Corporate Cash Holdings, Liquidity and Solvency Risks

Jean-Paul Décamps

(Toulouse School of Economics, IDEI & Europlace Institute of Finance)

Stéphane Villeneuve

(Toulouse School of Economics , IDEI & Europlace Institute of Finance)

Modeling and Managing Financial Risks, Paris January 12th 2011

Introduction

Let us agree on the meaning of the title

- In Market Finance, Liquidity Risk refers to the risk that arises from the difficulty to sell an asset.
- In Corporate Finance, Liquidity Risk refers to the difficulty to meet its operational needs.
- Cash holdings are the more liquid asset of a company. Liquidity risk models take cash level as state variable.
- Solvency risk refers to the unability to honour its debt commitment. This means that the insolvent company owes more than it owns. Solvency risk models take firm asset as state variable.

A Benchmark example

- Balance sheet of a firm

Asset A	Debt D
cash m	Equity E

- Assumptions
  - Earnings of the productive assets $dX_t = \mu dt + \sigma dW_t$ where $W = (W_t, t \geq 0)$ Brownian motion.
  - profititability of the firm asset μdt. μ **is perfectly known**
  - Debt Coupon cdt
  - Shareholders are risk neutral and discount the future earnings at $r > 0$.
  - There is **no transaction costs**: firm can issue securities at no costs whenever it is needed.

A Benchmark example

- Results
  - Cash reserves are useless.
  - Debt Value

$$D = \mathbb{E} \int_0^{\infty} e^{-rt} c \, dt = \frac{c}{r}.$$

  - Equity Value

$$E = m + \mathbb{E} \int_0^{\infty} e^{-rt} \left(dX_t \, dt - c \, dt \right) = m + \frac{\mu - c}{r}.$$

- Modigliani-Miller Theorem
  - Firm value= $E + D = m + \frac{\mu}{r}$
  - coupon level c is irrelevant
  - the sign of $\mu - c$ gives the solvency of the firm.

Liquidity vs Solvency

- Liquidity risk models focus on optimal cash management (dividend and issuance policy, inside financing) when external financing is costly.
- Financial frictions generate risk aversion even for well-diversified firm. Too little cash leads to liquidation of productive assets but holding too much cash reduces profitability.
- Optimal policy: firms have target cash levels (cash in excess of certain threshold is returned to shareholders).
- Math. Fin. literature on optimal liquidity management policies: Jeanblanc and Shiryaev (1995); Asmussen, Højgaard and Taksar (1999); Sethi and Taksar (2002); Choulli, Taksar and Zhou (2003);Lokka and Zervos, (2005); Cadenillas, Choulli, Taksar and Zhang (2006), Décamps, Mariotti, Rochet and Villeneuve (2007) .

Liquidity vs Solvency

- Solvency risk models focus on random profitability. μ is stochastic.
- But Solvency risk models assume costless external financing, Cash reserves do not matter.
- As long as the firm asset is higher than the firm liability, shareholders inject cash at no costs to meet the operational needs.
- The liquidation of the firm asset is endogeneous $\rightarrow$ optimal stopping problem.
- Literature: Leland (1994); Leland and Toft (1998); Hilberink and Rogers (2002); Detemple, Tian and Xiong (2010); Décamps and Villeneuve (2010).

Mathematical Modeling of Liquidity vs Solvency

Following Gryglewicz (JFE 2010), we consider a model that takes into account both random profitability and costly external financing. Cash reserves process X_t evolves up to a liquidation time as

$$dX_t = (r - \lambda)X_t + \mu\, dt + \sigma\, dW_t - dZ_t.$$

- $W = (W_t)_{t \geq 0}$ is a Brownian motion and μ can take two values $\underline{\mu} < 0 < \overline{\mu}$.
- Z_t represents the cumulative amount of dividends paid out up to time t. $Z = (Z_t)_{t \geq 0}$ is nonnegative, nondecreasing and adapted to the filtration generated by $R_t = \mu t + \sigma W_t$.
- The liquidation time is defined as $\tau \wedge \tau_0$ with $\tau_0 = \inf\{t \geq 0 \,:\, X_t < 0\}$.

The shareholder value function is given by

$$V^* = \sup_{Z,\tau} \mathbb{E} \left(\int_0^{\tau_0 \wedge \tau} e^{-rs}\, dZ_s + e^{-r(\tau_0 \wedge \tau)} X_{(\tau_0 \wedge \tau)^-} \right).$$

Mathematical Modeling of Liquidity vs Solvency

- Filtering techniques

$$\mu_t = \mathbb{E}[\mu \,|\, \mathcal{F}_t^R], \text{ and } d\mu_t = \frac{1}{\sigma}(\mu_t - \underline{\mu})(\overline{\mu} - \mu_t)dB_t$$

- Cash reserves in its own filtration

$$dX_t = (r - \lambda)X_t + \mu_t\, dt + \sigma\, dB_t - dZ_t.$$

- Relation between cumulative cash-flows R_t and belief process μ_t

$$\phi(\mu) = \frac{\sigma^2}{\overline{\mu} - \underline{\mu}} \ln\left(\frac{\mu - \underline{\mu}}{\overline{\mu} - \mu}\right),$$

$$R_t - R_0 = \phi(\mu_t) - \phi(\mu_0) + \frac{\overline{\mu} + \underline{\mu}}{2} t.$$

- Value function

$$V^*(x, \mu_0) = x + \sup_{Z,\tau} \mathbb{E}\left(\int_0^{\tau_0 \wedge \tau} e^{-rs}(-\lambda X_s + \mu_s)\, ds\right).$$

Mathematical Modeling of Liquidity vs Solvency

▷ A benchmark

- There is no costly external financing $\rightarrow$ cash reserves is useless
- Solvency risk comes from the uncertainty on the profitability $V^*(x) = x + \hat{V}(\mu_0)$, where

$$\hat{V}(\mu_0) = \sup_{\tau} \mathbb{E}\left(\int_0^{\tau} e^{-rs}\mu_s\, ds\right)$$

- The optimal liquidation time is $\tau^* = \inf\{t \geq 0\,,\ \mu_t = \mu^*\}$ with $\mu^* = \frac{\underline{\mu}\overline{\mu}}{(1-\gamma)\underline{\mu}+\gamma\overline{\mu}} < 0$ and $\gamma = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8r\sigma^2}{(\overline{\mu}-\underline{\mu})^2}}$.

Towards a unifying description

- As in Gryglewicz, let us first assume that $\lambda = 0$ and $\overline{\mu} + \underline{\mu} \geq 0$.

$$V^*(\mu_0) = x + \sup_{\tau} \mathbb{E}\left(\int_0^{\tau \wedge \tau_0} e^{-rs} \mu_s \, ds \right)$$

- The cash reserve **inside** the firm is remunerated at the risk free rate r thus hoarding cash is optimal.
- Consider the strategy *distribute nothing and liquidate when* $\mu_t = \mu^*$ whose value function is

$$\mathcal{V}(x, \mu_0) = \mathbb{E}\left[e^{-r(\tau_0 \wedge \tau^*)} X_{(\tau_0 \wedge \tau^*)^-} \right]$$

Towards a unifying description

- Gryglewicz noted that when $x = \phi(\mu_0) - \phi(\mu^*)$, the strategy $Z_t = \int_0^t rX_s\,ds + \frac{\overline{\mu}+\underline{\mu}}{2}t$ maintains $X_t = \phi(\mu_t) - \phi(\mu^*)$ and thus $V^*(x, \mu_0) = x + \hat{V}(\mu_0)$.
- Results
  1. We have $V^* = \mathcal{V}$.
  2. Define $\sigma = \inf\{t \,:\, X_t > \phi(\mu_t) - \phi(\mu^*)\}$. Every policy $(Z_t^* \mathbb{1}_{t \geq \sigma}, \tau^*)$ where $Z_t^* \leq \displaystyle\int_0^t rX_s\,ds + \frac{1}{2}(\overline{\mu} + \underline{\mu})t$ for all $t \geq 0$ is optimal.
- Comments
  1. Non-uniqueness of the optimal strategy of dividend payment.
  2. Firm is only closed for solvency reasons if and only if $X_t \geq \phi(\mu_t) - \phi(\mu^*)$.

Towards a unifying description

- Sketch of the proof: To apply the usual verification procedure, we have to check that $\mathcal{V}_x \geq 1$
- We have

$$\mathcal{V}(x, \mu_0) = x + \hat{V}(\mu_0) - \mathbb{E}_x \left[e^{-r(\tau_0 \wedge \tau^*)} \hat{V}(\mu_{\tau_0 \wedge \tau^*}) \right].$$

- We prove that the function

$$x \to \mathbb{E}_x \left[e^{-r(\tau_0 \wedge \tau^*)} \hat{V}(\mu_{\tau_0 \wedge \tau^*}) \right]$$

is decreasing by noting that

$$\mu_{\tau_0} = \phi^{-1} \left(\phi(\mu_0) - x - \int_0^{\tau_0} r X_s^0 \, ds - \frac{\underline{\mu} + \overline{\mu}}{2} \tau_0 \right)$$

is decreasing in x.

Towards a unifying description

- Assume $\lambda = r$ and $\overline{\mu} + \underline{\mu} = 0$ such that $dR_t = d\phi(\mu_t)$.
- Intuitively, optimal dividend payment if $X_t \geq x^*(\mu_t)$.
- Set $Y_t = X_t - \phi(\mu_t) = R_0 - \phi(\mu_0) - Z_t$ and $y^* = -\phi(\mu^*)$.
- Results: There is a solution (U^*, g^*) solution to the free boundary problem

  (i) $\frac{1}{2\sigma^2}(\mu - \underline{\mu})^2(\overline{\mu} - \mu)^2 U_{\mu\mu}(y, \mu) - rU(y, \mu) = 0$ on $\{\mu < g(y)\}$,
  (ii) $U_y(y, g(y)) = 1$,
  (iii) $U_{y\mu}(y, g(y)) = 0$,

  with the initial condition $U(y, \phi^{-1}(-y)) = 0$.
- The function g satisfies an explicit o.d.e. with boundary condition $g(y^*) = \mu^*$
- $U^* = V^*$.
- Uniqueness of the optimal strategy.
- Firm is closed for liquidity reasons.