A Consistent Pricing Model for Index Options and Volatility Derivatives

Modeling and Managing Financial Risks

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Motivation

- The coexistence of a liquid market for options and volatility derivatives such as VIX options, VIX futures
- a well developed over-the-counter market for options on variance swaps, and
- the use of variance swaps and volatility index futures as hedging instruments

have led to the need for a pricing framework in which volatility derivatives and derivatives on the underlying asset can be priced in a consistent manner.
In order to yield derivative prices in line with their hedging costs, such models should be based on a realistic and consistent joint dynamics of the underlying asset and their variance swaps and match the observed prices of liquid derivatives –futures, calls, puts and variance swaps– used as hedging instruments.

Motivation: Market Models of Volatility

- In principle, any continuous-time model with stochastic volatility and/or jumps implies some joint dynamics for variance swaps and the underlying asset price but in practice this joint dynamics can be highly intractable and/or unrealistic (Bergomi 2004).

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- In principle, any continuous-time model with stochastic volatility and/or jumps implies some joint dynamics for variance swaps and the underlying asset price but in practice this joint dynamics can be highly intractable and/or unrealistic (Bergomi 2004).
- Opposed to the modeling of instantaneous (unobservable) volatility, a modeling approach motivated by the availability of variance swap/VIX quotes is proposed in Dupire (1993) and recently developed in Bergomi (2005,2008), Buehler (2006), and Gatheral (2008), in which volatility risk is modelled through observable volatility indicators, such as spot and forward variance swap rates (or spot VIX and VIX futures),

Motivation: Objectives

We propose an arbitrage-free modeling framework for the joint dynamics of forward variance swap rates along with the underlying index, which

1. captures the information in index option prices by matching the index implied volatility smiles.
2. can reproduce the term structure of variance swap rates
3. captures the information in options on VIX futures by matching their prices/smiles.
4. is compatible with empirical properties of index/ variance swap dynamics, allowing in particular for jumps in volatility and returns (see e.g. Todorov and Tauchen (2010), Jacod and Todorov (2009)) and the type of correlations observed in data.
5. enables efficient pricing of vanilla options, a key point for calibration and implementation of the model.

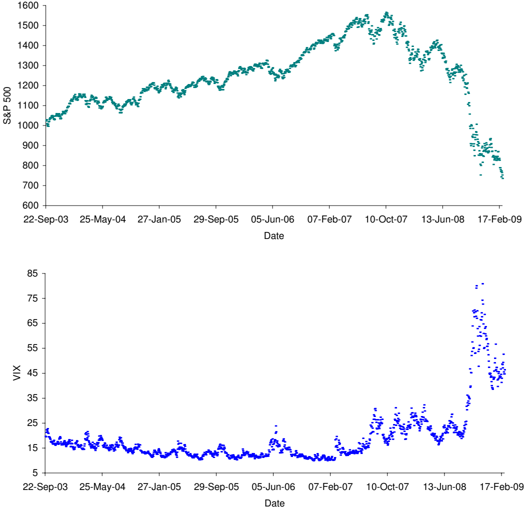

Figure: Time series of the VIX index (bottom) depicted together with the S&P 500 (top) covering the period from September 22nd, 2003 to February 27th, 2009.

Conditional Correlation

Table: Conditional correlation between the daily returns on S&P 500 and the VIX from September 22nd, 2003 to February 27th, 2009, given the index return r_t is below a threshold.

Unconditional	$r_t < -6.5\%$	$r_t < -5\%$	$r_t < -4\%$	$r_t < -3\%$	$\lvert r_t \rvert < 0.5\%$
-0.74	-0.88	-0.55	-0.45	-0.24	-0.45

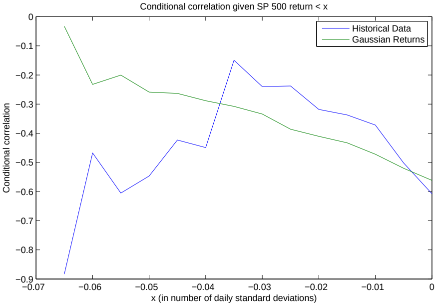

Figure: Conditional correlation implied by data on S&P 500 and the VIX compared to simulated correlated Gaussian returns with same unconditional correlation of -0.74.

Variance Swaps and Forward Variances

Variance swaps (VS) offer investors an efficient way to take positions in pure volatility/variance.

- At maturity T a VS pays the difference between the annualized *realized variance* of the log-returns $RV_{t,T}$ less the *VS rate* V_t^T

$$RV_{t,T} - V_t^T = \frac{M}{k} \sum_{i=1}^{k} \left(\log \frac{S_{t_i}}{S_{t_{i-1}}} \right)^2 - V_t^T \ .$$

  where M is the total number of measurement points in one year (i.e. trading days per year (252) if k is the number of trading days between t and T).

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  where M is the total number of measurement points in one year (i.e. trading days per year (252) if k is the number of trading days between t and T).
- As $\sup\left(t_{i+1} - t_i\right) \to 0$ the realized variance converges towards the quadratic variation of the log-price

$$\frac{M}{n} \sum_{i=1}^{n} \left(\log \frac{S_{t_i}}{S_{t_i-1}} \right)^2 \xrightarrow{\mathbb{Q}} \frac{1}{T-t} \left([\log S]_T - [\log S]_t \right) \ . \qquad (1)$$

V_t^T is determined such that the VS has zero price at initiation, so taking risk neutral expectation on RHS in (1)

$$V_t^T = \frac{1}{T-t}\mathbb{E}\left([\log S]_T - [\log S]_t \mid \mathcal{F}_t\right) . \tag{2}$$

The *forward variance* between time T_1 and T_2 is defined as

$$V_t^{T_1,T_2} = \frac{1}{T_2 - T_1}\mathbb{E}\left([\log S]_{T_2} - [\log S]_{T_1} \mid \mathcal{F}_t\right) \tag{3}$$

$$= \frac{(T_2 - t)\, V_t^{T_2} - (T_1 - t)\, V_t^{T_1}}{T_2 - T_1} , \tag{4}$$

where $t < T_1 < T_2$. Notice, $V_t^{T_1,T_2}$ market data since $V_t^{T_1}$ and $V_t^{T_2}$ are. Take a tenor structure with $T_{i+1} - T_i = \tau$ and define

$$V_t^i \equiv V_t^{T_i,T_{i+1}}.$$

- Forward variances are martingales under the risk neutral measure.
- We model *the observables* V_t^i.

Model: Variance Swap Dynamics

We model the forward variance swap rate as an exponential martingale with a diffusion and jump component:

$$V_t^i = V_0^i e^{X_t^i}$$
$$= V_0^i \exp\left\{ \int_0^t \mu_s^i ds + \int_0^t \omega e^{-k_1(T_i - s)} dZ_s + \int_0^t \int_{\mathbb{R}} e^{-k_2(T_i - s)} x J(dxds) \right\}, \tag{5}$$

where $J(dxdt)$ is a random measure with non-random compensator $\nu(dxdt) = \nu(dx)\,dt$, Z a Wiener process, independent of the jump term. To ensure that the above is a martingale, the drift equals

$$\mu_t^i = -\frac{1}{2}\omega^2 e^{-2k_1(T_i - t)} - \int_{\mathbb{R}} \nu(dx) \left(\exp\left\{ e^{-k_2(T_i - t)} x \right\} - 1 \right).$$

For $t > T_i$ we let $V_t^i = V_{T_i}^i$.

- For proper choice of ν, we know the characteristic function of $X_{T_i}^i$ so options on VSs can be priced by fast Fourier transform methods (Carr and Madan 1999) $\rightarrow$ Computationally very efficient.

Model: Index Dynamics

Once the dynamics of forward variance swaps V_t^i for a discrete set of maturities $T_i, i = 1..n$ has been specified, we look for a specification of the (risk neutral) dynamics of the underlying asset $(S_t)_{t \geq 0}$ such that

1. it is consistent with variance swap dynamics:

$$\forall i = 1..n, \qquad \frac{1}{T_{i+1} - T_i} E[\ [\log S]_{T_{i+1}} - [\log S]_{T_i}\ |\mathcal{F}_t] = V_t^i \quad (6)$$

2. the model values of calls/puts on S match the observed prices across strikes and maturities.

Typically we need at least two distinct parameters/degrees of freedom in the dynamics of the underlying asset in order to accommodate points 1) and 2).

Bergomi (2005,2008) proposes to achieve this by introducing a random "local volatility" function which is reset at each tenor date T_i to match the observed value of $V_{T_i}^i$. This leads to a loss of tractability: even vanilla call options need to be priced by Monte Carlo simulation when their maturity $T > T_1$.

Our choice for the stock dynamics is then for $t = T_m, m = 1, ..., n$

$$S_{T_m} = S_0 \exp\left\{ \int_0^{T_m} (r_s - q_s)\, ds + \sum_{i=0}^{m-1} \mu_i \left(T_{i+1} - T_i \right) + \sigma_i \left(W_{T_{i+1}} - W_{T_i} \right) \right.$$
$$\left. + \sum_{i=0}^{m-1} \int_{T_i}^{T_{i+1}} \int_{\mathbb{R}} u_i \left(x, V^i_{T_i} \right) J\left(dxds \right) \right\} \ ,$$

where $\mu_i = -\frac{1}{2}\sigma_i^2 - \int_{\mathbb{R}} \nu\left(dx\right) \left(e^{u_i\left(x, V^i_{T_i}\right)} - 1 \right)$, the σ_is are stochastic and fixed/revealed at time T_i to match the known $V^i_{T_i}$. The drift terms μ_i are also stochastic and $\mathcal{F}_{T_i}$-measurable. J in the stock index dynamics is the same as that in the VS dynamics, so the two jump simultaneously but in opposite directions. u_i is a deterministic function of x and $V^i_{T_i}$ chosen to match the observed implied volatility smiles. W is independent of J but $dW_t dZ_t = \rho dt$.

- Presence of a jump component as well as a diffusion component in the underlying asset allows us to satisfy the points 1) and 2).

Fitting the Variance Swaps

Remember

$$V_t^i = \frac{1}{T_{i+1} - T_i} \mathbb{E}\left([\log S]_{T_{i+1}} - [\log S]_{T_i} \mid \mathcal{F}_t \right) .$$

In our model we have

$$V_t^i = \mathbb{E}\left[\sigma_i^2 \mid \mathcal{F}_t\right] + \mathbb{E}\left[\int_{\mathbb{R}} u_i\left(x, V_{T_i}^i\right)^2 \nu\left(dx\right) \mid \mathcal{F}_t \right] ,$$

but since V_t^i is a martingale we just have to ensure at time T_i that

$$V_{T_i}^i = \sigma_i^2 + \int_{\mathbb{R}} u_i\left(x, V_{T_i}^i\right)^2 \nu\left(dx\right) . \tag{7}$$

The observed forward variances at times T_is can be matched by appropriate choices of the σ_is, which leaves the parameters in u_i free to calibrate to option prices.

Pricing of Vanilla Options

- For the model to be consistent with market prices of call/put options we need to be able to compute efficiently

$$C(0, S_0, T_m, K) = e^{-\int_0^{T_m} r_s ds} E[(S_{T_m} - K)^+ | \mathcal{F}_0]. \tag{8}$$

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- Denote by $\mathcal{F}_t^{(Z,J)}$ the filtration generated by the Wiener process Z and the Poisson random measure J. By first conditioning on the factors driving the variance swap curve and using the iterated expectation property

$$C(0, S_0, T_m, K) = e^{-\int_0^{T_m} r_s ds} E[E[(S_{T_m} - K)^+ | \mathcal{F}_{T_m}^{(Z,J)}] \, | \mathcal{F}_0] \tag{9}$$

we obtain a *mixing formula* à la Hull-White for valuing call options:

Proposition

The value $C(0, S_0, K, T_m)$ *of a European call option with maturity* T_m *and strike* K *is given by*

$$C(0, S_0, K, T_m) = E^{Z,J}[C^{BS}(S_0 e^{u_m}, K, T_m; \sigma_*)], \tag{10}$$

where $C^{BS}(S, K, T; \sigma)$ *denotes the Black-Scholes formula and*

$$\sigma_*^2 = \frac{1}{T_m} \sum_{i=0}^{m-1} \sigma_i^2 \left(1 - \rho^2\right) \left(T_{i+1} - T_i\right), \tag{11}$$

$$u_m = \Bigg\{ \sum_{i=0}^{m-1} - \left(\frac{1}{2} \sigma_i^2 \rho^2 + \int_{\mathbb{R}} \left(e^{u_i\left(x, V_{T_i}^i\right)} - 1 \right) \nu\left(dx\right) \right) \left(T_{i+1} - T_i\right) \\ \rho\left(Z_{T_{i+1}} - Z_{T_i}\right)\sigma_i + \int_{T_i}^{T_{i+1}} \int_{\mathbb{R}} u_i(x, V_{T_i}^i) J(dx\ ds) \Bigg\}$$

- Note that the outer expectation can be computed by Monte Carlo simulation of the Z and J: with N simulated sample paths for Z and J we obtain the following approximation

$$C\left(0, S_0, K, T_m\right) \simeq \frac{1}{N} \sum_{k=1}^{N} C^{BS}\left(S_0 e^{u_m^{(k)}}, K, T_m; \sigma^*\left(k\right)\right). \qquad (12)$$

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- Equation (12) is important since it shows that we are able, in a cost efficient way, to calibrate the model to the *entire* implied volatility smile for various maturities. In the Bergomi models it is only possible to calibrate to at-the-money slope of the implied volatility (ATM skew).

- We specify the Lévy measure as $\nu(dx) = \lambda f(x)\, dx$, where f is the density for the normal distribution with mean m and variance δ^2 and λ the intensity of the jumps.
- We let the u_is be given by

$$u_i\left(x, V^i_{T_i}\right) = \left(\frac{V^i_{T_i}}{V^i_0}\right)^{\frac{1}{2}} b_i x \ . \tag{13}$$

- This gives us the σ_is at time T_i

$$\sigma_i^2 = V^i_{T_i} - \lambda \frac{V^i_{T_i}}{V^i_0}\left(b_i^2 m^2 + b_i^2 \delta^2\right).$$

In order to achieve non-negative values for σ_i^2 we require that

$$\lambda\left(b_i^2 m^2 + b_i^2 \delta^2\right) \leq V^i_0. \tag{14}$$

Example: Double-Exponential Jumps

- The jump size density is chosen as

$$f(x) = \left(p\alpha_+ e^{-\alpha_+ x}\mathbf{1}_{x\geq 0} + (1-p)\,\alpha_- e^{-\alpha_-|x|}\mathbf{1}_{x<0}\right) \tag{15}$$

  where p denote the probability of a positive jump and $1/\alpha_+$ and $1/\alpha_-$ the mean positive and negative jump sizes.
- We take as before

$$u_i\left(x, V^i_{T_i}\right) = \left(\frac{V^i_{T_i}}{V^i_0}\right)^{\frac{1}{2}} b_i x\ , \tag{16}$$

- which yields

$$\sigma_i^2 = V^i_{T_i} - \lambda\frac{V^i_{T_i}}{V^i_0}\left(\frac{2pb_i^2}{\alpha_+^2} + \frac{2\,(1-p)\,b_i^2}{\alpha_-^2}\right).$$

  To ensure positive σ_is we constrain the calibration by

$$\lambda\left(\frac{2pb_i^2}{\alpha_+^2} + \frac{2\,(1-p)\,b_i^2}{\alpha_-^2}\right) \leq V^i_0. \tag{17}$$

Data

In total, we have data from August 20th, 2008 on a range of:

- VIX put and call options for five maturities.
- call and put options on S&P 500 for six maturities.
- dividend yield and futures prices on S&P 500, from which we also derive a discount curve.
- forward 1 month VS rates for various maturities extracted from Bloomberg.

Calibration

The calibration of the model consists of three steps:

1. First, determine the parameters controlling the VS dynamics by calibration to VIX options using fast Fourier transform methods (here a convexity approximation is performed in order to go from forward VS dynamics to VIX futures dynamics).

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2. Then, use the parameters from first step simulate N paths of the VSs and store the increments of Z, the jump times and jump sizes along with the $V^i_{T_i}$s.
3. Now calibrate to options on the stock index recursively by use of (12)

$$C\left(S_0, K, T; u\right) = \frac{1}{N}\sum_{k=1}^{N} C^{BS}\left(S_0 e^{u_m^{(k)}}, K, T; \sigma^*\left(k\right)\right).$$

In the calibration steps we minimize the objective function on out-of-the-money options

$$\mathsf{SE} = \sum_{\text{options}} \frac{1}{Q_{Ask} - Q_{Bid}} \left(Q_{Market,Mid} - Q_{Model}\right)^2 \tag{18}$$

and we report the corresponding resulting calibration error given by

$$\text{Error} = \frac{1}{\#\left\{\text{options}\right\}} \sum_{\text{options}} \frac{\max\left\{\left(Q_{Model} - Q_{Ask}\right)^+, \left(Q_{Bid} - Q_{Model}\right)^+\right\}}{Q_{Market,Mid}} . \tag{19}$$

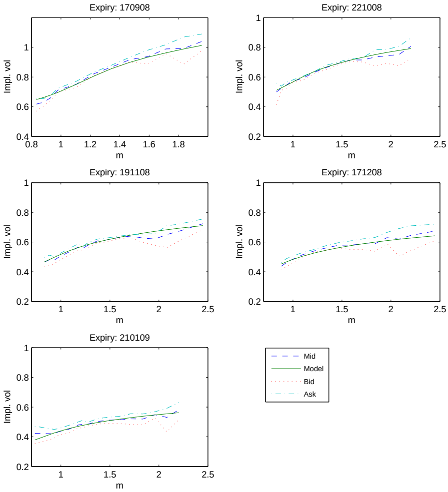

Figure: VIX implied volatility smiles on August 20th 2008 for the model with normally distributed jumps plotted against moneyness $m = K / VIX_t$ on the x axis. Compare with downward sloping in the Heston model.

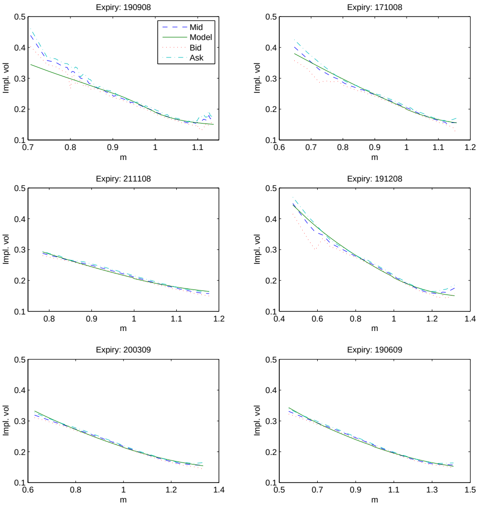

Figure: S&P 500 implied volatility smiles on August 20th 2008 for the model with normally distributed jumps plotted against moneyness $m = K/S_t$ on the x axis.

Table: Calibrated parameters for the two models from the VIX volatility smiles on August 20th, 2008 together with the resulting calibration error. The top panel corresponds to the normally distributed jumps and the bottom to the double exponentially distributed jumps.

Normal jumps							
λ	ω	k_1	k_2	m	δ		Error (%)
3.5201	2.0389	21.9623	2.0743	0.5394	0.2468		0.64
Double exponential jumps							
λ	ω	k_1	k_2	p	α_+	α_-	Error (%)
13.5938	1.9765	22.3033	2.2020	0.8663	4.2457	19.9055	0.85

Table: Model parameters calibrated from the S&P 500 volatility smiles on August 20th, 2008 together with the resulting calibration error. The correlation between the two Brownian components set to -0.45. The second and third row in each panel correspond to the mean and variance of the jumps before scaling with $\left(V^i_{T_i}/V^i_0\right)^{\frac{1}{2}}$.

i	0	1	2	3	4	5
Gaussian jumps						
b_i	-0.140	-0.161	-0.162	-0.187	-0.198	-0.199
$b_i m$	-0.075	-0.087	-0.088	-0.101	-0.107	-0.107
$\lvert b_i \delta \rvert$	0.034	0.040	0.040	0.046	0.049	0.049
Error (%)	3.9	0.6	0.6	1.5	1.2	1.3
Double exponential jumps						
b_i	-0.141	-0.159	-0.158	-0.187	-0.195	-0.192
$\left(\frac{b_i p}{\alpha_+} - \frac{b_i(1-p)}{\alpha_-}\right)$	-0.028	-0.031	-0.031	-0.037	-0.039	-0.038
$\left(\frac{b_i^2 p}{\alpha_+^2} + \frac{b_i^2(1-p)}{\alpha_-^2}\right)^{\frac{1}{2}}$	0.031	0.035	0.035	0.041	0.043	0.042
Error (%)	2.7	0.7	1.1	1.8	1.3	1.8

Stability: Calibration to Another Date

Table: Parameters calibrated to options on July 16th, 2008. Top panel is to VIX options and bottom S&P 500 options.

Gaussian jumps							
λ	ω	k_1	k_2	m	δ		Error (%)
3.52*	2.04*	19.9	1.22	0.45	0.21		0.43
Double exponential jumps							
λ	ω	k_1	k_2	p	α_+	α_-	Error (%)
13.6*	1.98*	19.8	1.36	0.86*	4.90	15.8	0.38

* Fixed parameter from the calibration on August 20th 2008.

i	0	1	2	3	4	5
Gaussian jumps						
b_i	-0.201	-0.233	-0.237	-0.237	-0.259	-0.234
Error (%)	2.9	1.5	0.4	1.4	0.6	1.2
Double exponential jumps						
b_i	-0.203	-0.232	-0.234	-0.235	-0.250	-0.226
Error (%)	2.2	1.7	0.8	1.9	0.7	2.0

Contribution of Jumps to the Forward Variance Swap Rate

The error from neglecting jumps is given by

$$\varepsilon_i = -2\mathbb{E}\left[\int_{\mathbb{R}}\left(e^{u_i\left(x,V^i_{T_i}\right)} - 1 - u_i\left(x, V^i_{T_i}\right) - \frac{u_i\left(x, V^i_{T_i}\right)^2}{2}\right)\nu\left(dx\right) \mid \mathcal{F}_0\right].$$

Table: The error contribution of jumps to the forward variance swap rates, relative to the forward variance swap rate.

Start (months) End	0 1	1 2	2 3	3 4	4 7	7 10
Gaussian jumps						
$\frac{\varepsilon_i}{V^i_0}$ (%)	1.9	2.3	2.9	3.4	4.3	4.5
Double exponential jumps						
$\frac{\varepsilon_i}{V^i_0}$ (%)	1.9	2.4	2.8	3.6	4.3	4.5

Exotic Derivatives Examples

The forward straddle has time T_2 payoff

$$|S_{T_2} - S_{T_1}|,$$

where we in the pricing example choose the time points equal to $T_1 = 5$ months and $T_2 = 10$ months.

The reverse cliquet has a final time T_n payoff of

$$\max\left\{0, C + \sum_{i=1}^{n} \min\left\{\frac{S_{T_i} - S_{T_{i-1}}}{S_{T_{i-1}}}, 0\right\}\right\},$$

where the returns are observed monthly, $T_n = 10$ months and $C = 30\%$.

Table: Confidence intervals of prices computed with 2 million simulations.

	Gaussian jumps	Double exponential jumps
Forward Straddle	$[139.51, 139.83]$	$[139.70, 140.01]$
Reverse Cliquet	$[0.1065, 0.1068]$	$[0.1033, 0.1036]$

Conclusion

- A model for the joint dynamics of a set of forward variance swap rates and the underlying index.
- Using Lévy processes as building blocks leads to tractable pricing for VIX futures and options (Fourier) and vanilla call/put options (Hull-White type formula).
- This tractability makes calibration to such instruments feasible and distinguishes our model from (Bergomi 2005,2008, Gatheral 2008) which require full Monte Carlo pricing of vanilla options.
- Our model reproduces salient empirical features of variance swap dynamics- strong negative correlation of large index moves with VIX moves, positive skew observed in implied volatilities of VIX options- by introducing a common jump component in the variance swaps and the underlying asset.
- Enables to price and hedge payoffs sensitive to forward volatility, consistently with market prices of calls, puts or variance swaps

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