Pricing and Hedging Basis Risk under No Good Deal Assumption

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Introduction

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- Show that the "right NGD" price is greater than the one previously compute in the literature.
- Find a hedging strategy.
- Show numerically the NGD efficiency.

No Good Deal : the model

- Non risky asset : $dS_t^0 = S_t^0 r dt$.

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  $h_S = \frac{\mu_S - r}{\sigma_S}$ Sharpe ratio of S.

No Good Deal : Sharpe ratio

Global Sharpe ratio

Let X be a contingent claim and $\mathbb{Q} \in \mathcal{M}^2(\mathbb{P})$:

$SR^2(X,\mathbb{Q}) = \frac{\mathbb{E}(X)-\mathbb{E}^{\mathbb{Q}}(X)}{\sqrt{\mathrm{Var}(X)}}$

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Proposition : Klöppel-Schweizer (2007)
Let $\mathbb{Q} \in \mathcal{M}^2(\mathbb{P})$ then $\sup_{X \text{ “admi”}} SR^2(X,\mathbb{Q}) = \sqrt{\mathrm{Var}\, Z_T}$.

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NGD Assumption

There exists $\mathbb{Q} \in \mathcal{M}^2(\mathbb{P})$ and $\beta > 0$, such that $\forall X$, $SR^2(X,Q) \leq \beta$.

Proposition NGD Assumption $\Longleftrightarrow$

$$\mathcal{M}^{2,\beta}(\mathbb{P}) := \left\{ \mathbb{Q} \in \mathcal{M}^2(\mathbb{P}) \ : \ \|Z_T\|_{L^2(\mathbb{P})} \leq \sqrt{1+\beta^2} \right\} \neq \emptyset.$$

No Good Deal pricing

Assume that $\frac{1}{T}\ln(1+\beta^2) \geq h_S^2$ and $\lambda^{max} = \sqrt{\frac{1}{T}\ln(1+\beta^2) - h_S^2}$.

Cochrane-Saá-Requejo and Björk-Slinko NGD price

$\tilde{p}_0(H) = \sup_{\lambda_t(\omega)\in[-\lambda^{max},\lambda^{max}]} \mathbb{E}\left[Z_T^{\lambda}\frac{H}{S_T^0}\right].$

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Pricing via coherent measure of risk

$$\begin{aligned} p_0(H) &= \inf\left\{m \in \mathbb{R} \mid \ \exists \Phi \text{ s.t. } \inf_{\mathbb{Q}\in\mathcal{M}^{2,\beta}(\mathbb{P})} \mathbb{E}^{\mathbb{Q}}\left[\frac{X_T^{m,\Phi} - H}{S_T^0}\right] \geq 0\right\} \\ &= \sup_{\mathbb{Q}\in\mathcal{M}^{2,\beta}(\mathbb{P})} \mathbb{E}^{\mathbb{Q}}\left[\frac{H}{S_T^0}\right]. \end{aligned}$$

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Remark : There is no natural hedging strategies associated to this notion of NGD.

No Good Deal : Sharpe ratio and No Good Deal Pricing

Instantaneous Sharpe ratio

- Let X_t be the value of a self-financed strategy :

$$SR^1(X_t) = \frac{\frac{1}{dt}\mathbb{E}\left(\frac{dX_t}{X_t}/\mathcal{F}_t\right) - r}{\frac{1}{dt}\sqrt{\mathrm{Var}\left(\frac{dX_t}{X_t}/\mathcal{F}_t\right)}}.$$

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- A bound on λ implies a bound on $|SR^1(X_t)|$ or $|SR^2(X_t)|$ but the converse does not hold true.
- $\tilde{p}_0(H) \leq p_0(H)$ and is not related to the NGD principle.

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  - p_0^{LB} : λ is assume to be independent of W : explicit computation of p_0 is possible when relaxing the positivity assumption on Z_T^{λ}.

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Theorem

Assume that $H = (V_T - K)_+$ and there is NGD. Then

$$p_0^{UB} \geq p_0 \geq p_0^{LB} \geq \tilde{p}_0.$$

In some cases, $p_0^{LB} > \tilde{p}_0$.

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$$\begin{aligned}
\tilde{p}_0 &= e^{-rT} BS(V_0, T, K, \mu_V - \sigma_V \rho h_S + \sigma_V \lambda^{max}\sqrt{1-\rho^2}, \sigma_V), \\
p_0^{LB} &= \varepsilon \tilde{p}_0 + (1-\varepsilon) e^{-rT}\mathbb{E}(Z_T^0 Y^{down} H),\ \varepsilon \in (0,1), \\
p_0^{UB} &= \mathbb{E}((Z_T^0 + e^{h_S^2 T/2}\bar{\beta}\frac{H - \mathbb{E}(H \mid \mathcal{F}_T^W)}{\sqrt{\mathbb{E}[H^2 - \mathbb{E}(H \mid \mathcal{F}_T^W)^2]}})H),\ \bar{\beta} = \sqrt{(1+\beta^2)e^{-h_S^2 T} - 1}.
\end{aligned}$$

No Good Deal Pricing : numerical results

Assume the following market conditions :

μ_V	σ_V	V_0	μ_S	σ_S	S_0	r	T
0.04	0.32	15	0.0272	0.256	100	2%	0.25

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Example

If $K = 15$, $\rho = 0.8$, $\beta = 2$ then our lower bound is 8.4% higher than "NGD-CSR". The difference can reach 25% in other market conditions.

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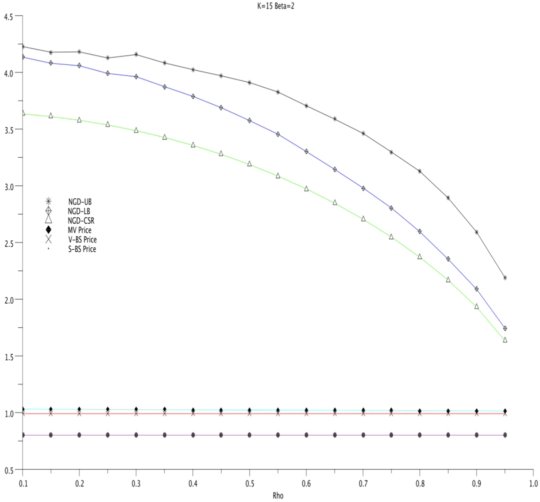

- NGD Price $\downarrow$ when $\rho \uparrow$
  (OK theoretically for "NGD-CSR" but not for "NGD-LB" and 'NGD-UB").
- NGD Price $\longrightarrow$ "MV-Price" when $\rho \longrightarrow 1$
  (OK theoretically).
- The following items are not always true :
  - "V-BS Price" is near "MV-Price",
  - "S-BS Price" is very low.

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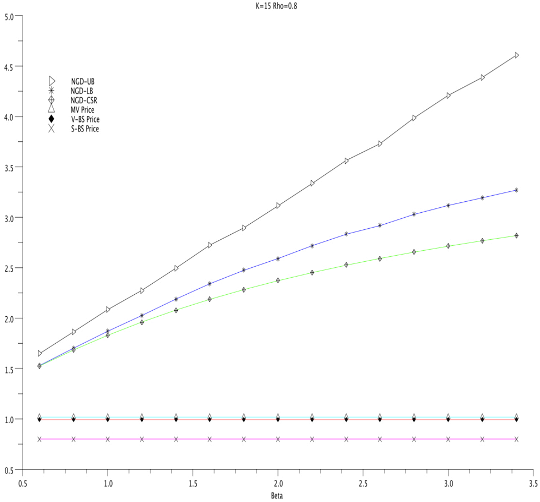

- NGD Price $\uparrow$ when $\beta \uparrow$

  (OK theoretically for "NGD-CSR" and "NGD-UB", but not for "NGD-LB"),

- NGD Price $\rightarrow$ "MV-Price" when $\beta \rightarrow \sqrt{e^{h_S^2 T} - 1}$

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- Compute explicitly the strategy in *exchangeable assets* that minimize the quadratic error under the historic probability when starting with an initial capital equal to NGD price.

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  - with other hedging strategy.

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- Comparaison
  - with other hedging strategy.
  - with other initial wealths.
  - according to several risk measures.

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Methodology

Solve :

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    - show directly that the martingale property of the risky assets implies a particular form for the numéraire with no use of the so-called optimal variance measure.

No Good Deal : hedging

Theorem

Let $H = (V_T - K)_+$, the solution of the preceding problem is given by

$$\Phi_t^{0,H} = \frac{U_t}{S_t^0}\left[\frac{\sigma_S + h_S}{\sigma_S}\left(X_0 + \int_0^t \left(h_S K_l + \rho \frac{L_l}{U_l}\right) dW_l^U\right) - \frac{1}{\sigma_S}\left(h_S K_t + \rho \frac{L_t}{U_t}\right)\right],$$

$$\Phi_t^{1,H} = \frac{U_t}{\sigma_S S_t}\left[\left(h_S K_t + \rho \frac{L_t}{U_t}\right) - h_S\left(X_0 + \int_0^t \left(h_S K_l + \rho \frac{L_l}{U_l}\right) dW_l^U\right)\right].$$

The minimum is equal to

$$v_\alpha(H) = e^{(2r - h_S^2)T}\Bigg[\left(e^{-rT} BS(V_0, T, K, \mu_V - \sigma_V h_S \rho, \sigma_V) - X_0\right)^2 + (1-\rho^2)\mathbb{E}^{Q^U}\left(\int_0^T \left(\frac{L_t}{U_t}\right)^2 dt\right)\Bigg],$$

$$U_t = e^{-h_S W_t + (r - 3/2 h_S^2)t}$$

$$K_t = \frac{e^{-r(T-t)}}{U_t} BS(V_t, T-t, K, \mu_V - \sigma_V h_S \rho, \sigma_V)$$

$$L_t = \sigma_V e^{-r(T-t) + (\mu_V - \sigma_V h_S \rho)(T-t)} V_t \mathcal{N}(d_1(V_t, T-t, K, \mu_V - \sigma_V h_S \rho, \sigma_V)),$$

$W_t^U = W_t + 2h_S t$ and $W_t^{*,U} = W_t^*$ are brownien motions under Q^U.

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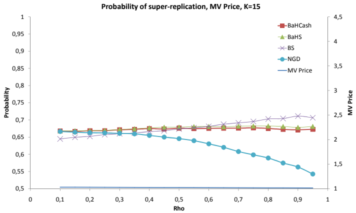
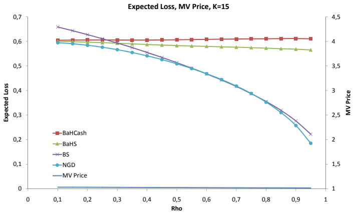
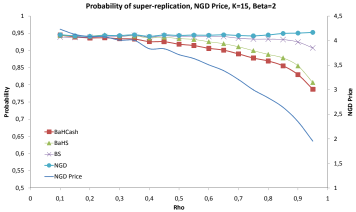
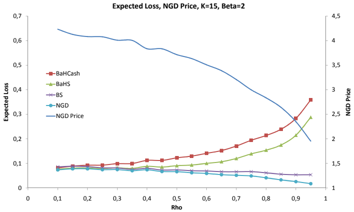
The best situations are those where sur-replication probability is near 1, expected losses are small and VaR is low.

Numerical results

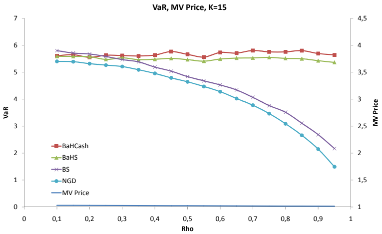


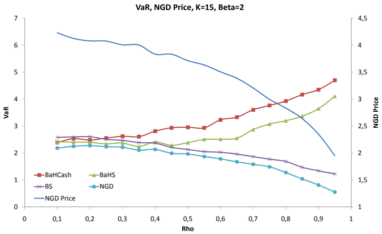






- Naives strategies : BaHCash and BaHS :
  - $\rho \uparrow \Rightarrow$ SRP$\downarrow$, EL $\uparrow$ and VaR $\uparrow$,
  - because ρ only appears in X_0 and "NGD-CSR", "NGD" and "MV" $\downarrow$ when $\rho \uparrow$.





- More elaborated strategies : minimum variance “NGD” and “BS ”, those strategies approximate H :
  - When $\rho\uparrow$, $S \sim V$ from a risk perspective $\Rightarrow$ SRP $\uparrow$, EL $\downarrow$ and VaR $\downarrow$.