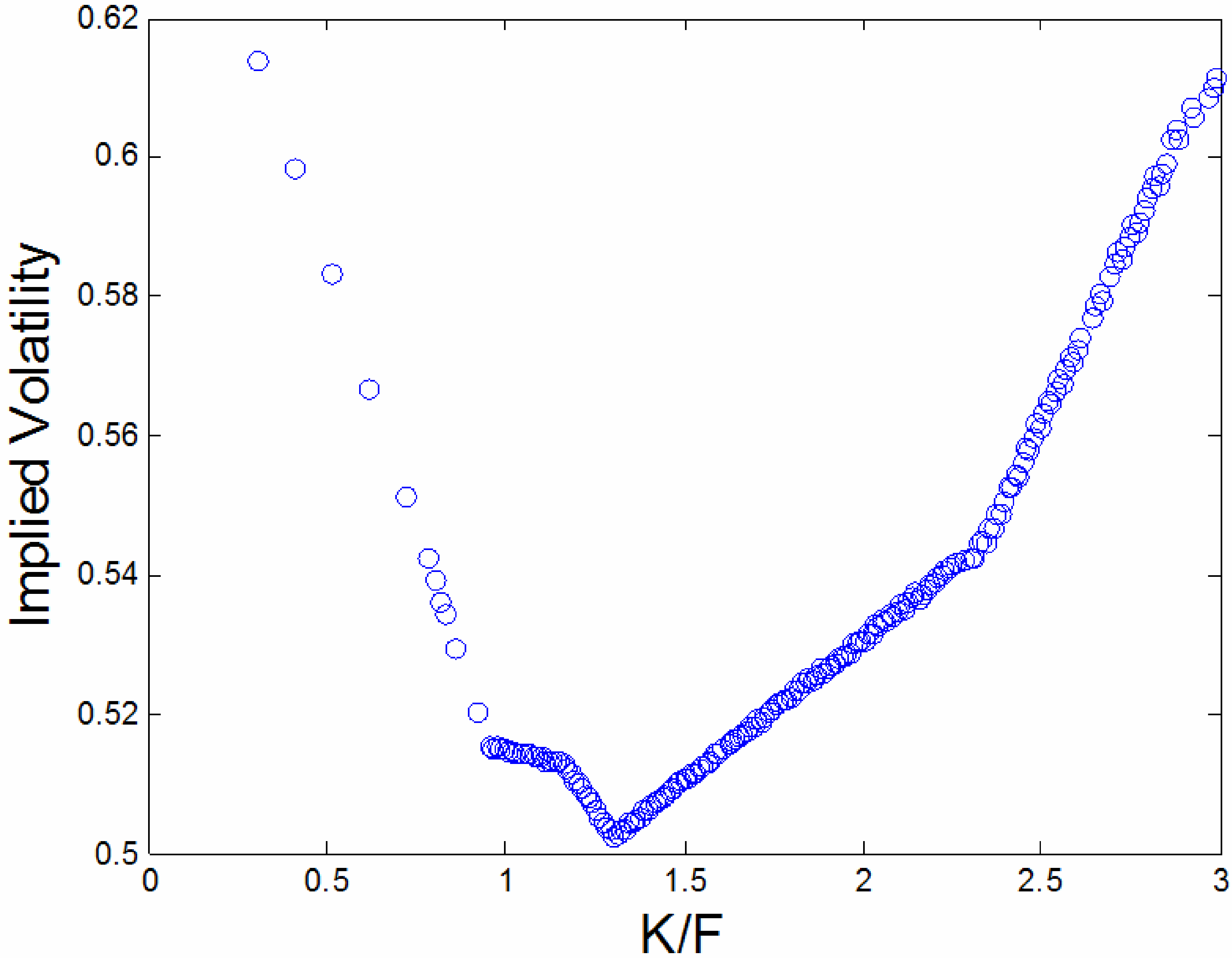

In 2003, the NYMEX natural gas volatility smile is skewed to the right:

*Inverse Leverage Effect*

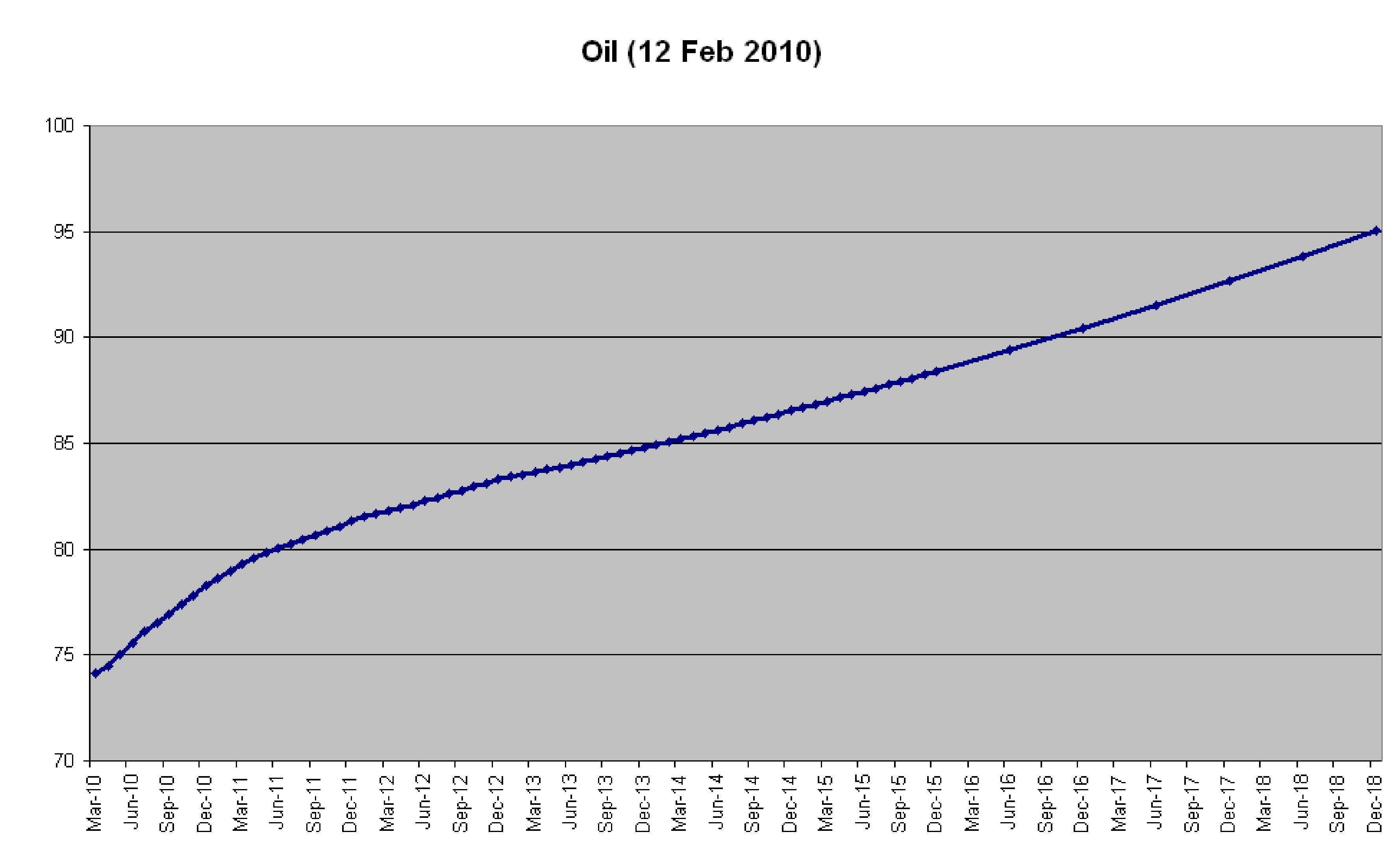
# Crude oil Forward Curve, Oct 2009

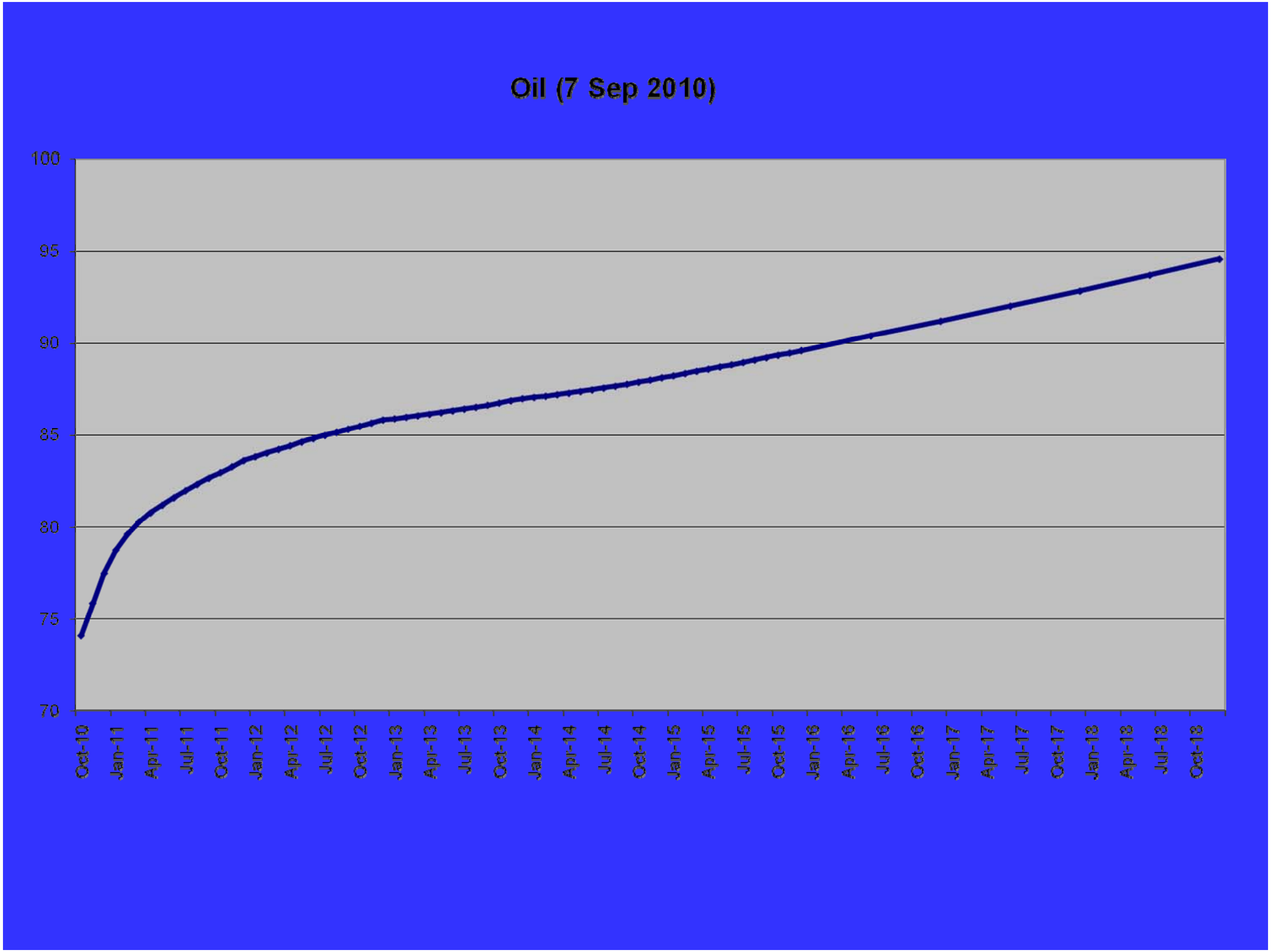


# Feb 23, 09 Oil 6 Months implied Volatility



# Crude Oil Forward Curve – Feb 2010




Oil (7 Sep 2010)
100
95
90
85
80
75
70
Oct-10
Jan-11
Apr-11
Jul-11
Oct-11
Jan-12
Apr-12
Jul-12
Oct-12
Jan-13
Apr-13
Jul-13
Oct-13
Jan-14
Apr-14
Jul-14
Oct-14
Jan-15
Apr-15
Jul-15
Oct-15
Jan-16
Apr-16
Jul-16
Oct-16
Jan-17
Apr-17
Jul-17
Oct-17
Jan-18
Apr-18
Jul-18
Oct-18

# A Possible Model for Oil Futures and Options

→ The first state variable is naturally the spot price of the commodity

→ Stochastic volatility is a good candidate for the second state variable, as it will account for inventory (besides being very popular in equity option models)

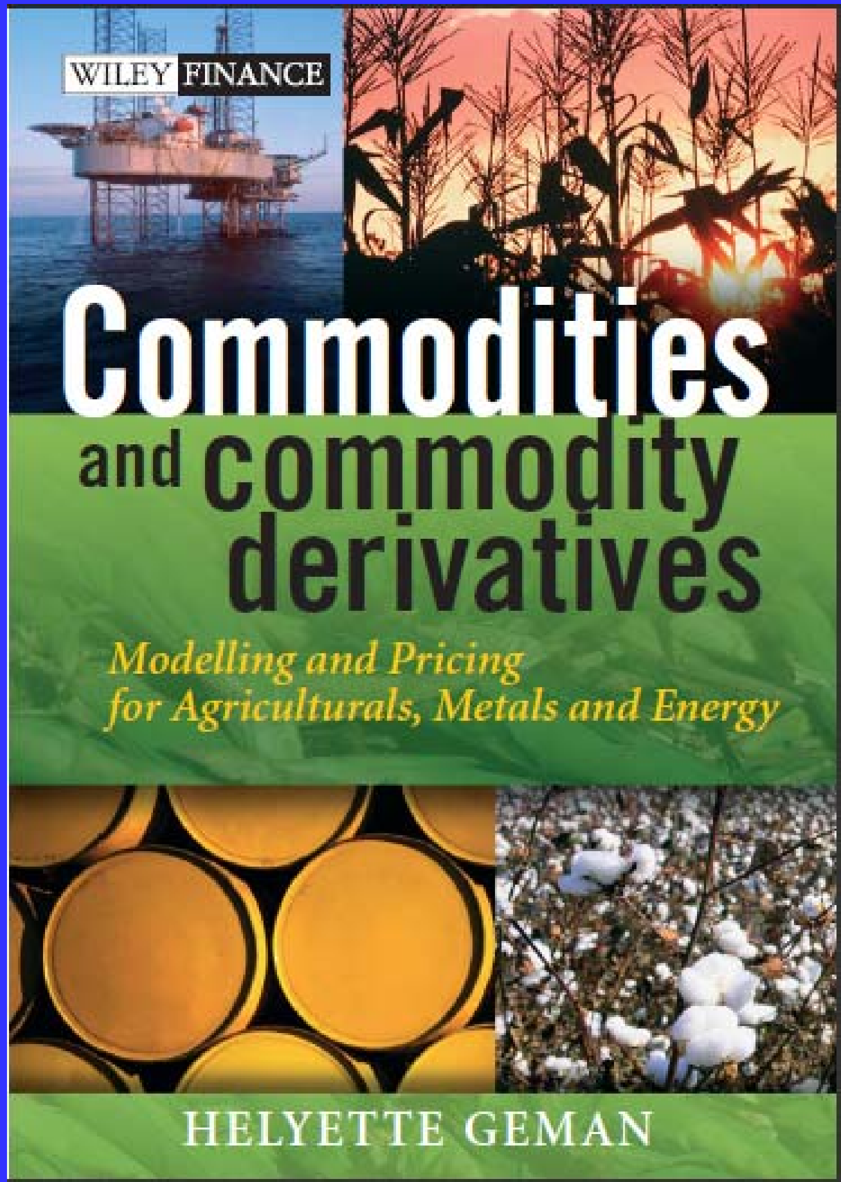
→ The third state variable may be the long-term value of the commodity, translating in particular the forecast on long-term supply

→ Hence, a three-factor model with stochastic volatility and a rising long-term price ($\mu > 0$)

$$dS_t = a(\ln L_t - \ln S_t)\, S_t dt + \sigma_t S_t\, dW_t^1$$

$$dU_t = \alpha(b - U_t)dt + \eta \sqrt{U_t}\, dW_t^2 \qquad \text{where } U_t = \sigma_t^2$$

$$dL_t = \mu L_t dt + \xi L_t\, dW_t^3$$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Hélyette GEMAN is a Professor of Finance at Birkbeck, University of London and ESSEC Graduate Business School. She is a graduate of Ecole Normale Superieure in mathematics, holds a Masters degree in theoretical physics and a PhD in mathematics from the University Pierre et Marie Curie and a PhD in Finance from the University Pantheon Sorbonne. Professor Geman has been a scientific advisor to a number of major energy companies for the last decade, covering the spectrum of oil, natural gas and electricity as well as agricultural commodities origination and trading ; and was previously the head of Research and Development at Caisse des Depots. She has published more than 80 papers in major finance journals including the Journal of Finance, Mathematical Finance, Journal of Financial Economics, Journal of Banking and Finance and Journal of Business. She has also written a book entitled *Insurance and Weather Derivatives* and is a Member of Honor of the French Society of Actuaries. Professor Geman's research includes asset price modelling using jump-diffusions and Lévy processes, commodity forward curve modelling and exotic option pricing for which she won the first prize of the Merrill Lynch Awards. She was named in 2004 in the Hall of Fame of Energy Risk. Her latest book *Commodities and Commodity Derivatives* was published by Wiley Finance in January 2005. Professor Geman is a Member of the Board of the UBS-Bloomberg Commodity Index.