Dynamic Valuation and Hedging of Counterparty Credit Exposure

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- Bielecki, T., Crépey, S., Jeanblanc, M. and Zargari, B.: Valuation and Hedging of CDS Counterparty Exposure in a Markov Copula Model. *Submitted*.
- Assefa, S., Bielecki, T. R., Crepey, S. and Jeanblanc, M.: CVA computation for counterparty risk assessment in credit portfolios. Forthcoming in *Recent Advancements in Theory and Practice of Credit Derivatives*, Bielecki, T. R., Brigo, D. and Patras, F., eds, Wiley.
- Bielecki, T. R. and Crépey, S.: Dynamic Hedging of Counterparty Exposure. Forthcoming in *The Musiela Festschrift*, Zariphopoulou, T., Rutkowski, M. and Kabanov, Y., eds, Springer.

Outline

Basic Concept

Risk that some value is lost by a party in OTC derivatives contracts due to the default of the other party [Canabarro and Duffie 03, Brigo et al. *]

- Early termination of a contract with positive value at time of default of the other party
  - Cum-dividend value, including promised payment not paid at default time

The primary form of financial (credit) risk

- Vulnerability
- Counterparty credit risk as opposed to reference credit risk

Very significant during the crisis
An important dynamic modeling issue/challenge, particularly in connection with credit derivatives

- Pricing at any future time
- Defaults dependence modeling
  - Wrong way risk

General Set-Up

$(\Omega, \mathbb{F}, \mathbb{P}), \mathbb{F} = (\mathcal{F}_t)_{t \in [0,T]}$ risk-neutral pricing model (with $r = 0$ for notational simplicity, except in the numerical part)

$\mathbb{E}_t$ Conditional expectation under $\mathbb{P}$ given $\mathcal{F}_t$

τ_{-1} and τ_0 Default times of the two parties, referred to henceforth as the investor, labeled -1, and its counterparty, labeled 0

- $[0, +\infty]$-valued $\mathbb{F}$-stopping times
- Bilateral counterparty risk $\leftrightarrow$ counterparty risk on both sides is considered $\leftrightarrow \tau_{-1} < +\infty, \ \tau_0 < +\infty$
  - Whenever it makes sense: benefiting from its own default??
- Unilateral counterparty risk$\leftrightarrow \tau_{-1} = +\infty$

R_{-1} and R_0 Recovery rates, given as $\mathcal{F}_{\tau_{-1}}$- and $\mathcal{F}_{\tau_0}$- measurable $[0,1]$-valued random variables

τ $\tau_{-1} \wedge \tau_0$, with related default and non-default indicator processes denoted by H and J, so $H_t = \mathbb{1}_{\tau \le t}$ and $J = 1 - H$.

- No actual cash flow after τ

All cash flows and prices considered from the perspective of the investor

Cash Flows

General case reduced by additivity to that of a

Fully netted and collateralized portfolio

Δ Counterparty risky cumulative cash flows

D Counterparty clean cumulative cash flows

$$\Longrightarrow \Delta = JD + HD_{\tau-} + H\left(\Gamma_\tau + \mathbb{1}_{\tau=\tau_+}\left(R_+\chi^+ - \chi^-\right) - \mathbb{1}_{\tau=\tau_-}\left(R_-\chi^- - \chi^+\right) - \mathbb{1}_{\tau_+=\tau_-}\chi\right)$$

Γ_τ Value of the collateral (or margin account) at time τ

$\chi = P_{(\tau)} + (D_\tau - D_{\tau-}) - \Gamma_\tau$ Algebraic 'debt' of the counterparty to the investor at time τ

$P_{(\tau)}$ 'Fair (ex-dividend) value' of the portfolio at τ

$D_\tau - D_{\tau-}$ Promised cash flow at τ

Collateral Formation Qualification

An Important Modeling Issue

AIG's bailout largely triggered by its inability to face increasing margin calls on its sell-protection CDS positions

- On the distressed Lehman in particular

For simplicity we only use in this presentation highly stylized models for the collateral process. We refer to the papers for such aspects of the collateral formation as

- margin call frequency + margin cure period = margin period of risk,
- collateral thresholds,
- minimum transfer amounts,
- haircut provisions.

Representation Formula I

$\Pi_t := \mathbb{E}_t\left[\Delta_T - \Delta_t\right]$ Counterparty Risky Value of the portfolio

$P_t := \mathbb{E}_t\left[D_T - D_t\right]$ Counterparty Clean Value of the portfolio

Market 'Legal Value' standard $P_{(\tau)} = P_\tau$ assumed for simplicity

CVA (Credit Valuation Adjustment)

$$\mathrm{CVA}_t := J_t(P_t - \Pi_t)$$

can be represented as

$$\mathrm{CVA}_t = J_t\mathbb{E}_t\left[\xi\right] ,$$

where the $\mathcal{F}_\tau$-measurable Potential Future Exposure at Default (PFED) ξ is given by

$$\xi = (1 - R_0)\mathbb{1}_{\tau=\tau_0<T}\chi^+ - (1 - R_{-1})\mathbb{1}_{\tau=\tau_{-1}<T}\chi^- = \xi^+ - \xi^-$$

$\rightarrow$ Need of a dynamic, tractable model for P_t, Γ_t

Proof

Let $D^* = JD + HD_{\tau-}$ denote the dividend process corresponding to the cash flows of D 'stopped at $\tau-$'. One has,

$$J_t \mathbb{E}_\tau \int_t^T \left(dD_s - dD_s^*\right) = J_t \mathbb{E}_\tau \int_{[\tau,T]} dD_s$$
$$= J_t \mathbb{1}_{\tau<T} \left(P_\tau + D_\tau - D_{\tau-}\right) = J_t \mathbb{1}_{\tau<T} \left(\chi + \Gamma_\tau\right) \ .$$

So, taking conditional expectation given $\mathcal{F}_t$,

$$J_t(P_t - \Pi_t) = J_t \mathbb{E}_t \left\{ \mathbb{1}_{\tau<T} \left[\chi + \Gamma_\tau - \left(\Gamma_\tau + \mathbb{1}_{\tau=\tau_+}(R_+\chi^+ - \chi^-) - \mathbb{1}_{\tau=\tau_-}(R_-\chi^- - \chi^+) - \mathbb{1}_{\tau_+=\tau_-}\chi \right) \right] \right\}$$
$$= J_t \mathbb{E}_t \left[\xi \right] \ .$$

Representation Formula II

Expected Exposures (EPEs) and CVA

$$\mathrm{CVA}_0 = \int_0^T \mathrm{EPE}_+(s)\mathbb{P}(\tau_0 \in ds,\, \tau_{-1} \geq s)$$
$$- \int_0^T \mathrm{EPE}_-(s)\mathbb{P}(\tau_{-1} \in ds,\, \tau_0 \geq s)$$

where the Expected Positive Exposures $\mathrm{EPE}_\pm$, also known as the Asset Charge and the Liability Benefit, respectively, are the functions of time defined by, for $t \in [0, T]$,

$$\mathrm{EPE}_+(t) = \mathbb{E}\left[(1 - R_0)\chi^+ | \tau_0 = t \leq \tau_{-1}\right],$$

$$\mathrm{EPE}_-(t) = \mathbb{E}\left[(1 - R_{-1})\chi^- | \tau_{-1} = t \leq \tau_0\right].$$

Counterparty Credit Risk

A crucial issue in connection with valuation and risk management of credit derivatives in the crisis

- Wrong Way Risk [Redon 06]
  - Cycle and contagion effects $\rightarrow$ Time of default of a counterparty selling credit protection typically given as a moment of high value of credit protection
- 'Joint Defaults Component' of the PFED hardly collateralizable

$\rightarrow$ Need of a dynamic but tractable model of defaults' dependence

More Set-Up

- $\mathbb{N}_n$ $\{-1, 0, \ldots, n\}$
- τ_is Default times (stopping times) of the investor, its counterparty and n credit names underlying a portfolio of credit derivatives
- H^is Default indicator processes, so $H^i_t = \mathbb{1}_{\tau_i \leq t}$
- R_is Recovery rates, assumed to be constant for simplicity ($= 0$ here for notational simplicity, except in the numerics)

Outline

Common Shocks Copula [Elouerkhaoui 07, Brigo et al. 07]

Let $\mathcal{I} = \{I_1, \dots, I_m\}$ denote a bunch of pre-specified subsets of $\mathbb{N}_n$

Sets of obligors susceptible to default simultaneously

Set $Y = \mathbb{N}_n \cup \mathcal{I}$
Define, for $\imath \in Y$, an intensity function $\lambda_\imath(t)$, and

$$\widehat{\tau}_\imath = \inf\{t > 0; \int_0^t \lambda_\imath(s)ds \geq E_\imath\} \ ,$$

for IID exponential random variables $E_\imath$s
One then sets, for every $i \in \mathbb{N}_n$

$$\widetilde{\tau}_i = \widehat{\tau}_i \wedge \bigwedge_{I \in \mathcal{I};\, I \ni i} \widehat{\tau}_I$$

Immediate extension to stochastic intensities $\lambda_\imath(t, \mathrm{X}_t)$, for $i \in \mathbb{N}_n$, for a factor Markov process $\mathrm{X} = (X^i)_{i \in \mathbb{N}_n}$ independent of the $E_\imath$s.

Markovian Copula model

$\mathrm{X} = (X^i)_{i \in \mathbb{N}_n}$, $\mathrm{H} = (H^i)_{i \in \mathbb{N}_n}$

Main Properties

(i) The pair (X, H) is a Markov process.

(ii) For $i \in \mathbb{N}_n$, the pair (X^i, H^i) is a Markov process

- No direct contagion effects

(iii) Common shocks copula interpretation $(\tau_i)_{i \in \mathbb{N}_n} \stackrel{(l)}{=} (\widetilde{\tau}_i)_{i \in \mathbb{N}_n}$

- Common shocks interpretation also available conditionally on any given state of the Markovian Copula model
- Defaults dependence and Wrong way risk via Joint Defaults

Toy Example: Two Names with Deterministic Intensities

The counterpart labelled ^{0} and a reference firm labelled ^{1} (default-free investor $\rightarrow$ unilateral counterparty risk)
Pair $H = (H^0, H^1)$ modeled as continuous-time Markov chain
State space $E = \{(0,0), (0,1), (1,0), (1,1)\}$

Generator-matrix of $H = (H^0, H^1)$

$$A(t) = \begin{bmatrix} -l(t) & l_0(t) & l_1(t) & l_2(t) \\ 0 & -q_0(t) & 0 & q_0(t) \\ 0 & 0 & -q_1(t) & q_1(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with

$$l(t) = l_0(t) + l_1(t) + l_2(t)\,,\quad q_0(t) = l_0(t) + l_2(t)\,,\quad q_1(t) = l_1(t) + l_2(t)$$

Special case of Constant Intensities l_is

Marshall-Olkin Copula

$$\mathbb{P}(\tau_0 > s, \tau_1 > t) = C(\mathbb{P}(\tau_0 > s), \mathbb{P}(\tau_1 > t))$$

where the Marshall-Olkin survival copula function C is defined by, for $p, q \in [0, 1]$,

$$C(p, q) = pq \min(p^{-\alpha_0}, q^{-\alpha_1})$$

with $\alpha_i = \frac{l_2}{l_i + l_2}$.

A Tractable Model of Counterparty Credit Risk

- Semi-explicit pricing formulas for (clean) single-name credit derivatives like individual CDSs (assuming, say, affine processes X^is) at any time t,
- Fast recursive convolution pricing schemes for (clean) portfolio loss credit derivatives like CDO tranches at any time t,
- Independent calibration of the model marginals and dependence structure
- Model simulation very fast
- Consistent dynamic hedging
  - Though market incompleteness
    - Martingale dimension of the model $= O(2^n)$

Outline

Hedging the CVA in the Markovian Copula Model

- Unilateral counterparty credit risk
  - $\xi = \chi^+$
- Rolling CDS on the counterparty used as a hedging instrument
  - Wealth of a self-financing trading strategy in market CDSs on the counterparty
  - Much like with futures contracts
    - Value $Q = 0$
    - Yet due to the trading gains ('dividends') of the strategy the related cumulative value process $\widehat{Q} \neq 0$
- Riskless (constant) asset used for making the hedging strategy self-financed
- Min-variance hedging the counterparty jump-to-default component of the delta-hedged P&L

ζ_t Number of rolling CDS held in the hedging strategy at time t

So $P\&L_0^{\zeta} = 0$ and, for $t \in [0, T]$,

$$dP\&L_t^{\zeta} = d\mathrm{CVA}_t - \zeta_t d\widehat{Q}_t$$

Theorem

The strategy which minimizes the risk-neutral variance of the counterparty jump-to-default risk component of process $P\&L^{\zeta}$, is given by, for $t \leq \tau$ (and $\zeta^{jd} = 0$ on $(\tau, T]$)

$$\zeta_t^{jd} = EPE_t - CVA_{t-}$$

where the Expected Positive Exposure is defined by

$$EPE_t = \mathbb{E}(\xi \mid \mathcal{F}_{\tau-}) \mid_{\tau=t}$$

- Rigorous connection between the CCR 'price' CVA and its 'delta' EPE
- Related notions of EPEs used in an ad-hoc way by practitioners for hedging CVA, but
  - Dynamic min-variance hedging strategy here, as opposed to hypothetical static replication strategies based on the unilateral CVA Representation Formula II

$$\mathrm{CVA}_0 = \int_0^T \mathrm{EPE}(s)\mathbb{P}(\tau \in ds) \text{ with } \mathrm{EPE}(t) = \mathbb{E}\left[\xi | \tau = t\right]$$

  - Process $\mathrm{EPE}_t = \mathbb{E}(\xi \,|\, \mathcal{F}_{\tau-}) \,|_{\tau=t} \neq$ Function $\mathrm{EPE}(t) = \mathbb{E}(\xi \,|\, \tau = t)$
- ζ^{jd} changes the counterparty jump-to-default exposure from ξ to $\mathrm{EPE}_\tau = \mathbb{E}(\xi \,|\, \mathcal{F}_{\tau-})$, the 'best guess' of ξ available right before τ
- Issue of hedging bilateral counterparty risk more involved
  - Using hedging instruments sensitive to the default times of the counterparty and the investor

Outline

1. Counterparty Risk
   - Counterparty Credit Risk
2. Markovian Copula Model and Common Shocks Copula Interpretation
3. Hedging the CVA in the Markovian Copula Model
4. Numerics
   - Case of one CDS
   - Case of a Portfolio of CDSs
   - Case of a CDO Tranche

Unilateral CCR on a Payer CDS

'AIG selling protection on LEH to You'

Investor	Buyer of default protection on a firm ('You')
Counterparty	Seller of default protection on the firm ('AIG')
Firm	Reference credit underlying the CDS ('LEH')

$\tau_{-1} = +\infty$, $\tau = \tau_0$, $n = 1$

[Huge and Lando 99, Hull and White 01, Jarrow and Yu 01, Leung and Kwok 05, Brigo and Chourdakis 08, Brigo and Capponi 08, Blanchet-Scalliet and Patras 08, Lipton and Sepp 09]

PFED (no margining)

$$\xi = (1 - R_0)\Big(\mathbb{1}_{\tau<\tau_1\wedge T} P_\tau^+ + \mathbb{1}_{\tau=\tau_1<T}(1-R_1)\Big)$$

P Clean Price of the CDS

T Maturity of the CDS

R_1 Recovery rate on the underlying firm

Assessing the impact on the counterparty risk of the investor of

- the (clean) CDS spread κ_0 of the counterparty
- the asset correlation ρ between the underlying firm and the counterparty

Limited impact of the factor process

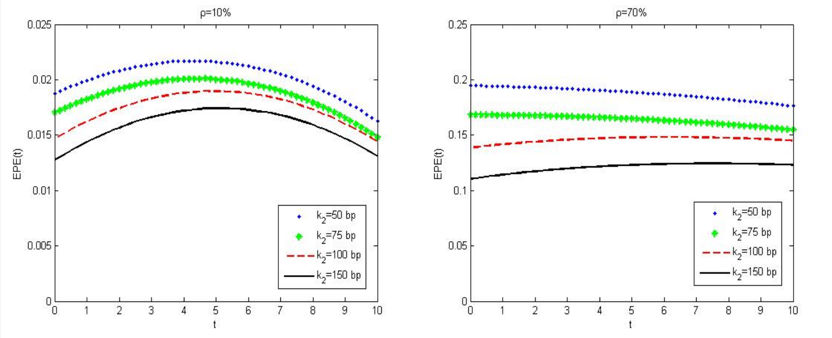
Toy model with deterministic intensities below (affine in time)

$$\mathrm{EPE}(t) = (1 - R_0)\left((1-R_1)\frac{l_2(t)}{q_0(t)} + P^+(t)\frac{l_0(t)}{q_0(t)}\right) e^{-\int_0^t l_1(x)dx}$$

$$\mathrm{CVA}(t) = \int_t^T (1-R_0)\Big((1-R_1)l_2(s) + P^+(s)l_0(s)\Big)e^{-\int_t^s l(x)dx}ds$$

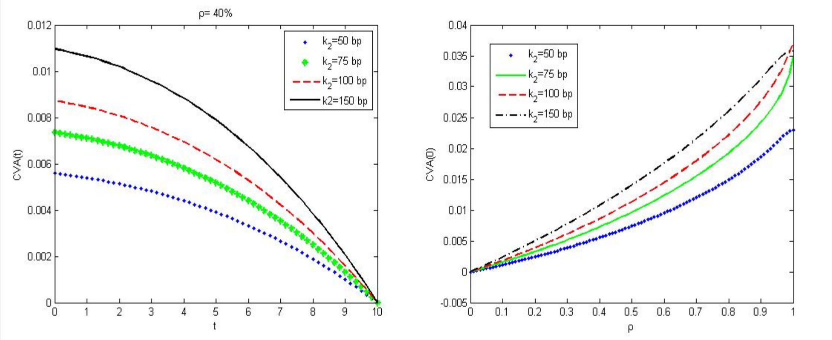
EPE(t)

Left: $\rho = 10\%$, Right: $\rho = 70\%$



CVA

Left: CVA(t) (ρ=40%), Right: CVA(0) as a function of ρ

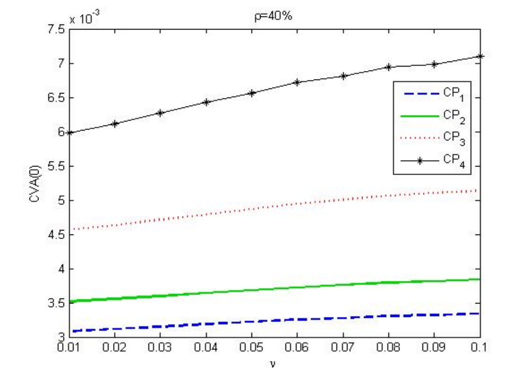


Stochastic Intensities

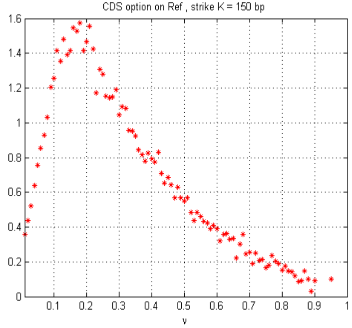
CPU times in seconds for deterministic, two-factor and three-factor CIR specifications of the intensities

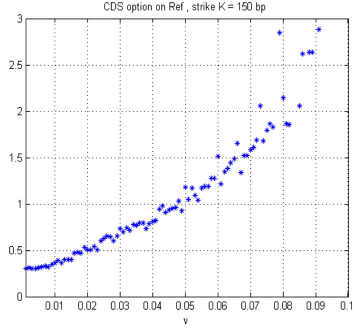
	0F	2F	3F
Calibration	0.01	0.30	0.35
EPE(t)	0.015	5.1	12
CVA(0)	0.015	5.0	12

CVA_0 versus ν for a CDS on a low risk reference entity in the case **2F**.

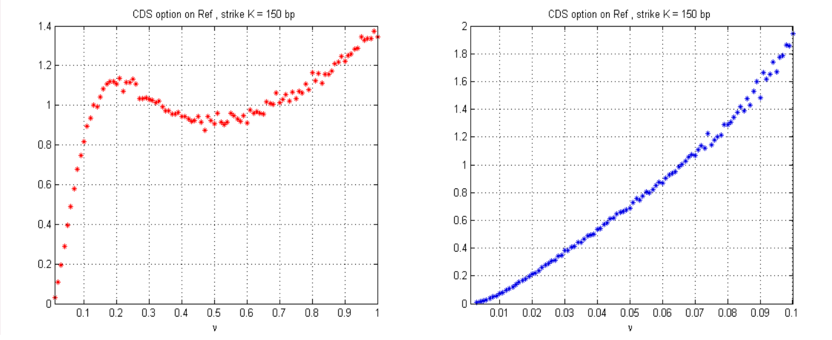


Implied volatility of a payer CDS option as a function of ν: **2F** (left) vs **3F** (right)





Implied volatility of a receiver CDS option as a function of ν: **2F** (left) vs **3F** (right)



Bilateral CCR on a Payer CDS

$\tau_{-1} \vee \tau_0 < +\infty$, $n = 1$

Proposition

For a counterparty risky payer CDS, one has,

$$\begin{aligned} \xi &= (1-R_0)\mathbb{1}_{\tau=\tau_0}\Big(P_\tau + \mathbb{1}_{\tau_1=\tau<T}(1-R_1) - \Gamma_\tau\Big)^+ \\ &\quad -(1-R_{-1})\mathbb{1}_{\tau=\tau_{-1}}\Big(P_\tau + \mathbb{1}_{\tau_1=\tau<T}(1-R_1) - \Gamma_\tau\Big)^- . \end{aligned}$$

So, in case of no collateralization ($\Gamma = 0$),

$$\xi = (1-R_0)\mathbb{1}_{\tau=\tau_0}\Big(P_\tau^+ + \mathbb{1}_{\tau_1=\tau<T}(1-R_1)\Big) - (1-R_{-1})\mathbb{1}_{\tau=\tau_{-1}}P_\tau^- ,$$

and in the case of extreme collateralization ($\Gamma_\tau = P_{\tau-}$),

$$\begin{aligned} \xi &= (1-R_0)\mathbb{1}_{\tau=\tau_0=\tau_1<T}(1-R_1-P_{\tau-})^+ \\ &\quad -(1-R_{-1})\mathbb{1}_{\tau=\tau_{-1}=\tau_1\le T}(1-R_1-P_{\tau-})^- . \end{aligned}$$

Bilateral CCR on a Payer CDS: Numerics (CIR Intensities)

Scenario: an investor with a very low risk profile, a counterparty which has middle credit risk profile and a reference name with high risk profile.

CVA in basis points for the case $\sigma^l_{-1} = \sigma^m_0 = 0.01$

	$\sigma^h_1 = 0.01$		$\sigma^h_1 = 0.20$	
α_I	No Marg.	Marg.	No Marg.	Marg.
0	0.03 (0.00)	0.0 (0.00)	0.03 (0.00)	0.0 (0.00)
0.1	0.034 (0.00)	0.0 (0.00)	0.034 (0.00)	0.0 (0.00)
0.2	0.038 (0.00)	0.0 (0.00)	0.037 (0.00)	0.0 (0.00)
0.3	0.042 (0.00)	0.0 (0.00)	0.041 (0.00)	0.0 (0.00)
0.4	0.046 (0.00)	0.0 (0.00)	0.045 (0.00)	0.0 (0.00)
0.5	0.050 (0.00)	0.0 (0.00)	0.049 (0.00)	0.0 (0.00)
0.6	0.055 (0.00)	0.0 (0.00)	0.053 (0.00)	0.0 (0.00)
0.7	0.059 (0.00)	0.0 (0.00)	0.056 (0.00)	0.0 (0.00)
0.8	0.063 (0.00)	0.0 (0.00)	0.060 (0.00)	0.0 (0.00)
0.9	0.068 (0.00)	0.0 (0.00)	0.064 (0.00)	0.0 (0.00)
1.0	0.072 (0.00)	0.0 (0.00)	0.068 (0.00)	0.0 (0.00)

CDS Portfolio Unilateral CCR

Proposition

For a portfolio of CDS with unilateral CCR, one has,

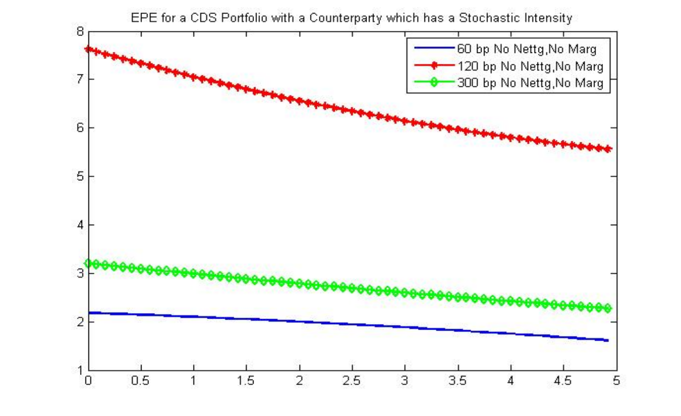
$$\xi = (1-R)\Big(P_\tau + \big(\sum_{i \ pay} - \sum_{i \ rec}\big)\mathbb{1}_{\tau_i = \tau < T_i}(1-R_i) - \Gamma_\tau\Big)^+ ,$$

with $\Gamma = 0$ in the no collateralization case and $\Gamma_\tau = P^+_{\tau-}$ in the unilateral extreme collateralization case.

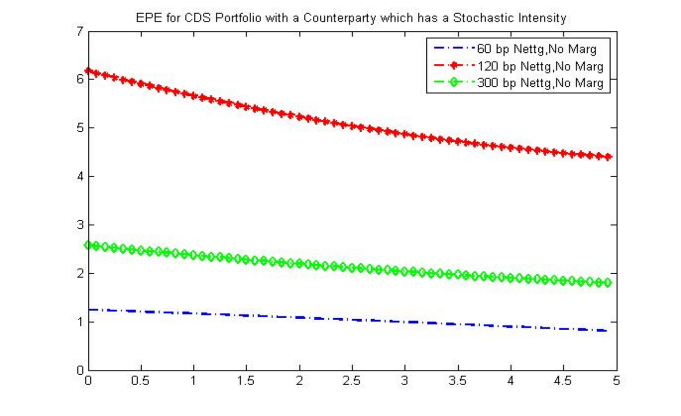
- Portfolio of 70 payer and 30 receiver CDSs
- Individual intensities of the form $a_i + X^i$ where a_i is a constant and X^i is a CIR process.
- Three homogenous groups of obligors
- Three nested groups of joint defaults

Counterparty risk type	κ_0	CVA No Nett. No Marg.	CVA Nett. No Marg.	CVA Nett. with Marg.
low	10	488.4 (7.0)	262.1 (3.8)	256.7 (3.8)
low	20	808.4 (8.9)	433.9 (4.9)	423.0 (4.8)
low	60	834.2 (8.9)	448.5 (4.8)	415.9 (4.8)
low	100	860.4 (8.8)	463.2 (4.8)	409.8 (4.8)
middle	120	5440.2 (21.4)	4338.1 (17.2)	4256.6 (17.1)
middle	300	5364.1 (21.0)	4243.1 (16.9)	4076.8 (16.8)
high	400	8749.9 (22.1)	7211.3 (18.1)	6943.0 (18.0)
high	500	8543.5 (21.8)	7017.9 (17.8)	6713.8 (17.7)

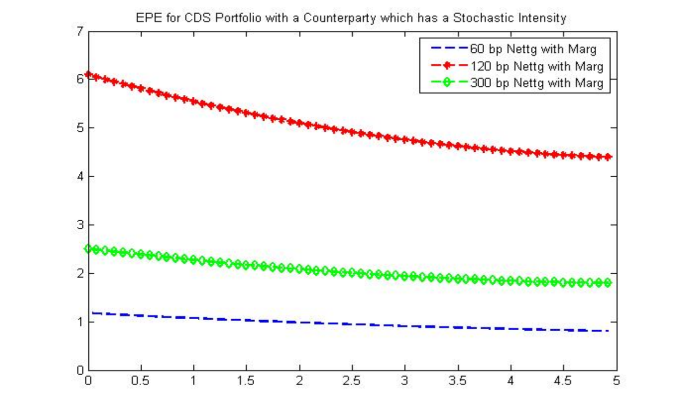
EPE(t) for portfolio – No Netting, No Margining



EPE(t) for portfolio – Netting, No Margining



EPE(t) for portfolio – Netting and Margining



CCR of a Payer CDO

Investor or credit name -1 (default free) Buyer of default protection on the default of firms $\{1, \ldots, m\}$ via tranche(s) of a synthetic CDO e.g. iTraxx Europe

Counterparty or credit name 0 Seller of default protection on the firms $\{1, \ldots, n\}$ via tranche(s) of a synthetic CDO e.g. iTraxx Europe

Unilateral CCR $\quad \tau_{-1} = \infty, \quad \tau = \tau_0 < +\infty, \; n = 125$

$\mathrm{CVA}_0 = \mathbb{E}\chi^+$ where $\chi = P_\tau + (D_\tau - D_{\tau-}) - \Gamma_\tau$

- P_τ is the CDO tranche clean price at τ which is available through the conditional common shocks copula interpretation of the Markovian copula model
- $D_\tau - D_{\tau-}$ is the CDO tranche promised cash flow at τ_0 which is the default payment due to the joint default of the counterparty and a subset of the names $\{1, 2, \ldots, n\}$
- Let collateral $\Gamma_\tau = 0$

$\rightarrow$ Exact CVA Monte Carlo Valuation Scheme

- Up to the MC statistical error

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CVA of CDO with joint default and no joint default

CDS^0 (bp)	Tranches					
	1	2	3	4	5	6
1	2.441 (0.547)	0.051 (0.011)	0.027 (0.007)	0.008 (0.003)	0.004 (0.001)	0.059 (0.012)
10	27.092 (1.816)	0.656 (0.0412)	0.276 (0.0223)	0.0898 (0.0112)	0.0975 (0.0194)	0.752 (0.0428)
100	279.871 (5.773)	6.389 (0.125)	2.863 (0.0732)	0.772 (0.0327)	1.024 (0.0771)	8.4116 (0.4056)
	279.86 (5.773)	6.338 (0.124)	2.803 (0.0708)	0.712 (0.0266)	0.792 (0.0378)	7.086 (0.127)
1000	2073.953 (13.635)	55.147 (0.3818)	34.855 (0.342)	18.739 (0.2945)	15.206 (0.318)	106.662 (1.201)
	2064.147 (13.652)	36.932 (0.261)	19.26 (0.187)	7.933 (0.1309)	9.968 (0.178)	46.487 (0.392)

Hedging the CVA of a CDO tranche

- Markov copula model calibrated assuming 5 groups of obligors
- CDX 2007-12-17 data
- Calibration to the five 5y-tranches and to the 61 riskiest underlying CDSs
- All recoveries for the CDO names are equal to 0.4.
- $R = 0$
- 50000 monte carlo paths
- Index nominal scaled to unity

Hedged vs Unhedged Exposure on the Equity Tranche in case of a 500 bps Counterparty

(in bps)	ξ	θ	$\xi - \theta$	$\frac{\sigma(\xi)}{\sigma(\xi-\theta)}$	$\frac{\sigma(\xi-\theta)}{CVA}$
Mean	31	28	2.3	2.7684	0.0040
Stderr	0.3	0.3	0.1		

To Sum-up

- Simplicity and Consistency of a 'dynamized copula' set-up
  - Fast single-name and static basket credit derivatives pricing schemes
  - Decoupled Calibration Methodology
    - Automatically calibrated marginals
    - Model dependence parameters calibrated independently
  - Model simulation very fast
- Adequation of the model's CVA and EPE with stylized features
- Dynamic Consistency between price (CVA) and hedging (EPE revisited)

Perspectives

Devising a Cross – Asset Classes Model of Counterparty Risk

- Including in particular funding costs/benefits in order to cope with the various bases emerged since the crisis on interest-rates markets
  - Liquidity Issues

Facing the simulation computational challenge of CCR on real-life porfolios with tens of thousands of contracts

- More intensive than (Credit-)VaR or other risk measure computations
  - Value the portfolio at every time point of every simulated trajectory
- Devise appropriate variance reduction techniques
  - Importance Sampling exploiting the Markovian structure of the model
  - Particle methods
- Devise appropriate approximate or simulation/regression procedures for non-analytic 'exotic' derivatives