Optimal observation for wave equations

Y. Privat, E. Trélat[1], E. Zuazua

[1] Univ. Paris 6 (Labo. J.-L. Lions) and Institut Universitaire de France

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A. Agrachev's 60th birthday, Cortona, 2012

Observation of wave equations

- $\Omega \subset \mathbb{R}^d$
- $T > 0$ fixed
- $\omega \subset \Omega$ subset of positive measure

Wave equation with Dirichlet boundary conditions

$$\begin{aligned} & y_{tt} - \Delta y = 0, \qquad (t,x) \in (0,T) \times \Omega, \\ & y(t,\cdot)_{|\partial\Omega} = 0, \\ & y(0,\cdot) = y^0 \in L^2(\Omega),\ y_t(0,\cdot) = y^1 \in H^{-1}(\Omega) \end{aligned}$$

$\forall (y^0, y^1) \in L^2(\Omega) \times H^{-1}(\Omega) \qquad \exists ! y \in C^0(0,T; L^2(\Omega)) \times C^1(0,T; H^{-1}(\Omega))$

Observable

$$z = \chi_\omega y$$

Observability

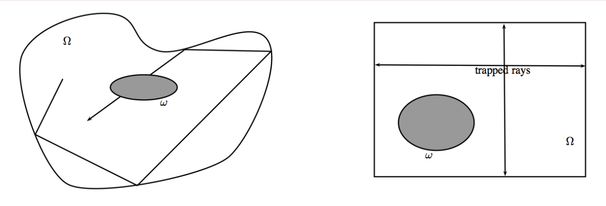
Observability inequality

The system is said observable (in time T) if there exists $C_T(\omega) > 0$ such that

$$\forall (y^0, y^1) \in L^2(\Omega) \times H^{-1}(\Omega) \qquad C_T(\omega) \| (y^0, y^1) \|^2_{L^2 \times H^{-1}} \leq \int_0^T \int_\omega y(t,x)^2 dxdt.$$

Bardos-Lebeau-Rauch (1992) : in the class of C^∞ domains, the observability inequality holds if and only if the pair (ω, T) satisfies the Geometric Control Condition (GCC) in Ω :

Every ray of geometrical optics that propagates in Ω and is reflected on its boundary $\partial\Omega$ intersects ω in time less than T.



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Bardos-Lebeau-Rauch (1992) : in the class of C^∞ domains, the observability inequality holds if and only if the pair (ω, T) satisfies the Geometric Control Condition (GCC) in Ω :

> *Every ray of geometrical optics that propagates in Ω and is reflected on its boundary $\partial\Omega$ intersects ω in time less than T.*

Question

What is the "best possible" control domain ω of fixed given measure ?

Two problems

More precisely, two questions arise.

Let $L \in (0,1)$ fixed.

First problem

Let $(y^0, y^1) \in L^2(\Omega) \times H^{-1}(\Omega)$ fixed. Maximize

$$G_T(\chi_\omega) = \int_0^T \int_\Omega \chi_\omega(x) y(t,x)^2 dxdt$$

over all possible subsets $\omega \subset \Omega$ of Lebesgue measure $|\omega| = L|\Omega|$.

(in what follows, denote $\mathcal{U}_L = \{\chi_\omega \mid \omega \subset \Omega, |\omega| = L|\Omega|\}$)

In this maximization problem, the optimal set ω, whenever it exists, depends on the initial data (y^0, y^1).
The aim of the following second problem is to discard this dependence.

Two problems

More precisely, two questions arise.

Let $L \in (0,1)$ fixed.

Second problem

Maximize

1) $$C_T(\omega) = \inf \left\{ \frac{G_T(\chi_\omega)}{\|(y^0,y^1)\|^2_{L^2\times H^{-1}}} \quad | \quad (y^0,y^1) \in L^2(\Omega)\times H^{-1}(\Omega) \right\}$$

or

2) $$\lim_{T\to+\infty} \frac{C_T(\omega)}{T}$$

over all possible subsets $\omega \subset \Omega$ of Lebesgue measure $|\omega| = L|\Omega|$.

Remark 1

In the first case, the optimal set ω, whenever it exists, depends on T, whereas it does not depend on T in the second case.

Related problems

1) What is the "best domain" for achieving HUM optimal control ?

$$y_{tt} - \Delta y = \chi_\omega u$$

2) What is the "best domain" domain for stabilization (with localized damping) ?

$$y_{tt} - \Delta y = -k\chi_\omega y_t$$

See works by
- P. Hébrard, A. Henrot : theoretical and numerical results in 1D for optimal stabilization (for all initial data).
- A. Münch, P. Pedregal, F. Periago : numerical investigations of the optimal domain (for one fixed initial data). Study of the relaxed problem.
- S. Cox, P. Freitas, F. Fahroo, K. Ito, ... : variational formulations and numerics.
- M.I. Frecker, C.S. Kubrusly, H. Malebranche, S. Kumar, J.H. Seinfeld, ... : numerical investigations (among a finite number of possible initial data).
- K. Morris, S.L. Padula, O. Sigmund, M. Van de Wal, ... : numerical investigations for actuator placements (predefined set of possible candidates), Riccati approaches.
- ...

Spectral expression of $G_T(\chi_\omega)$

$\lambda_j, \phi_j, j \in \mathbb{N}^*$: eigenelements

Every solution can be expanded as

$$y(t,x) = \sum_{j=1}^{+\infty} (a_j \cos(\lambda_j t) + b_j \sin(\lambda_j t))\phi_j(x)$$

with

$$a_j = \int_\Omega y^0(x)\phi_j(x)\,dx, \quad b_j = \frac{1}{\lambda_j}\int_\Omega y^1(x)\phi_j(x)\,dx,$$

for every $j \in \mathbb{N}^*$. Moreover, $\|(y^0,y^1)\|^2_{L^2\times H^{-1}} = \sum_{j=1}^{+\infty}(a_j^2+b_j^2)$.

Then :

$$G_T(\chi_\omega) = \int_0^T \int_\omega \left(\sum_{j=1}^{+\infty} (a_j \cos(\lambda_j t) + b_j \sin(\lambda_j t))\, \phi_j(x) \right)^2 dxdt = \sum_{i,j=1}^{+\infty} \alpha_{ij} \int_\omega \phi_i(x)\phi_j(x)\,dx$$

where

$$\alpha_{ij} = \int_0^T (a_i \cos(\lambda_i t) + b_i \sin(\lambda_i t))(a_j \cos(\lambda_j t) + b_j \sin(\lambda_j t))\,dt.$$

The coefficients α_{ij} depend only on the initial data (y^0, y^1).

Spectral expression of $G_T(\chi_\omega)$

Conclusion :

$$G_T(\chi_\omega) = \sum_{i,j=1}^{+\infty} \alpha_{ij} \int_\omega \phi_i(x)\phi_j(x)\,dx$$

with

$$\alpha_{ij} = \begin{cases} a_i a_j \left(\dfrac{\sin(\lambda_i+\lambda_j)T}{2(i+j)} + \dfrac{\sin(\lambda_i-\lambda_j)T}{2(\lambda_i-\lambda_j)} \right) + a_i b_j \left(\dfrac{1-\cos(\lambda_i+\lambda_j)T}{2(\lambda_i+\lambda_j)} - \dfrac{1-\cos(\lambda_i-\lambda_j)T}{2(\lambda_i-\lambda_j)} \right) & \\ \quad + a_j b_i \left(\dfrac{1-\cos(\lambda_i+\lambda_j)T}{2(\lambda_i+\lambda_j)} + \dfrac{1-\cos(\lambda_i-\lambda_j)T}{2(\lambda_i-\lambda_j)} \right) + b_i b_j \left(-\dfrac{\sin(\lambda_i+\lambda_j)T}{2(\lambda_i+\lambda_j)} + \dfrac{\sin(\lambda_i-\lambda_j)T}{2(\lambda_i-\lambda_j)} \right) & \text{if } \lambda_i \neq \lambda_j, \\ a_j^2 \left(\dfrac{T}{2} + \dfrac{\sin 2\lambda_j T}{4\lambda_j} \right) + a_j b_j \left(\dfrac{1-\cos 2\lambda_j T}{2\lambda_j} \right) + b_j^2 \left(\dfrac{T}{2} - \dfrac{\sin 2\lambda_j T}{4\lambda_j} \right) & \text{if } \lambda_i = \lambda_j. \end{cases}$$

The coefficients α_{ij} depend only on the initial data (y^0, y^1).

Solving of the first problem

Let $(y^0, y^1) \in L^2(\Omega) \times H^{-1}(\Omega$ be fixed initial data, and let α_{ij} be their associated coefficients defined as previously. For every $x \in \Omega$, define

$$\varphi(x) = \sum_{i,j=1}^{+\infty} \alpha_{ij}\phi_i(x)\phi_j(x). \tag{1}$$

Easily : φ is integrable on Ω. Moreover, $G_T(\chi_\omega) = \int_\omega \varphi(x)\,dx$ for every measurable subset ω of Ω.

First problem

$$\sup_{\substack{\omega\subset\Omega \\ |\omega|=L|\Omega|}} \int_\omega \varphi(x)\,dx$$

Hence, clearly :

- There exists an least one optimal measurable subset $\omega \subset \Omega$ of measure $L|\Omega|$.
- Characterization : there exists $\lambda \in \mathbb{R}$ such that every optimal set ω is contained in the level set $\{\varphi \geq \lambda\}$.

Solving of the first problem

Theorem

If $\exists M, \delta > 0$ such that
$$\forall i, j \in \mathbb{N}^* \quad |\alpha_{ij}| \leq M e^{-\delta(i+j)},$$
then the first problem has a unique solution χ_ω, where ω is a measurable subset of Ω of Lebesgue measure $L|\Omega|$. Moreover,

- ω has a finite number of connected components,
- if Ω has a symmetry hyperplane, then ω enjoys the same symmetry property.

– For instance : ok if y^0 and y^1 are analytic.
– If y^0 and y^1 have N nonzero coefficients, then the optimal set ω has at most $f(N)$ connected components (where the function f can be characterized).
– The result can be generalized with quasi-analyticity :
(see S. Mandelbrojt, *Quasi-analycité des séries de Fourier*)
– There exist C^∞ data (y^0, y^1) for which the optimal set ω has a fractal structure.
– Initial data (y^0, y^1) for which ω is not unique can be characterized.

Solving of the second problem

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} C_T(\omega) = \sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{\sum(a_j^2+b_j^2)=1} \sum_{i,j=1}^{+\infty} \alpha_{ij} \int_\omega \phi_i(x)\phi_j(x)\,dx$$

We do not know how to handle this problem in general because of the crossed terms.
If we remove the crossed terms then the second problem is

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\omega \phi_j(x)^2\,dx$$

There are two ways of getting rid of the crossed terms.

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There are two ways of getting rid of the crossed terms.
First way : we rather consider the problem

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \lim_{T\to+\infty} \frac{C_T(\omega)}{T}$$

Lemma

$$\lim_{T\to+\infty} \sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \frac{C_T(\omega)}{T} = \sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \lim_{T\to+\infty} \frac{C_T(\omega)}{T} = \sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\omega \phi_j(x)^2\,dx$$

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If we remove the crossed terms then the second problem is

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There are two ways of getting rid of the crossed terms.
Second way : we consider the observability inequality

$$C_{T,\mathrm{rand}}(\omega)\, \|(y^0,y^1)\|^2_{L^2\times H^{-1}} \leq \mathbb{E}\left(\int_0^T \int_\omega y(t,x)^2\, dxdt \right)$$

in a probabilistic sense.
Then crossed terms disappear (see Burq-Tzvetkov, Invent. Math. 2008).

Solving of the second problem

$$\sup_{\substack{\omega\subset\Omega\\ |\omega|=L|\Omega|}} C_T(\omega) = \sup_{\substack{\omega\subset\Omega\\ |\omega|=L|\Omega|}} \inf_{\sum(a_j^2+b_j^2)=1} \sum_{i,j=1}^{+\infty} \alpha_{ij} \int_\omega \phi_i(x)\phi_j(x)\,dx$$

We do not know how to handle this problem in general because of the crossed terms.
If we remove the crossed terms then the second problem is

$$\sup_{\substack{\omega\subset\Omega\\ |\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\omega \phi_j(x)^2\,dx$$

Remark 1 :

This is an energy concentration criterion.

Remark 2 :

The general problem with crossed terms is related with the (open) question of the existence of an optimal constant in Ingham's inequality.

Solving of the second problem

Second problem

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\Omega \chi_\omega(x)\phi_j^2(x)\,dx$$

1. Convexification procedure

$$\overline{\mathcal{U}}_L = \{a\in L^\infty(\Omega,(0,1)) \mid \int_\Omega a(x)\,dx = L|\Omega|\}.$$

$$\longrightarrow \quad \sup_{a\in\overline{\mathcal{U}}_L} \inf_{j\in\mathbb{N}^*} \int_\Omega a(x)\phi_j^2(x)\,dx$$

A priori :

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\omega \phi_j^2(x)\,dx \leq \sup_{a\in\overline{\mathcal{U}}_L} \inf_{j\in\mathbb{N}^*} \int_\Omega a(x)\phi_j^2(x)\,dx.$$

Solving of the second problem

Moreover, under the assumption

(weak Quantum Ergodicity) Assumption

There exists a subsequence such that $\phi_j^2 \rightharpoonup \frac{1}{|\Omega|}$ in weak star L^∞ topology.

we have

$$\sup_{a\in\overline{\mathcal{U}}_L} \inf_{j\in\mathbb{N}^*} \int_\Omega a(x)\phi_j^2(x)\,dx = L$$

(reached with $a \equiv L$)

Remarks :

- It is true in 1D, since $\phi_j(x) = \sqrt{\frac{2}{\pi}}\sin(jx)$ on $\Omega = [0,\pi]$.
  Moreover, this relaxed problem has an infinite number of solutions, given by

$$a(x) = L + \sum_j (a_j\cos(2jx) + b_j\sin(2jx)) \text{ with } a_j \leq 0$$

(and with $|a_j|$ and $|b_j|$ small enough so that $0 \leq a(\cdot) \leq 1$).

Solving of the second problem

Moreover, under the assumption

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There exists a subsequence such that $\phi_j^2 \rightharpoonup \frac{1}{|\Omega|}$ in weak star L^∞ topology.

we have

$$\sup_{a \in \overline{\mathcal{U}}_L} \inf_{j \in \mathbb{N}^*} \int_\Omega a(x)\phi_j^2(x)\,dx = L$$

(reached with $a \equiv L$)

Remarks :

- In multi-D : it is true under ergodicity assumptions :

If Ω is an ergodic billiard with $W^{2,\infty}$ boundary then $\phi_j^2 \rightharpoonup \frac{1}{|\Omega|}$ in weak star L^∞ for a subset of indices of density 1.

Gérard-Leichtnam (Duke Math. 1993), Zelditch-Zworski (CMP 1996)
(see also Shnirelman, Burq-Zworski, Colin de Verdière, etc)

Solving of the second problem

2. Gap or no-gap ?

A priori, under the weak QE assumption :

$$\sup_{\substack{\omega\subset\Omega\\|\omega|=L|\Omega|}} \inf_{j\in\mathbb{N}^*} \int_\omega \phi_j^2(x)\,dx \quad\leq\quad \sup_{a\in\overline{\mathcal{U}}_L} \inf_{j\in\mathbb{N}^*} \int_\Omega a(x)\phi_j^2(x)\,dx \quad=\quad L.$$

Remarks in 1D :

- Note that, for every ω, $\frac{2}{\pi}\int_\omega \sin^2(jx)\,dx \to L$ as $j \to +\infty$.
- No lower semi-continuity property of the criterion.
- With $\omega_N = \bigcup_{k=1}^N \left[\frac{k\pi}{N+1} - \frac{L\pi}{2N}, \frac{k\pi}{N+1} + \frac{L\pi}{2N}\right]$, one has $\chi_{\omega_N} \rightharpoonup L$ but $\lim_{N\to+\infty} \inf_{j\in\mathbb{N}^*} \frac{2}{\pi}\int_{\omega_N} \sin^2(jx)dx < L.$

Solving of the second problem

(Quantum Unique Ergodicity) Assumption

We assume that $\phi_j^2 \rightharpoonup \dfrac{1}{|\Omega|}$ in weak star L^∞ topology, as $j \to +\infty$.

(i.e. the whole sequence converges to the Liouville measure)

Theorem

Under the QUE assumption, there is no gap, that is :

$$\sup_{\chi_\omega \in \mathcal{U}_L} \inf_{j \in \mathbb{N}^*} \int_\Omega \chi_\omega(x)\phi_j(x)^2\, dx = \sup_{a \in \overline{\mathcal{U}}_L} \inf_{j \in \mathbb{N}^*} \int_\Omega a(x)\phi_j(x)^2\, dx = L.$$

Remark : it holds also true e.g. in a square domain Ω, for which however QUE is not satisfied.

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We assume that $\phi_j^2 \rightharpoonup \dfrac{1}{|\Omega|}$ in weak star L^∞ topology, as $j \to +\infty$.

(i.e. the whole sequence converges to the Liouville measure)

Comments on this assumption :

- It is true in 1D, since $\phi_j(x) = \sqrt{\frac{2}{\pi}} \sin(jx)$ on $\Omega = [0, \pi]$.
- Quantum Unique Ergodicity property (QUE) in multi-D :

  - Gérard-Leichtnam (Duke Math. 1993), Zelditch-Zworski (CMP 1996) :
  If Ω is an ergodic billiard with $W^{2,\infty}$ boundary then $\phi_j^2 \rightharpoonup \dfrac{1}{|\Omega|}$ in weak star L^∞ for a subset of indices of density 1.

  - Strictly convex billiards sufficiently regular are not ergodic (Lazutkin, 1973).
  Rational polygonal billiards are not ergodic.
  Generic polygonal billiards are ergodic (Kerckhoff-Masur-Smillie, Ann. Math. '86).

  - There exist some convex sets Ω (stadium shaped) that satisfy QE but not QUE (Hassell, Ann. Math. 2010)

  - QUE conjecture (Rudnick-Sarnak 1994) : every compact manifold having negative sectional curvature satisfies QUE.

Solving of the second problem

(Quantum Unique Ergodicity) Assumption

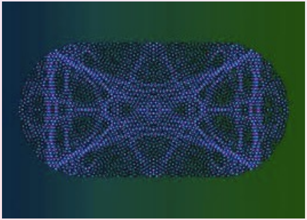
We assume that $\phi_j^2 \rightharpoonup \dfrac{1}{|\Omega|}$ in weak star L^∞ topology, as $j \to +\infty$.

(i.e. the whole sequence converges to the Liouville measure)

Hence in general this assumption is related with ergodic / concentration / entropy properties of eigenfunctions.

See Shnirelman, Sarnak, Bourgain-Lindenstrauss, Colin de Verdière, Anantharaman, Nonenmacher, De Bièvre,...

If this assumption fails, we may have scars : energy concentration phenomena (there can be exceptional subsequences converging to other invariant measures, like, for instance, measures carried by closed geodesics : scars)



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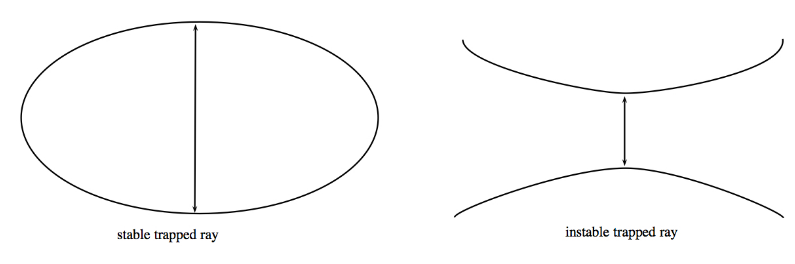

Solving of the second problem

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We assume that $\phi_j^2 \rightharpoonup \dfrac{1}{|\Omega|}$ in weak star L^∞ topology, as $j \to +\infty$.

(i.e. the whole sequence converges to the Liouville measure)

Come back to the theorem :

Under QUE, there is no gap, that is :

$$\sup_{\chi_\omega \in \mathcal{U}_L} \inf_{j \in \mathbb{N}^*} \int_\Omega \chi_\omega(x)\phi_j(x)^2\, dx = \sup_{a \in \overline{\mathcal{U}}_L} \inf_{j \in \mathbb{N}^*} \int_\Omega a(x)\phi_j(x)^2\, dx = L.$$

Moreover :
We are able to prove that, for certain sets Ω, the second problem does not have any solution (i.e., the supremum is not reached).
We conjecture that this property is generic.

Remark

QUE is not necessary. Example : 2D square.

Solving of the second problem

Last remark :

The proof of this no-gap result is based on a quite technical homogenization-like procedure. In dimension one, it happens that it is equivalent to the following harmonic analysis result :

Let $\mathcal{F}$ the set of functions

$$f(x) = L + \sum_{j=1}^{+\infty} (a_j \cos(2jx) + b_j \sin(2jx)), \qquad \text{with } a_j \leq 0 \quad \forall j \in \mathbb{N}^*.$$

Then :

$$d(\mathcal{F}, \mathcal{U}_L) = 0$$

but there is no $\chi_\omega \in \mathcal{F}$.

(where $\mathcal{U}_L = \{\chi_\omega \mid \omega \subset [0, \pi],\ |\omega| = L\pi\}$)

Truncated version of the second problem

Since the second problem may have no solution, it makes sense to consider as in

P. Hébrard, A. Henrot, *A spillover phenomenon in the optimal location of actuators*, SIAM J. Control Optim. **44** (2005), 349–366.

a truncated version of the second problem :

$$\sup_{\substack{\omega\subset\Omega \\ |\omega|=L|\Omega|}} \min_{1\leq j\leq N} \int_\omega \phi_j^2(x)\,dx$$

Truncated version of the second problem

$$\sup_{\substack{\omega\subset\Omega\\ |\omega|=L|\Omega|}} \ \min_{1\leq j\leq N} \int_\omega \phi_j^2(x)\, dx$$

Theorem

The problem has a unique solution ω^N.
Moreover, ω^N has a finite number of connected components.
If Ω has a symmetry hyperplane, then ω^N enjoys the same symmetry property.

Truncated version of the second problem

Theorem, specific to the 1D case

ω^N is symmetric with respect to $\pi/2$, is the union of at most N intervals, and : there exists $L_N \in (0,1]$ such that, for every $L \in (0, L_N]$,

$$\int_{\omega^N} \sin^2 x \, dx = \int_{\omega^N} \sin^2(2x) \, dx = \cdots = \int_{\omega^N} \sin^2(Nx) \, dx.$$

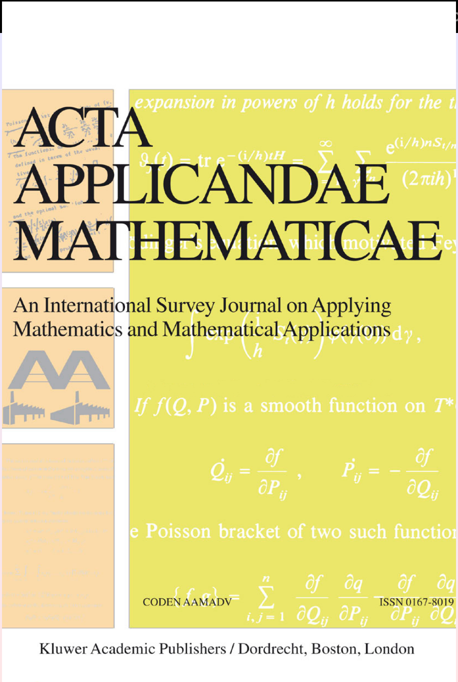
- Equality of the criteria $\Rightarrow$ the optimal domain ω^N concentrates around the points $\frac{k\pi}{N+1}$, $k = 1, \dots, N$.
- Spillover phenomenon : the best domain ω^N for the N first modes is the worst possible for the $N+1$ first modes.

Conclusion and perspectives

Next issues (ongoing work with Y. Privat and E. Zuazua)

- Same results for Schrödinger equations.
- Same kind of analysis for the optimal design of the (HUM) control domain.

  In particular, for the first problem : complete characterization of all initial data for which
  - there exists an optimal set with a finite number of components
  - there exists an optimal set of Cantor type
  - there exists no optimal set (relaxation phenomenon)
- Relations between shape optimization and ergodicity properties.
- Consider other kinds of spectral criteria permitting to avoid the spillover phenomenon.
- Investigation of other equations such as the heat equation.
- Discretization issues : do the numerical optimal designs converge to the continuous optimal design as the mesh size tends to 0 ?



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