Dispersions and "osmotic" pressure in sheared stokesian suspensions

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- **Stokesian suspensions**
- **State of the art**
- **Sheared induced pressure in a monodiperse suspension**
- **Sheared induced pressure in a bidiperse suspension**

Journée MMSN, Octobre, 2012

Stokesian suspensions

Non brownian particles (a>1μm) in a viscous fluid .

- **Fluid flow in inertialess regime:**

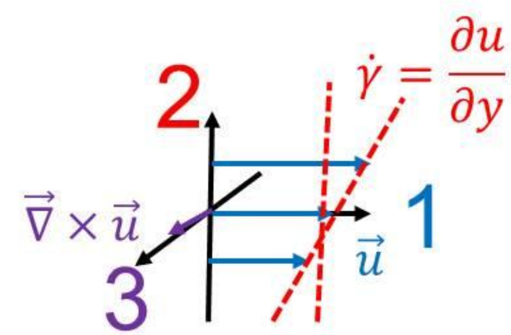
Navier-Stokes equation:

$$\rho_f\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\vec{\nabla})\vec{u}\right)=\vec{\nabla}.\overline{\overline{\sigma_f}}$$

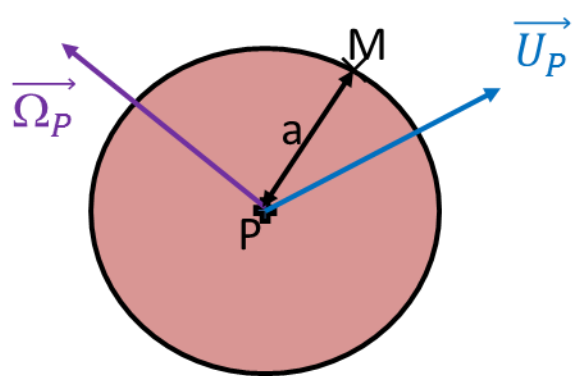
Stokes equation:

$$\vec{0}=\vec{\nabla}.\overline{\overline{\sigma_f}}$$

$$\overline{\overline{\sigma_f}}=-P\bar{\bar{I}}+\eta\overline{\overline{\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}}=\begin{pmatrix}-P & \eta\dot{\gamma} & 0\\ \eta\dot{\gamma} & -P & 0\\ 0 & 0 & -P\end{pmatrix}$$



- **With non-brownian inertialess particles:**



$$\vec{u}=\overrightarrow{U_P}+\overrightarrow{\Omega_P}\times\overrightarrow{PM}$$

$$\begin{cases}\overrightarrow{Ff/p}=\oiint\overline{\overline{\sigma_f}}.\overrightarrow{dS}\\ \overrightarrow{\Gamma^{f/p}}=\oiint\overrightarrow{PM}\times\overline{\overline{\sigma_f}}.\overrightarrow{dS}\end{cases}$$

Conditioning: $\left(\overrightarrow{U_P},\overrightarrow{\Omega_P}\right)$ **or** $\left(\overrightarrow{Ff/p},\overrightarrow{\Gamma^{f/p}}\right)$

Stokesian suspensions

Non brownian particles (a>1µm) in a viscous fluid .

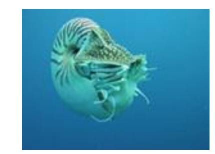
$$Re = \frac{a^2\dot{\gamma}}{\nu} \ll 1$$

$$St = \frac{\rho_p a^2 \dot{\gamma}}{\eta} \ll 1$$

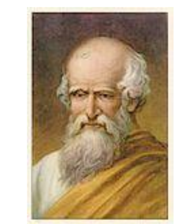
$$Pe = \frac{F_{\text{external}}}{\text{thermal agitation}} = \frac{6\Pi\eta\dot{\gamma}a^3}{k_B T} >> 1$$

$\boxed{\dot{\gamma} = 0 \;;\; \Delta\rho = 0}$ → **At rest forever**

(*Lazy nautilus -400000000*)



(*Archimède -250*)



$\boxed{\dot{\gamma} = 0 \;;\; \Delta\rho \neq 0}$ → **The particles settle:** (*Stokes1846, Richardson-Zaki 1954…*)

$$< \overrightarrow{u} >^p - \overrightarrow{u}^{susp} = f(\Phi)\overrightarrow{V_{Stokes}}$$

$$V_{Stokes} = \frac{2a^2\Delta\rho}{9\eta} g$$

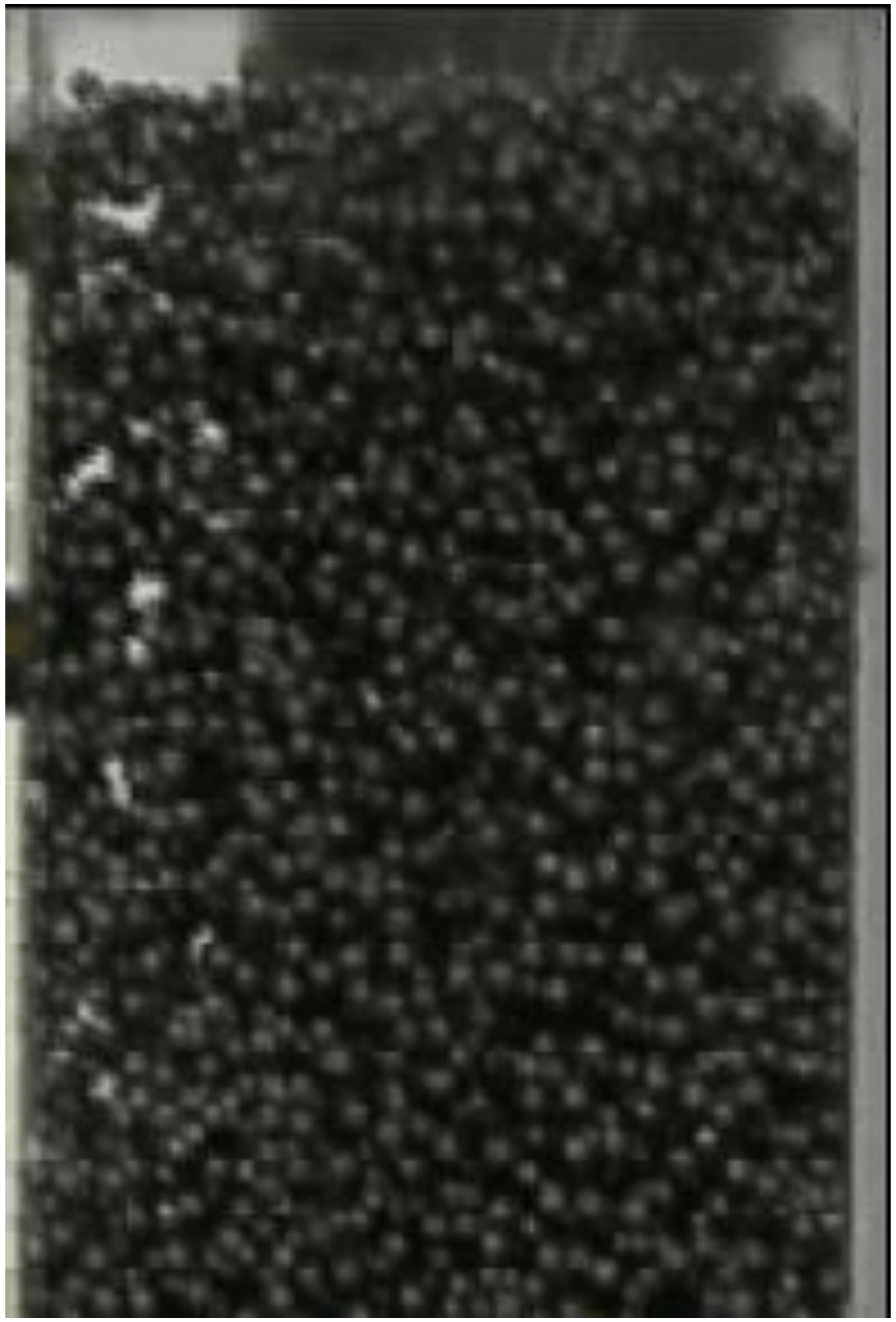
$$f(\phi) = (1-\phi)^n$$

$\boxed{\dot{\gamma} \neq 0 \;;\; \Delta\rho = 0}$ → **The viscosity increases:** (*Einstein 1905…Krieger & Dougherty 1959*)

$$\eta_{susp} = \frac{\eta}{(1-\chi)^{1,5}} \qquad \chi = \frac{\Phi}{\Phi^\star} \qquad \begin{cases} \chi = 0 \text{ at } \Phi = 0 \\ \chi = 1 \text{ at } \Phi = \Phi^\star \end{cases}$$

Buoyant particles

$\boxed{\dot{\gamma} = 0 \; ; \; \Delta\rho \neq 0}$ Sedimentation:



(Richardson-Zaki 1954…)

$$\boxed{< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}}}$$

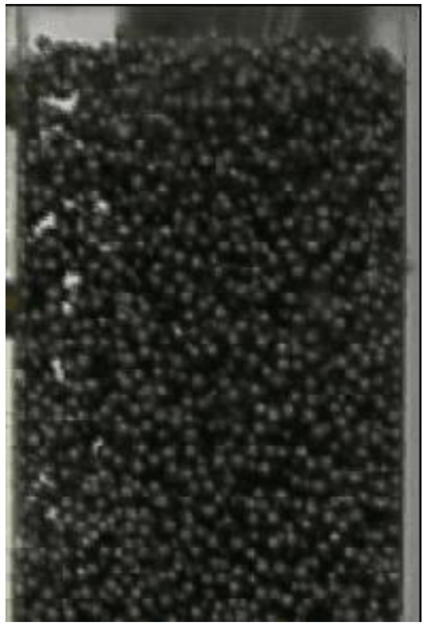
$$V_{Stokes} = \frac{2a^2\Delta\rho}{9\eta} g$$

$$\boxed{f(\Phi) = (1 - \Phi)^n}$$

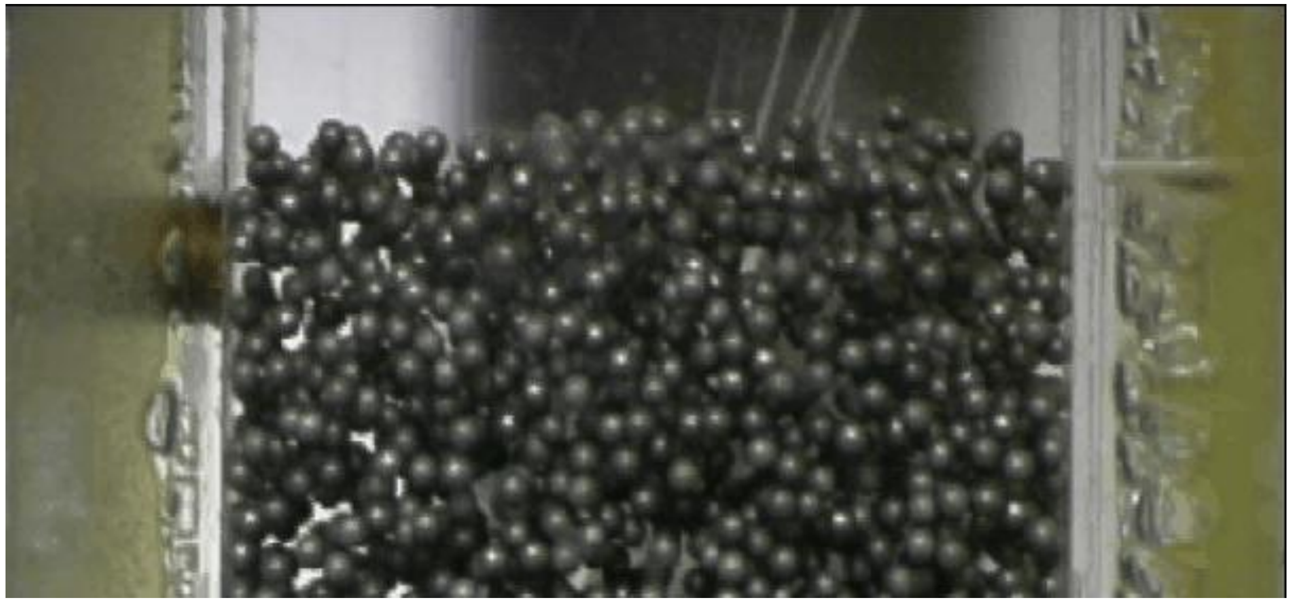
n≈5

Buoyant particles

Sedimentation:



Fluidization:



$\overrightarrow{V_{up}}$ ⬆

$$< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}}$$

$$f(\Phi) = (1 - \Phi)^n$$

n≈5

$$< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}} + \overrightarrow{V_{up}}$$

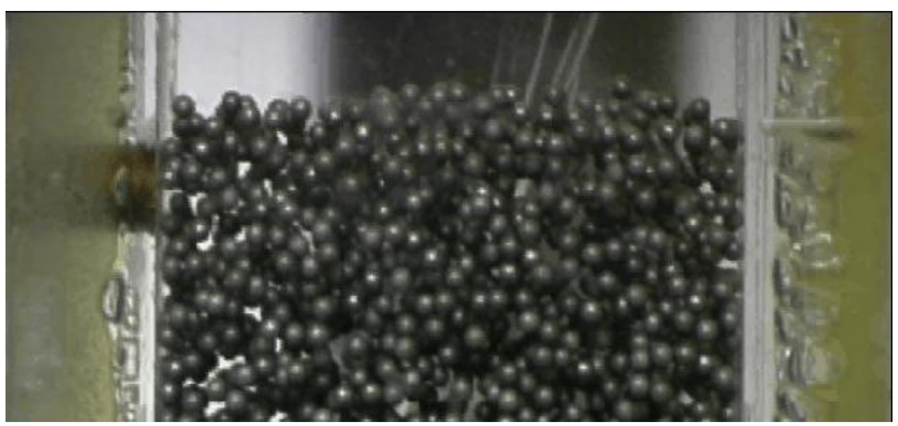
$$< \vec{u} >^p = \vec{0}$$
for $\Phi = \Phi°(V_{up})$

Suspension↔effective fluid?

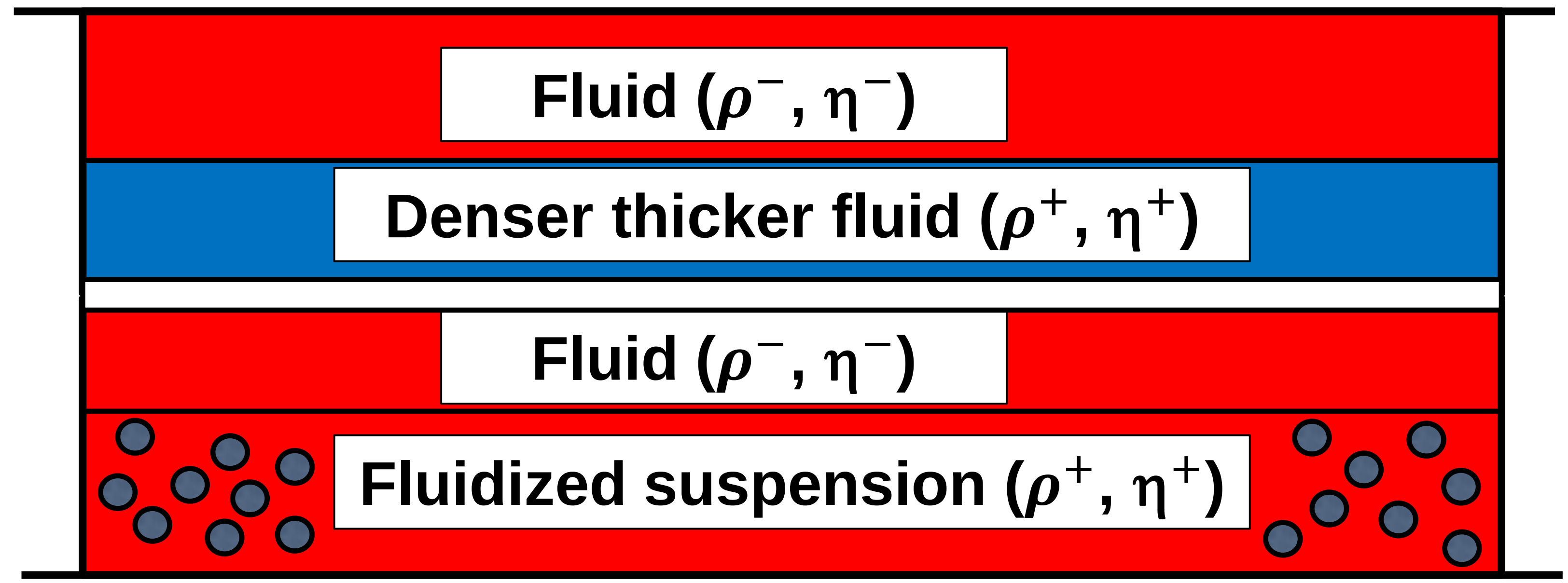
Fluidization:

$$< \vec{u} >^p = \vec{0}$$
for $\Phi = \Phi^\circ(V_{up})$

$$< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}} + \overrightarrow{V_{up}}$$



$\overrightarrow{V_{up}}$



Suspension=Effective dense viscous fluid?

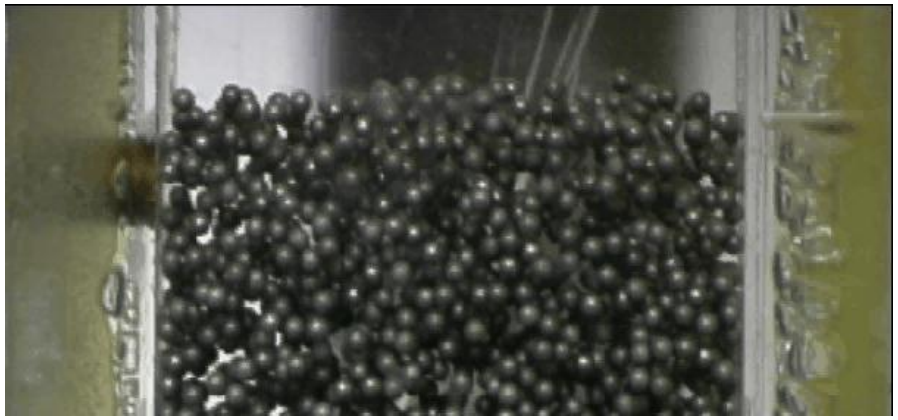
Gravity waves at miscible interfaces?

Suspension↔effective fluid?

Fluidization:

$$\boxed{\begin{array}{l} < \vec{u} >^p = \vec{0} \\ \text{for } \Phi = \Phi°(V_{up}) \end{array}}$$

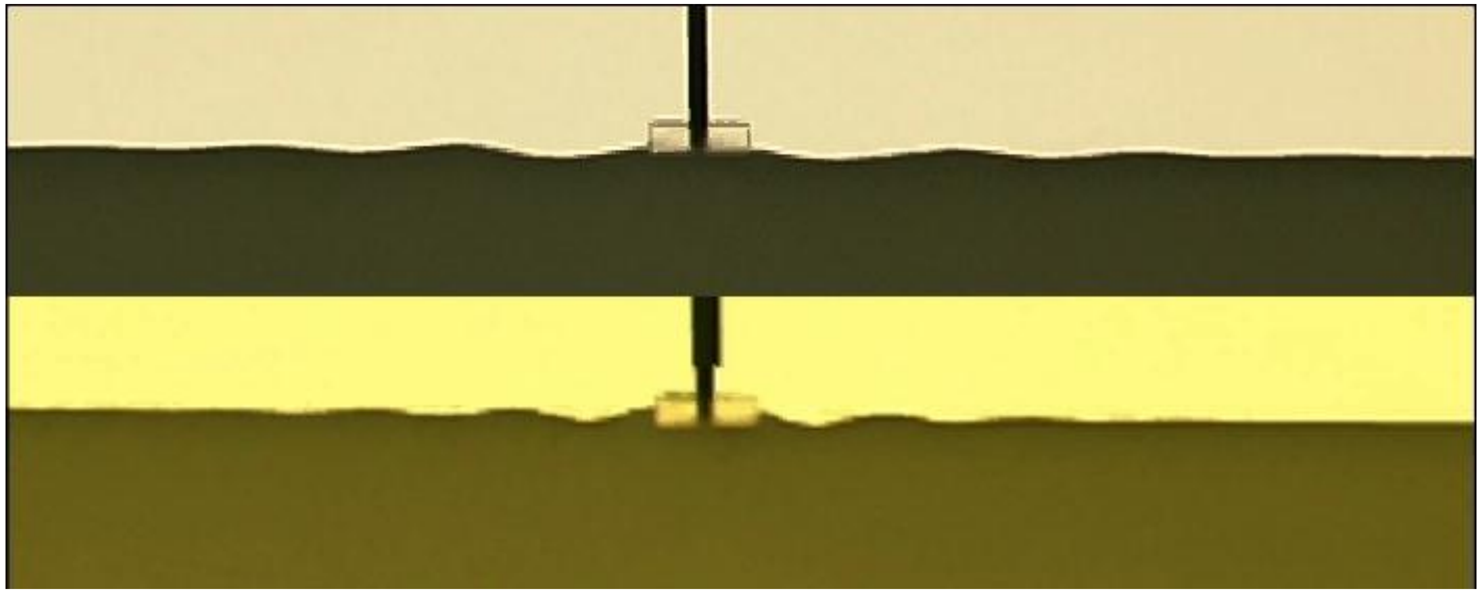
$$\boxed{< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}} + \overrightarrow{V_{up}}}$$



$\overrightarrow{V_{up}}$

Gravity waves at miscible interfaces:

(*Gauthier et al 2005*)



Suspension=Effective dense viscous fluid

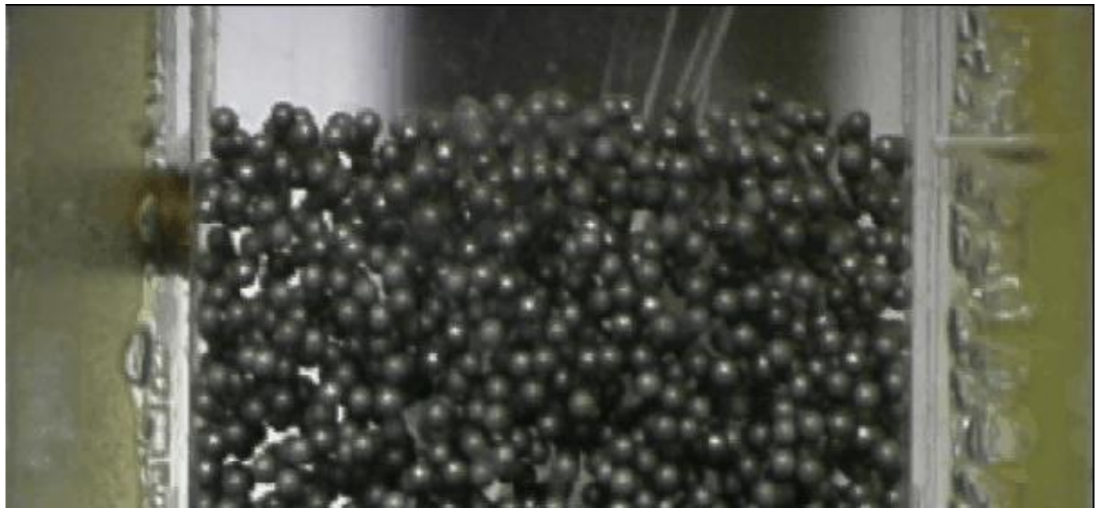
$$\boxed{\rho_{susp} = \Phi\rho_p + (1 - \Phi)\ \rho_f}$$

$$\boxed{\eta_{susp} = \frac{\eta}{(1 - \Phi/\Phi^*)^{1,5}}}$$

But also:

$\dot{\gamma} = 0$; $\Delta\rho \neq 0$

Fluidization:



$\overrightarrow{V_{up}}$

$< \vec{u} >^p = \vec{0}$
for $\Phi = \Phi^\circ(V_{up})$

$< \vec{u} >^p = f(\Phi)\overrightarrow{V_{Stokes}} + \overrightarrow{V_{up}}$

Hydrodynamic dispersion:

(*Martin et al 1995*)

Diffusive front :

$$\Phi\left(f(\Phi)\overrightarrow{V_{Stokes}} + \overrightarrow{V_{up}}\right) - D\vec{\nabla}\Phi = \vec{0}$$

(settling-induced) hydrodynamic diffusion:

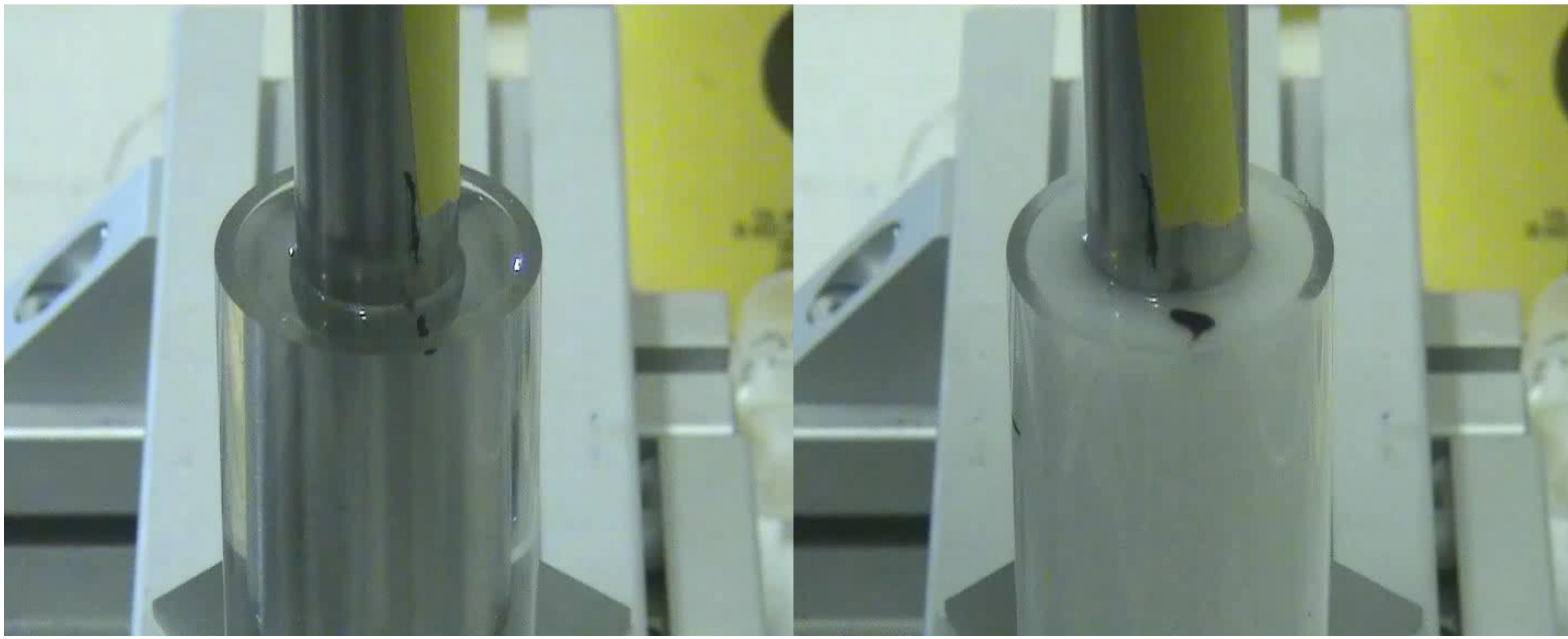
$\Delta\rho \neq 0$ → $D = d(\Phi)\, aV_{Stokes}$

$d(\Phi) \approx 60$

Stokes regime ⇒ Reversibility?

$$\boxed{\dot{\gamma} \neq 0 \; ; \; \Delta\rho = 0}$$

Taylor experiment with a stokesian suspension



Stokes regime ⇒ Reversibility?

$\boxed{\dot{\gamma} \neq 0 \; ; \; \Delta\rho = 0}$

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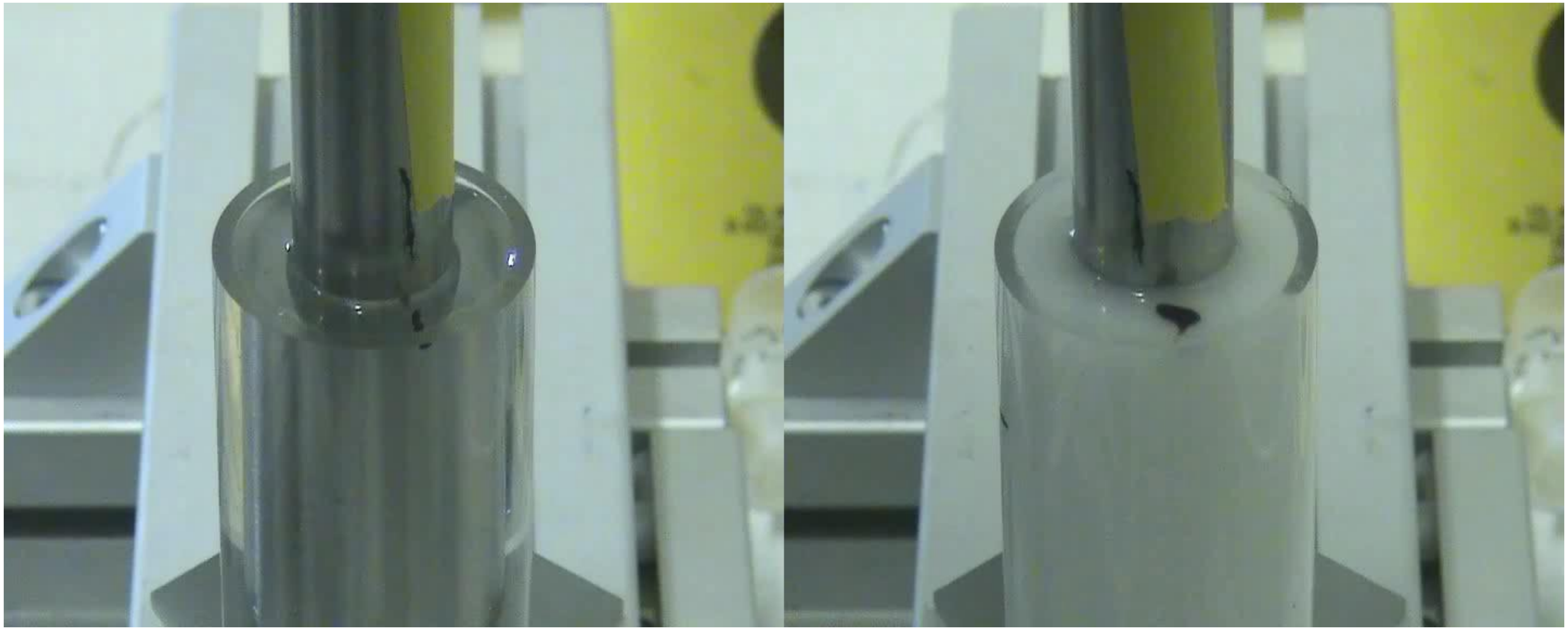
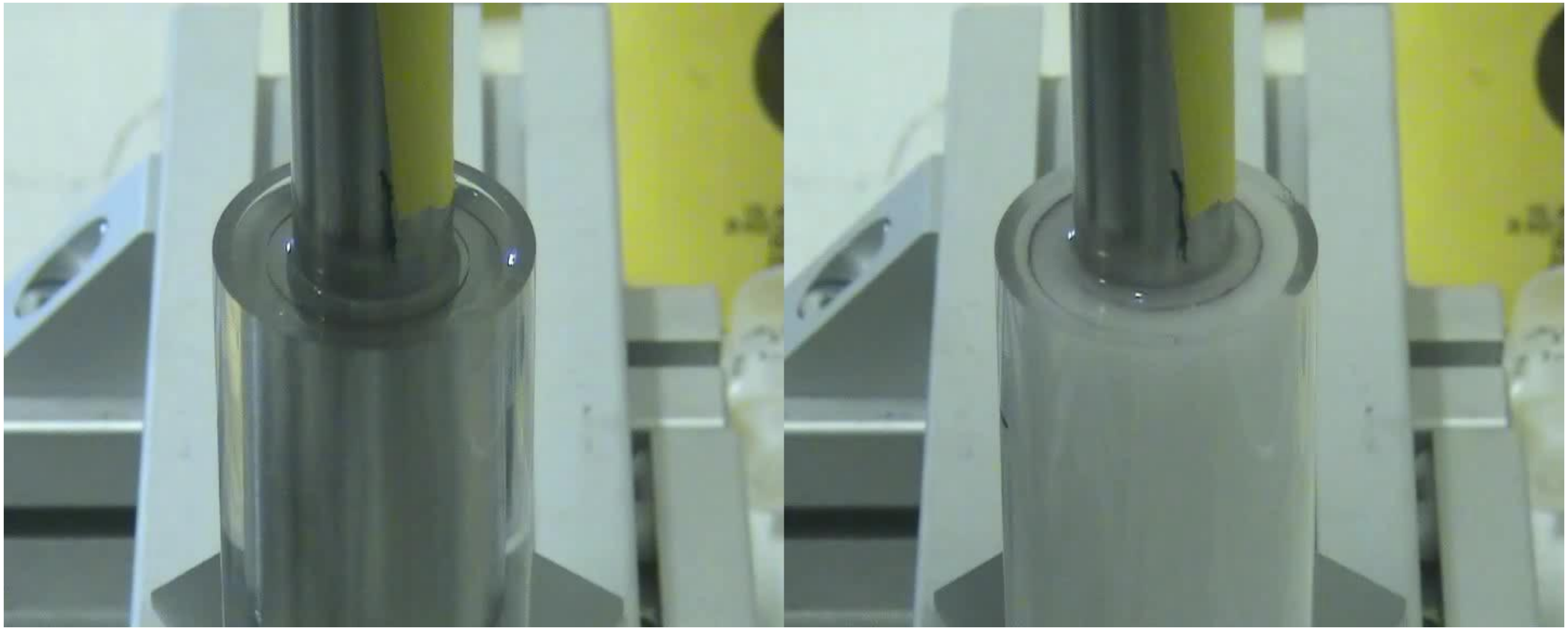


Stokes regime ⇒ Reversibility?

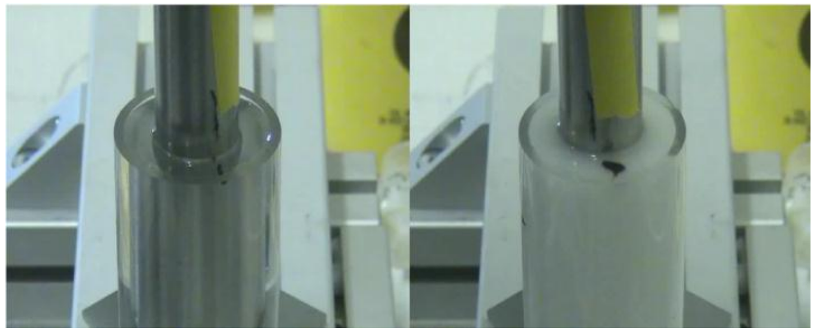
$\boxed{\dot{\gamma} \neq 0\ ;\ \Delta\rho = 0}$

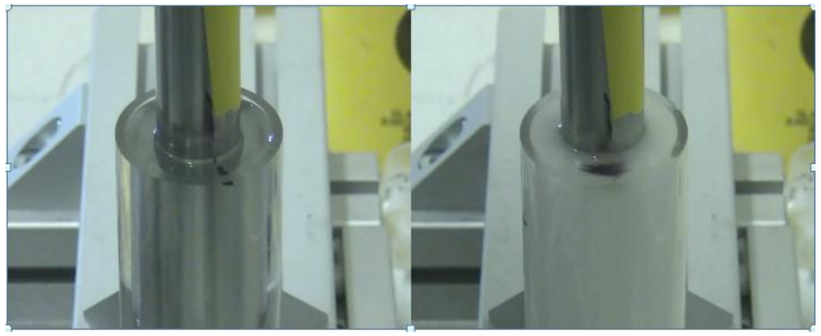
Taylor experiment with a stokesian suspension



Stokes regime but irreversibility!

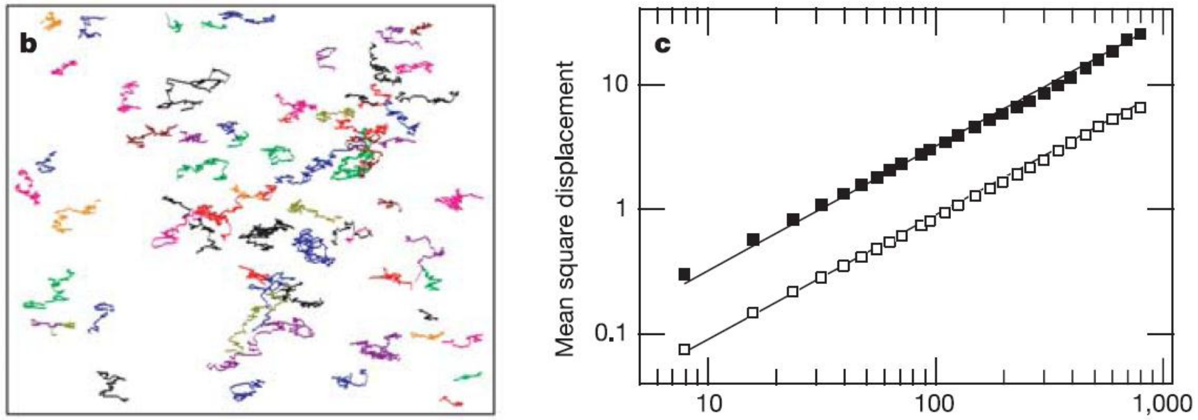
Taylor experiment with a stokesian suspension → Self diffusivities





Shear induced diffusion of the fluid

Periodic shear → random walk of particles: (Pine *et al.* 2005)



$D_1 > D_3$

Shear induced diffusion of the particles

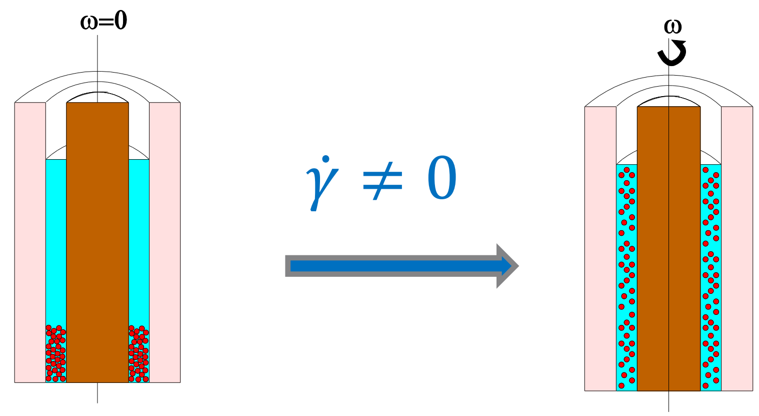
Agitation:
T(Brownian)⟺Shear (Stokesian)

Shear induced diffusion $D \propto a^2\dot{\gamma}$

Shear-induced diffusion →Collective migration

Viscous resuspension of buoyant macroscopic particles in a Couette device

(Gadala-Maria 1979,
Leighton & Acrivos 1987)

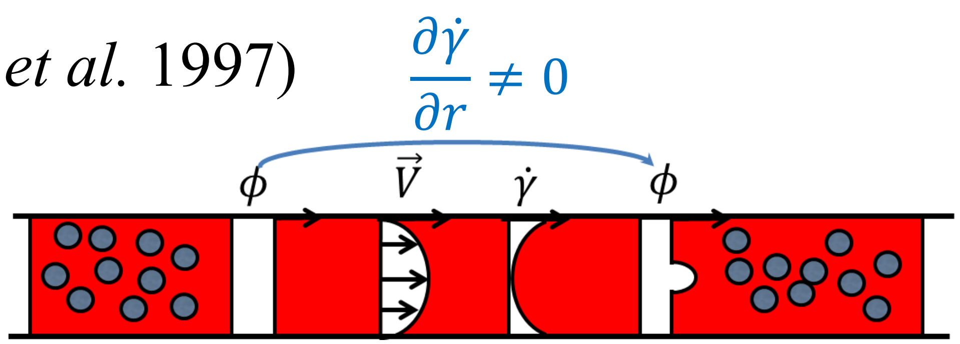


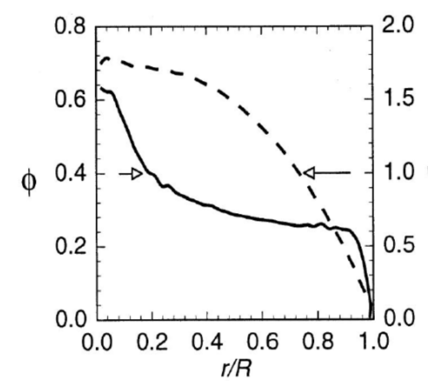
Fick:

$$\boxed{\text{Flux} \propto -D\vec{\nabla}\Phi}$$

Shear induced migration of neutrally buoyant particles in a Poiseuille flow:

(Hampton *et al.* 1997)





$$\boxed{\text{Flux} \propto -\vec{\nabla}\dot{\gamma}}$$

shear-induced osmotique pressure ?

$$\boxed{\text{Flux} \propto -\frac{2a^2}{9\eta} f(\Phi)\, \vec{\nabla}\Pi^{\,p}}$$

$$\boxed{\Pi^{\,p} \propto p(\Phi)\eta\dot{\gamma}}$$

… or shear-induced particle stress $\Sigma^{\,p}$?

Stresses in a sheared suspension

Phase-averaged equations for stokesian suspensions *(Jackson,1997)*

$$\boxed{\vec{\nabla}\cdot\vec{u}^{susp} = 0}$$

$$\vec{u}^{susp} = \Phi < \vec{u} >^p + (1-\Phi) < \vec{u} >^f$$

Fluid: $\vec{\nabla}\cdot((1-\Phi) < \overline{\overline{\sigma}} >^f) - \overrightarrow{F^{f/p}} + (1-\Phi)\rho_f\,\vec{g} = \vec{0}$

$$\overline{\overline{\sigma^f}} = -p^f\overline{\overline{I}} + 2\eta_f\overline{\overline{e}} = \begin{pmatrix} -p^f & \eta_f\dot{\gamma} & 0 \\ \eta_f\dot{\gamma} & -p^f & 0 \\ 0 & 0 & -p^f \end{pmatrix}$$

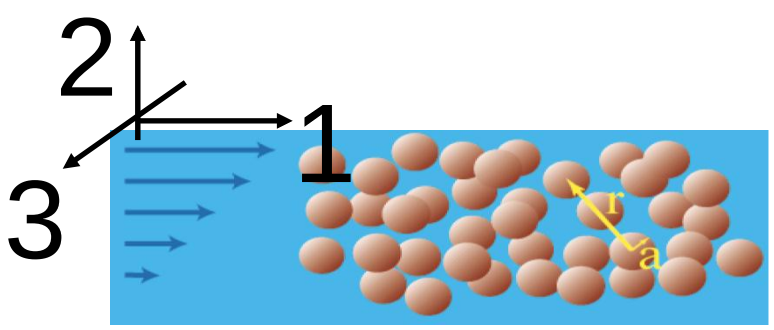
Particles: $\vec{\nabla}\cdot(\Phi < \overline{\overline{\sigma}} >^p) + \overrightarrow{F^{f/p}} + \Phi\rho_p\,\vec{g} = \vec{0}$

$$\Phi < \overline{\overline{\sigma}} >^p \quad ?$$

Suspension: $\boxed{\vec{\nabla}\cdot\overline{\overline{\Sigma}}^{susp} + \rho_{susp}\,\vec{g} = \vec{0}}$

$$\overline{\overline{\Sigma}}^{susp} = \Phi < \overline{\overline{\sigma}} >^p + (1-\Phi) < \overline{\overline{\sigma}} >^f$$

$$\rho_{susp} = \phi\rho_p + (1-\phi)\rho_f$$



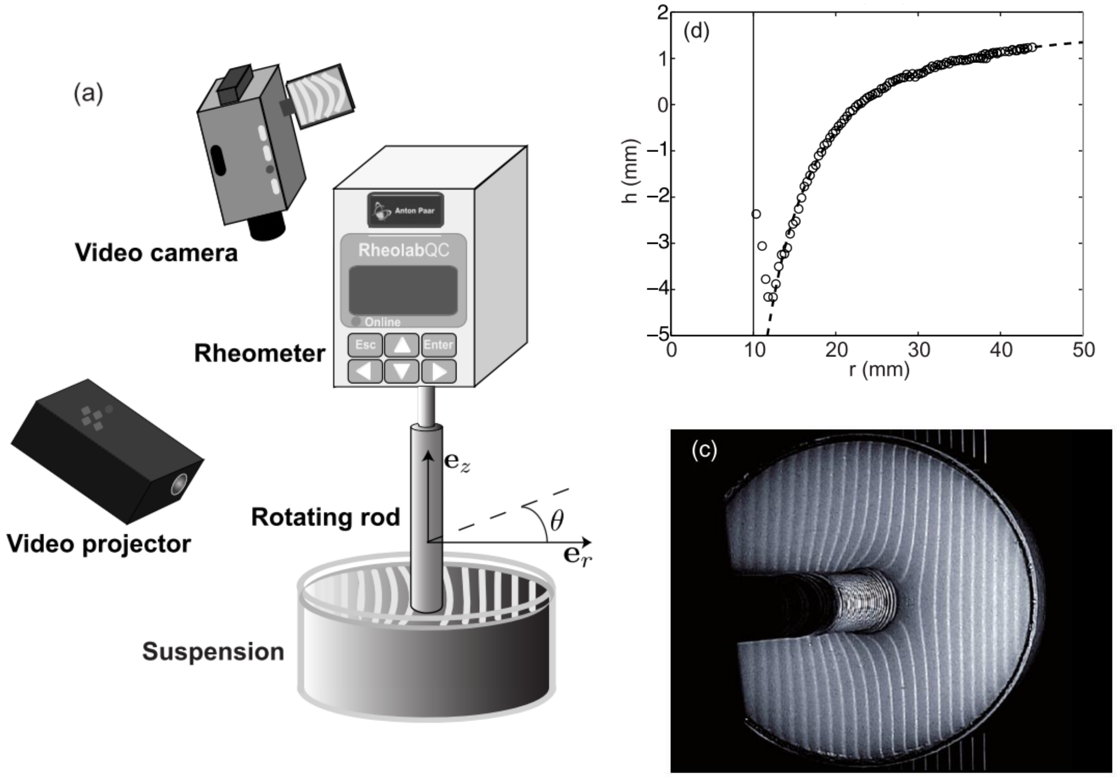
$$\overline{\overline{\Sigma}}^{susp} = \begin{pmatrix} \Sigma_{11}^{susp} & \eta_{susp}\dot{\gamma} & 0 \\ \eta_{susp}\dot{\gamma} & \Sigma_{22}^{susp} & 0 \\ 0 & 0 & \Sigma_{33}^{susp} \end{pmatrix}$$

Pressure or Normal Stress Differences ?

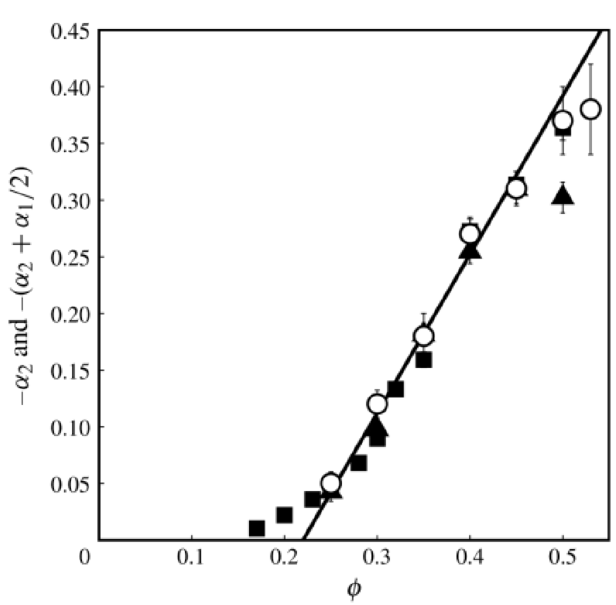
Non-newtonian behaviors !....A Bird's eye view

Anti-Weissenberg effect

(*Zarraga 2000, Boyer et al. 2011*)

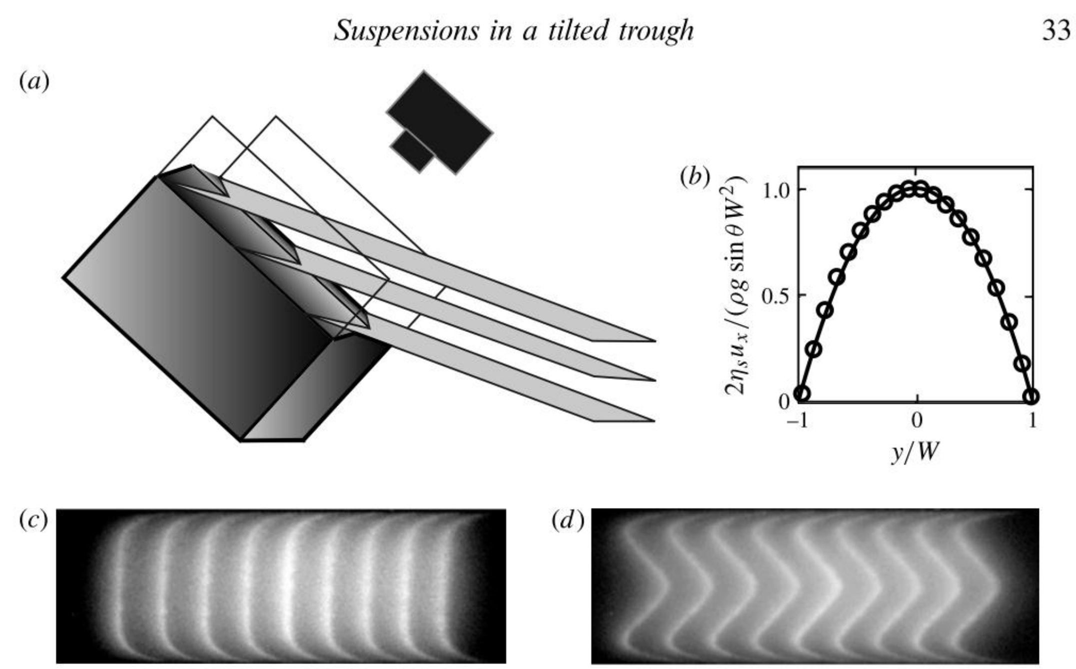


$$\boxed{N_2^{susp} + \frac{N_1^{susp}}{2}}$$



Free surface deformation in a tilted trough

(*Wineman & Pipkin 1966, Tanner 1970*)



$$\boxed{N_2^{susp}}$$

(*Couturier et al. 2011*)

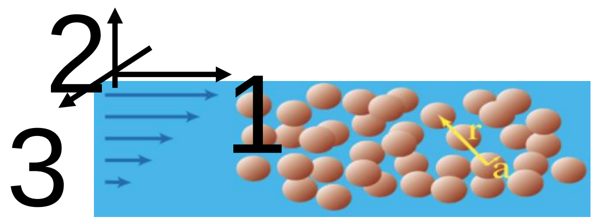
Normal Stress Differences!

$$N_2^{susp} = \Sigma_{22}^{susp} - \Sigma_{33}^{susp} = 1.4(\Phi - \Phi_c)\Sigma_{12}^{susp}$$

$$N_1^{susp} = \Sigma_{22}^{susp} - \Sigma_{11}^{susp} \approx 0 \qquad \boxed{\Phi_c = 0.22}$$

Particle migration $\Longleftarrow$ particle stress tensor

Suspension Balance Model *(Nott & Brady 1994, Morris & Boulay 1999)*



Particles: $\overrightarrow{\nabla} \cdot (\Phi < \overline{\overline{\sigma}} >^p) + \overrightarrow{F^{f/p}} + \Phi \rho_p \overrightarrow{g} = \overrightarrow{0}$

$$\overrightarrow{F^{f/p}} \Longrightarrow \begin{cases} \text{viscous friction} \\ \text{hydrosatic presure (or Archimède)} \\ + \text{other stress? (as solid friction)} \end{cases}$$

$$\overrightarrow{\nabla} \cdot \overline{\overline{\Sigma^p}} + \phi \Delta\rho \overrightarrow{g} + \frac{9\eta_f}{2a^2} \frac{\phi}{f(\phi)} (< \overrightarrow{u} >^p - \overrightarrow{u}^{susp}) = \overrightarrow{0}$$

$$\boxed{|\Sigma^p| \propto \eta\, \dot{\gamma} \mathrm{h}(\phi)}$$

Accounts for migration in the presence of gradients:

$$\overrightarrow{j} = \frac{-2a^2}{9\eta_f} f(\phi) \vec{\nabla}.\overline{\overline{\Sigma}}^p = \phi(< \overrightarrow{u} >^p - \overrightarrow{u}^{susp})$$

Rq: *(Lhuillier 2009)*

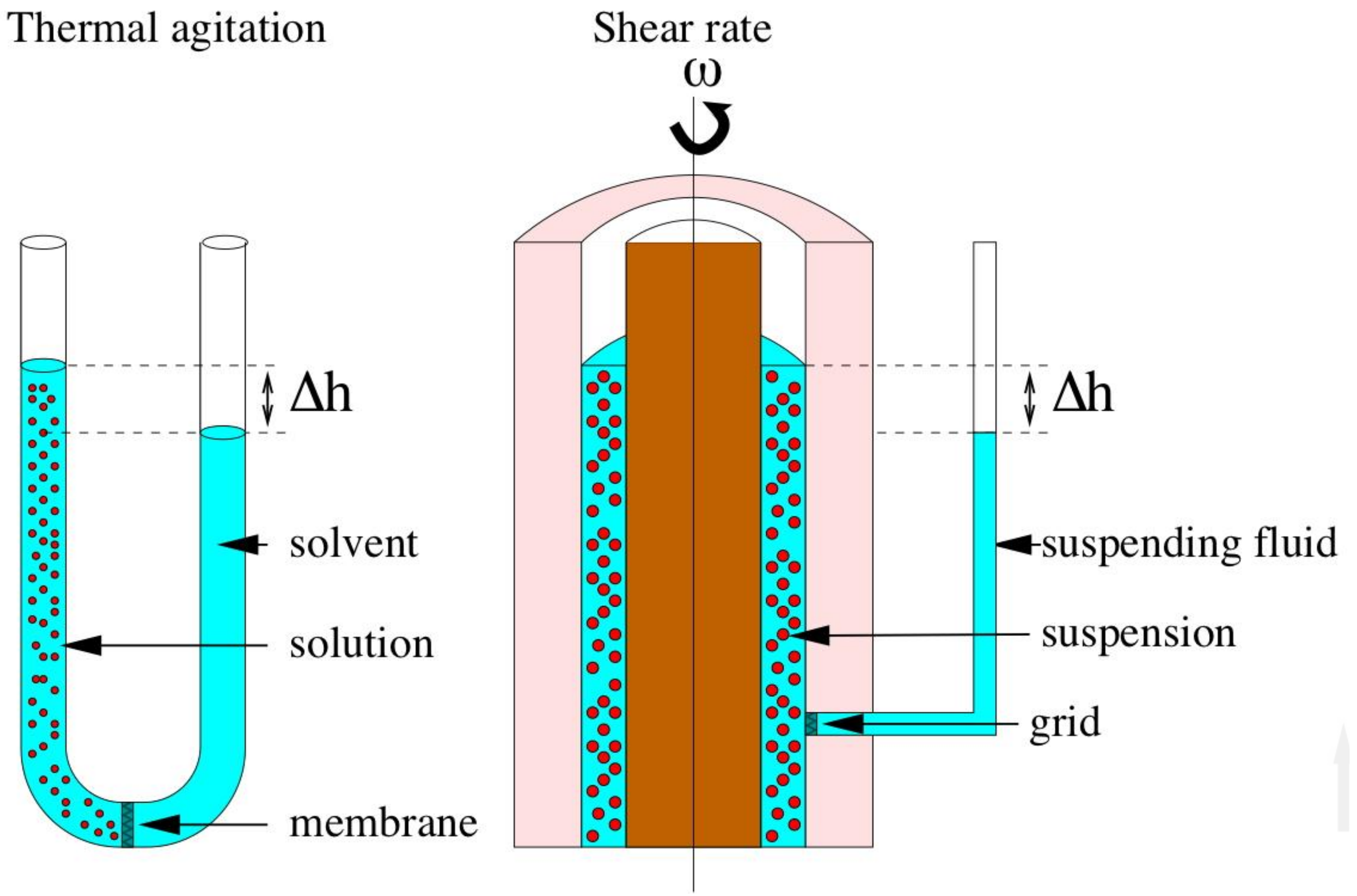
$$\overline{\overline{\Sigma}}^{susp} = \Phi < \overline{\overline{\sigma}} >^p + (1 - \Phi) < \overline{\overline{\sigma}} >^f = \overline{\overline{\Sigma}}^p + \overline{\overline{\Sigma}}^f \Longrightarrow$$

Direct measurement of the Particle stress?

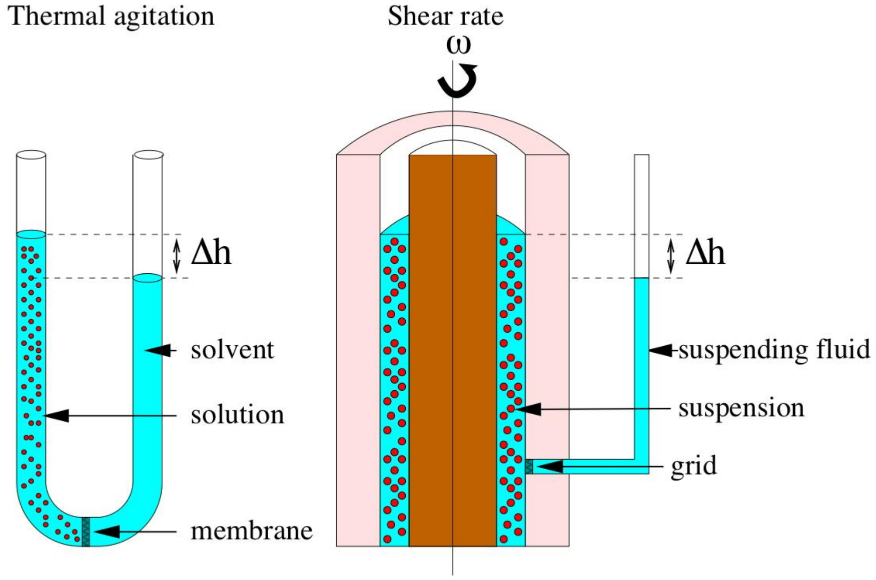
$$\overline{\overline{\Sigma}}^p \neq \Phi < \overline{\overline{\sigma}} >^p \qquad \mathrm{N}_i^p \neq \mathrm{N}_i^{susp}$$

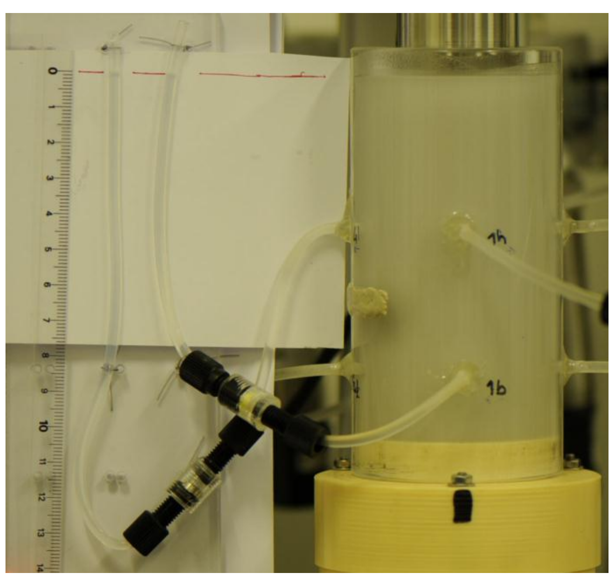
How to measure a sheared induced particle normal stress?

(Deboeuf et al., Phys Rev Lett. 2009)

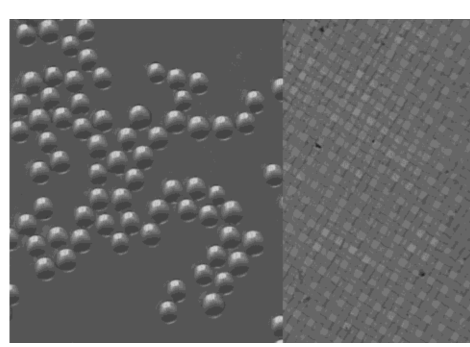


Sheared induced particle pressure?

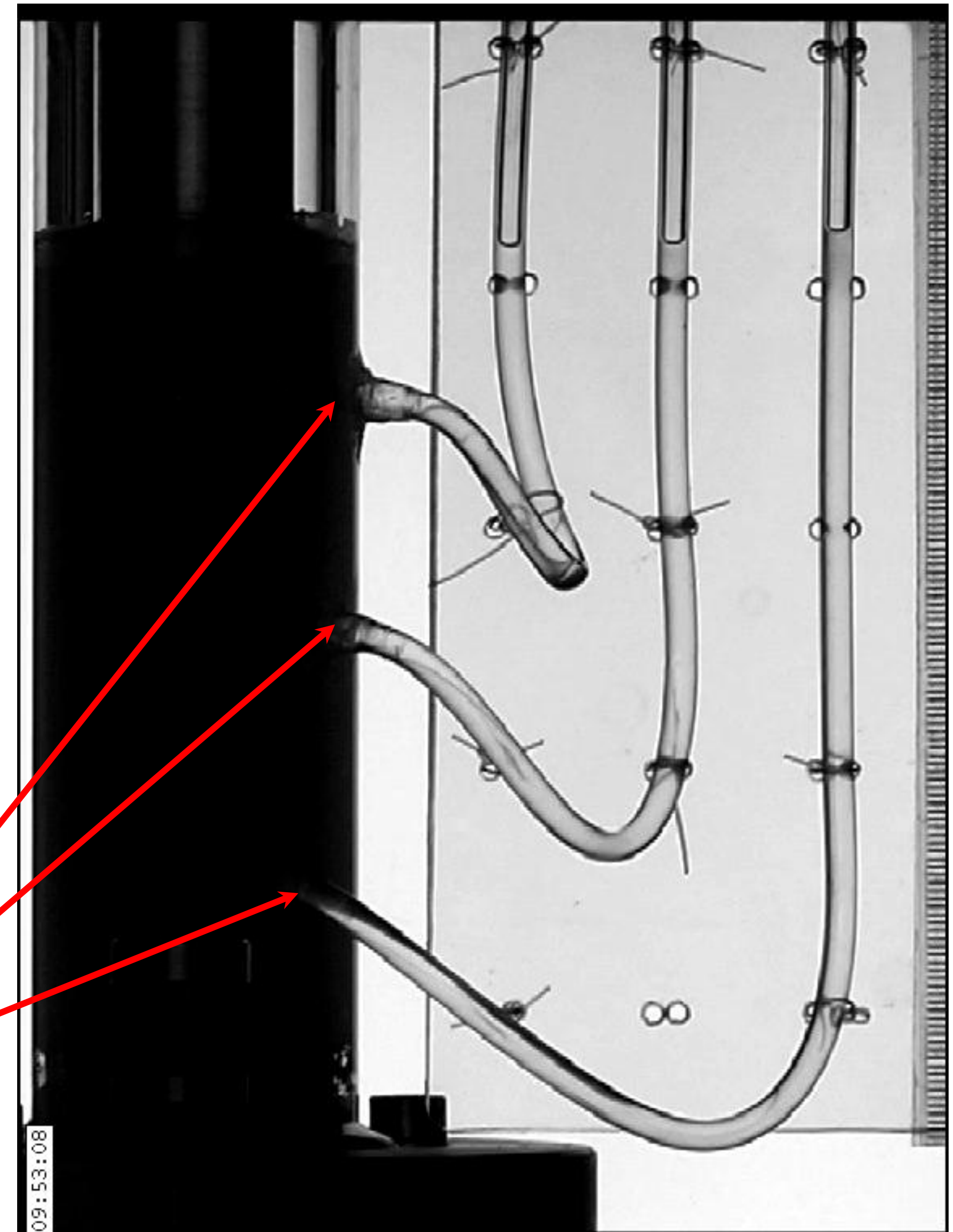




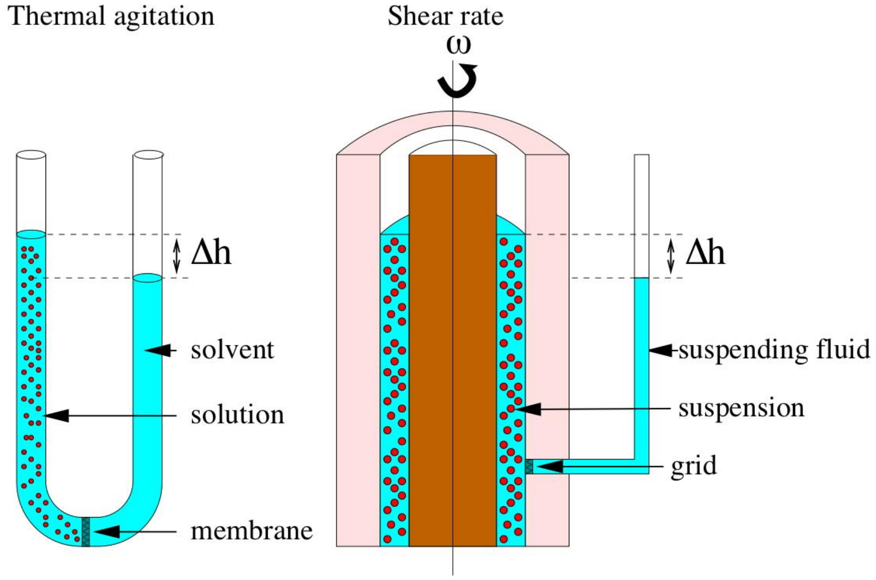
$$\begin{cases} a = 80\mu m \\ \eta_f = 3Pa.s \\ \phi = 0.4 \\ \dot{\gamma} = 10s^{-1} \end{cases}$$

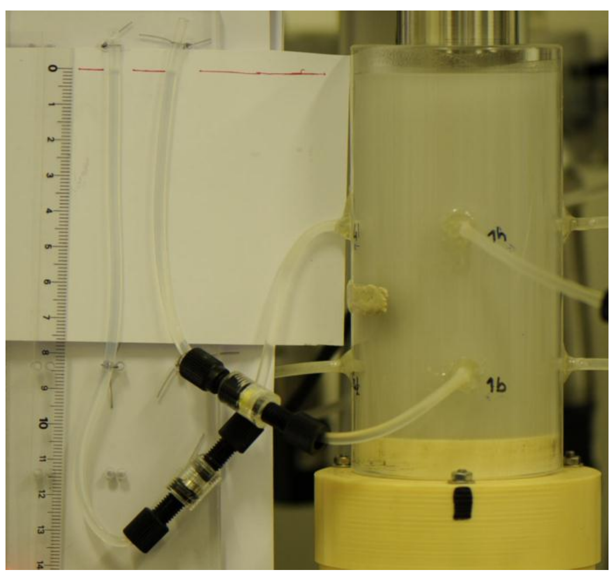


No shear: $\dot{\gamma} = 0$



Sheared induced particle pressure?





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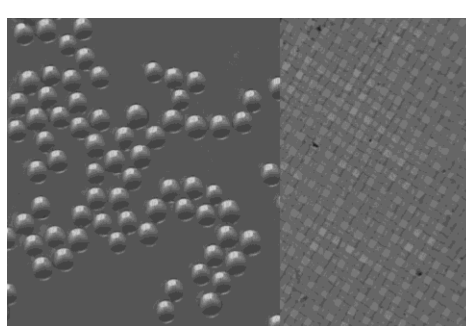
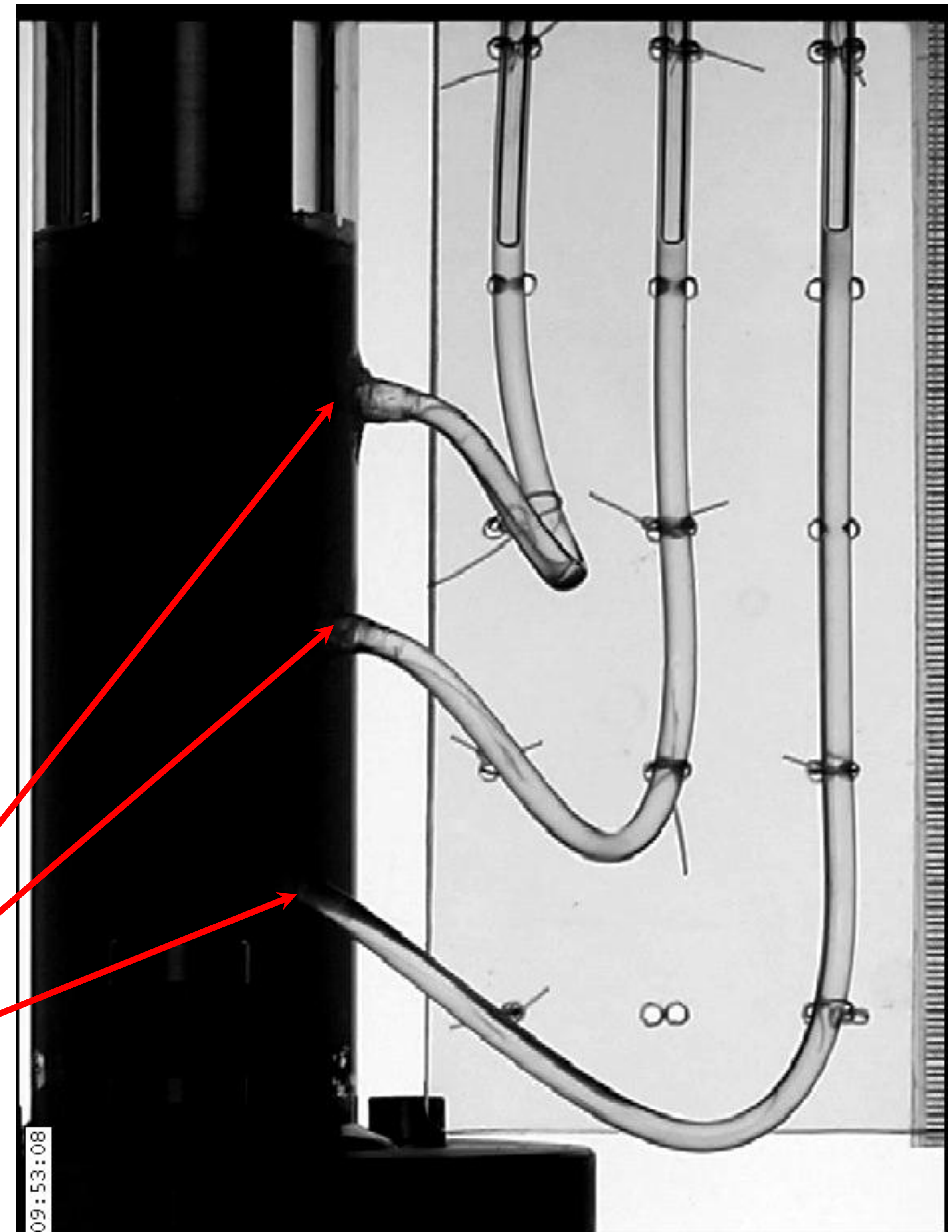
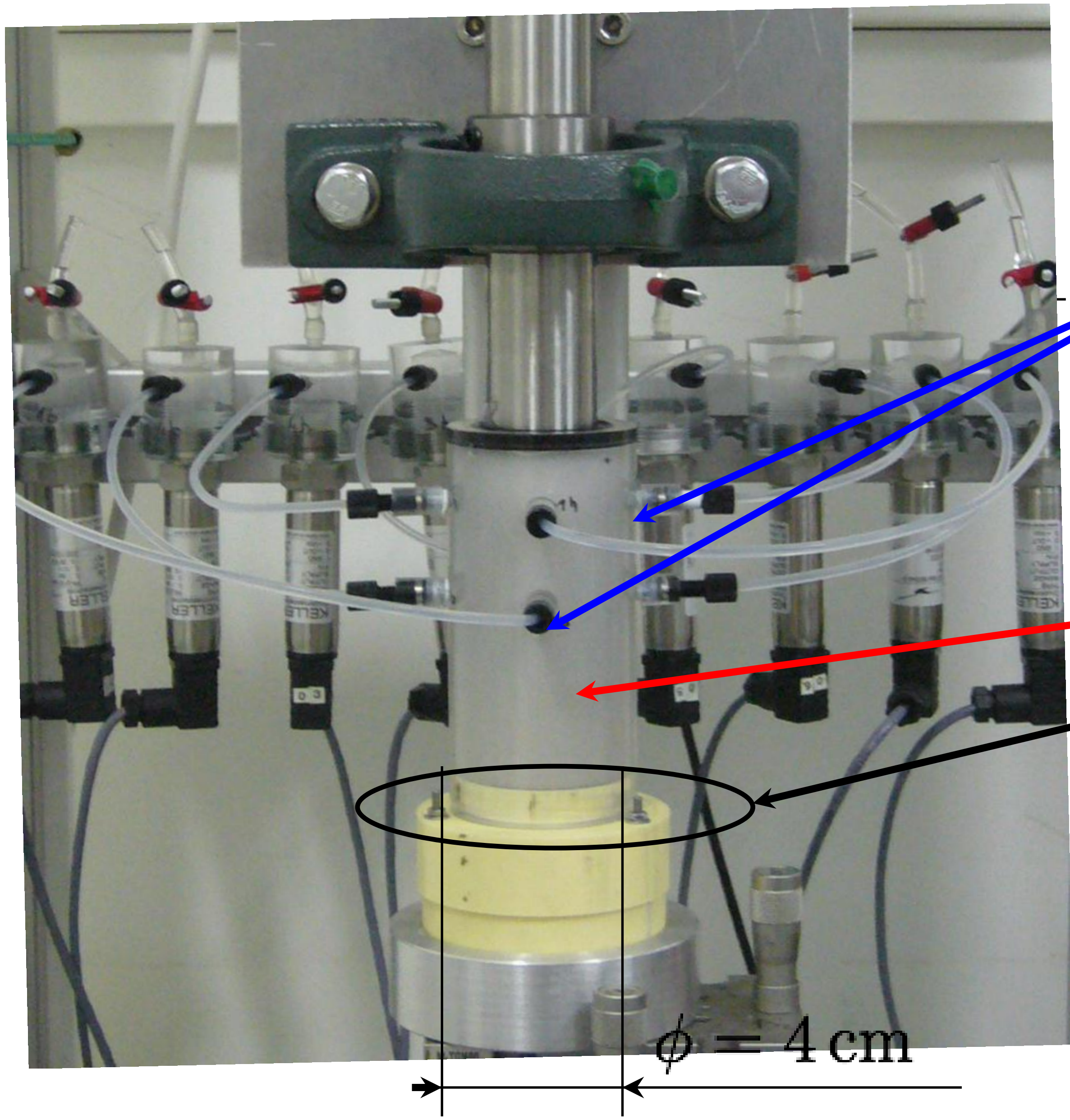


Shear: $\dot{\gamma} = 10s^{-1}$



Set up



8 grids
2 non permeable membranes
} connected to Pressure transducers

→ No flow across the grids

Suspension

Cone-cone geometry

→ No accumulation at the bottom

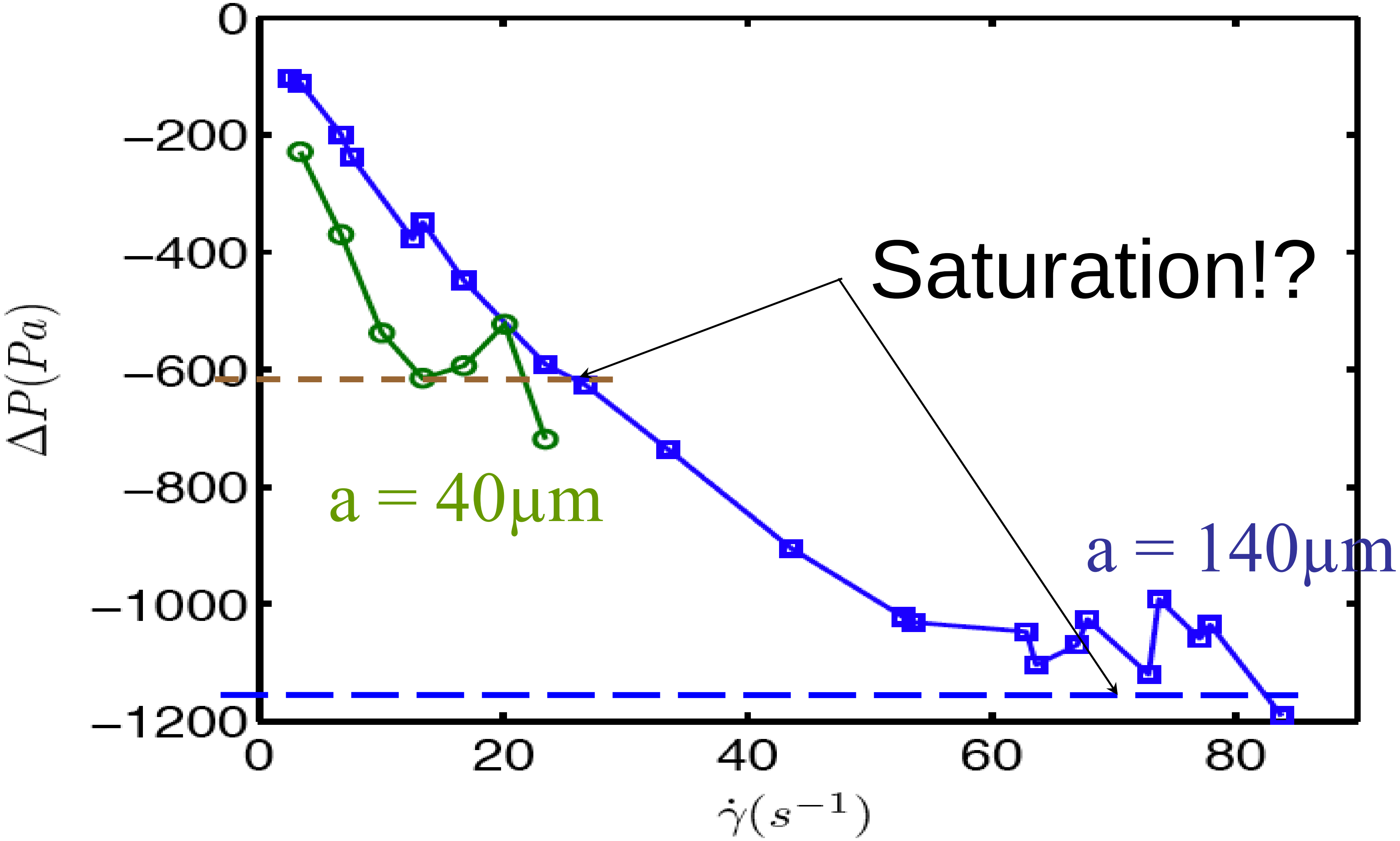
→ **No migration!**

$$\begin{cases} a \in [40, 140]\mu m \\ \eta_f = 3Pa.s \\ \phi \in [0.2, 0.5] \\ \dot\gamma \in [0, 10]s^{-1} \end{cases}$$

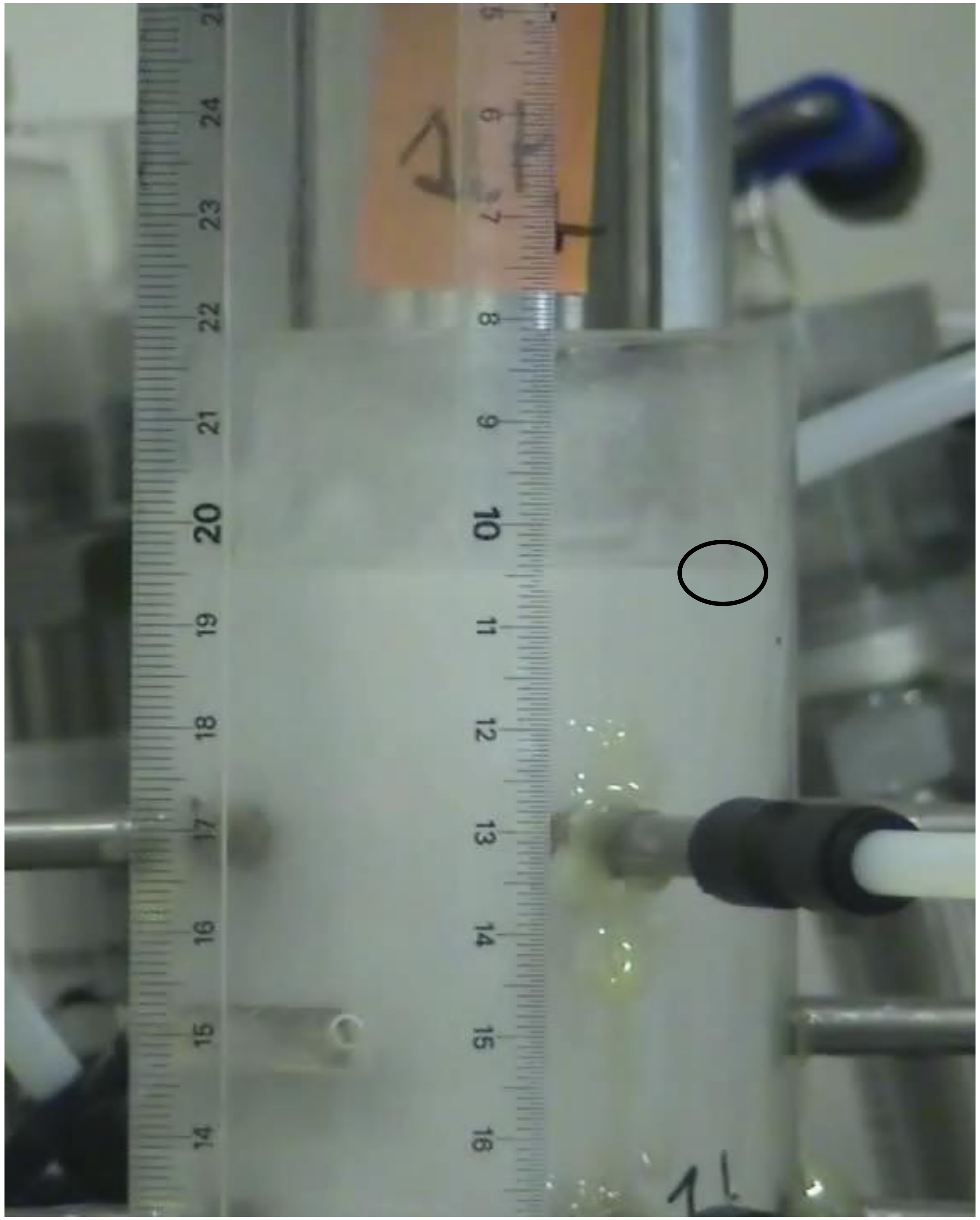
Rq : 8 grids captors used to align the set-up

$$\Sigma_{\text{grid}}(\dot\gamma = 0) = 0$$

Grid pressure measurements: Linear in shear rate?



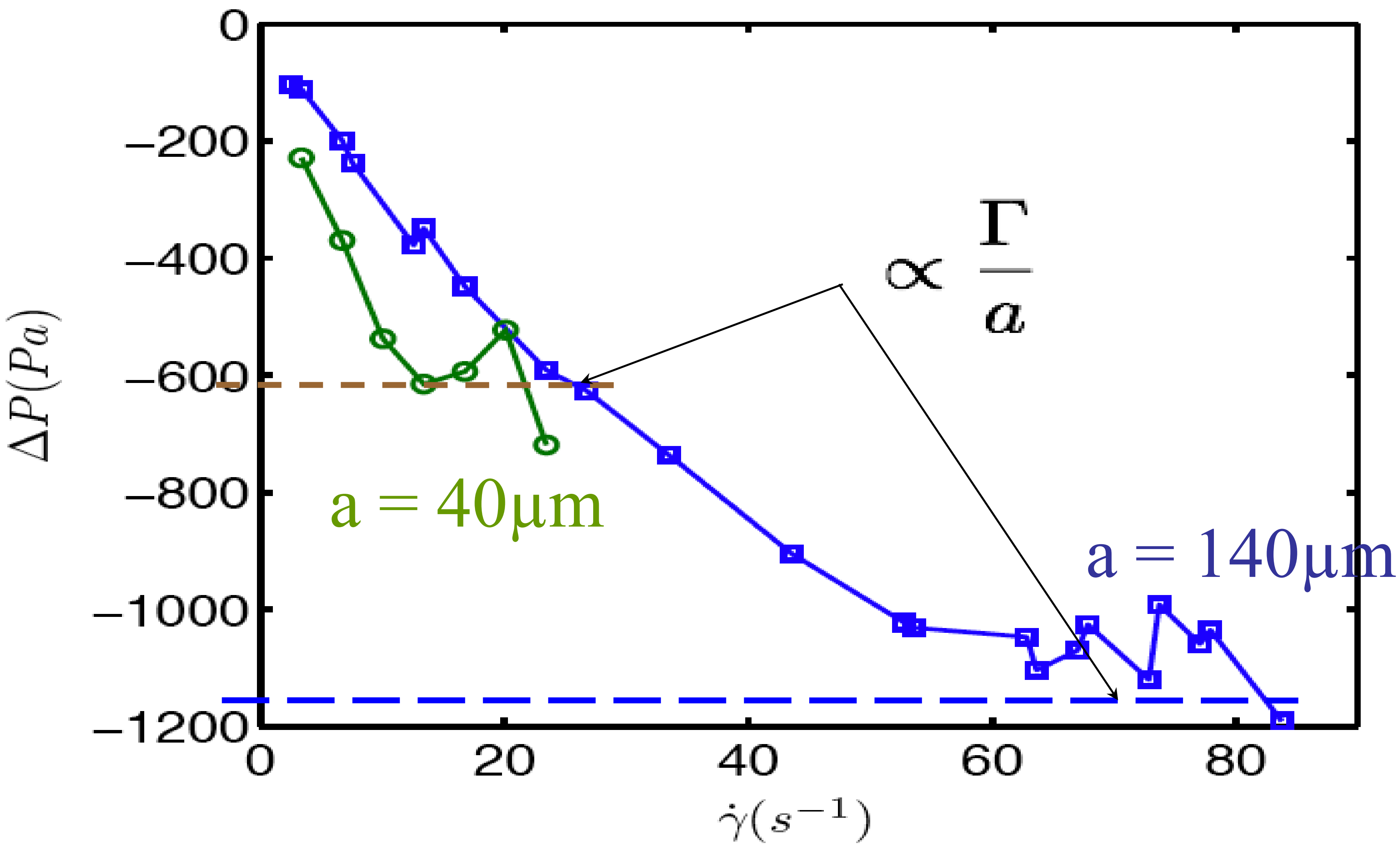
Case of saturated grid pressure



Grid pressure measurements: Linear in shear rate?

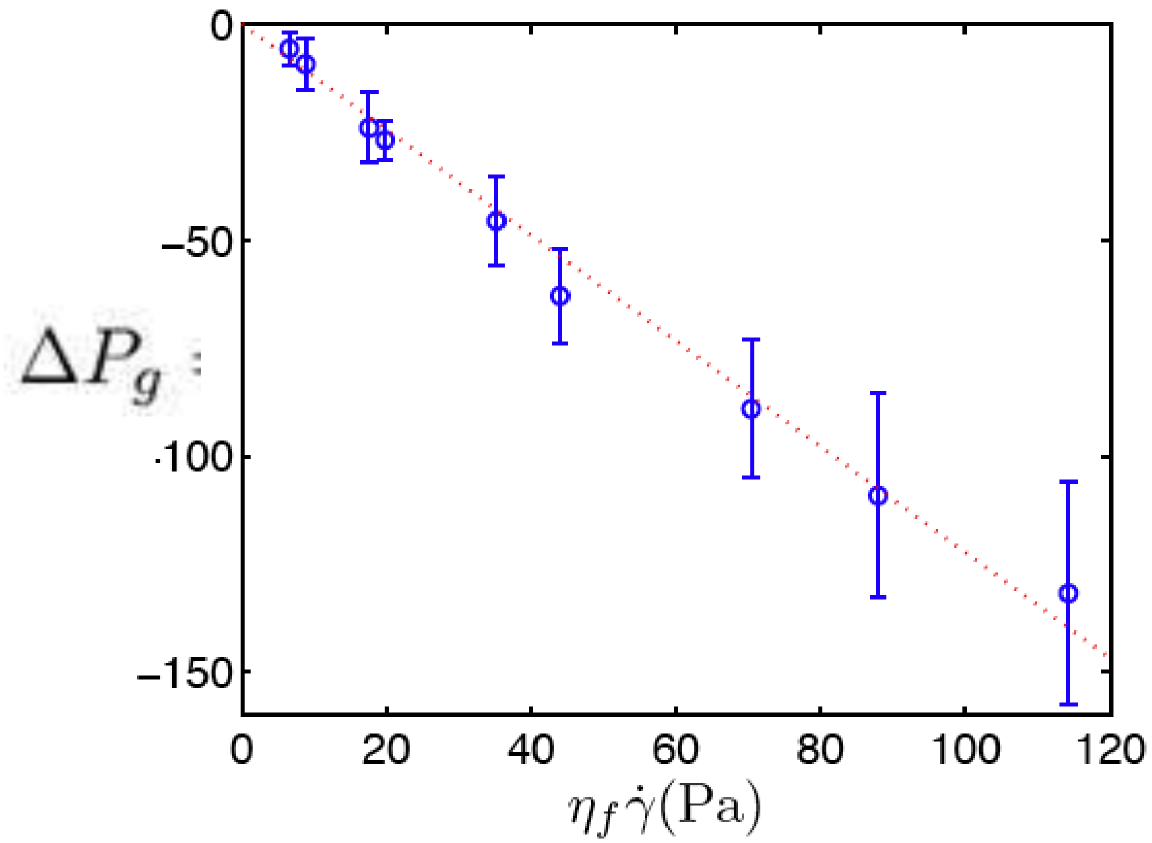
Creaming occurs when the particle pressure exceeds capillary pressure!

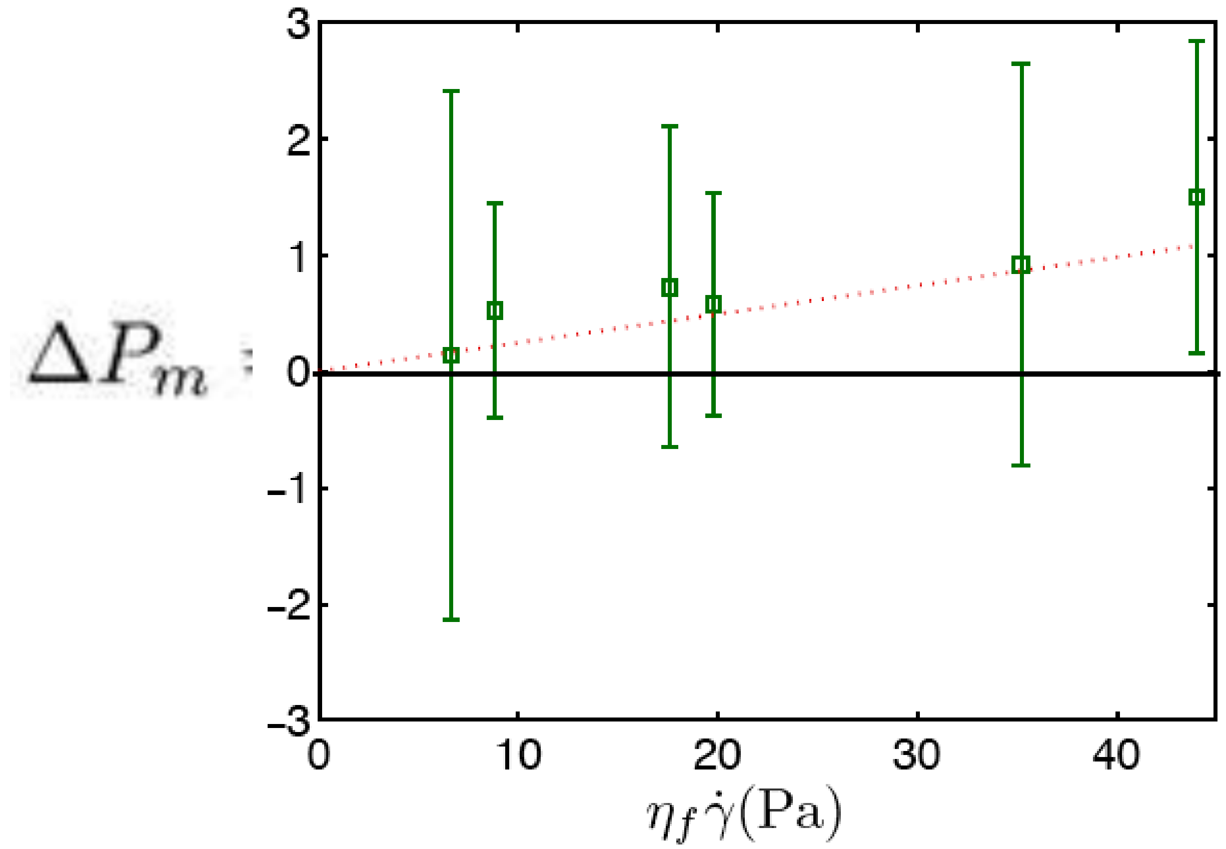
$$\Sigma_{zz}^{p} >_{\approx} \frac{\Gamma}{a}$$



Grid and membrane pressure measurements

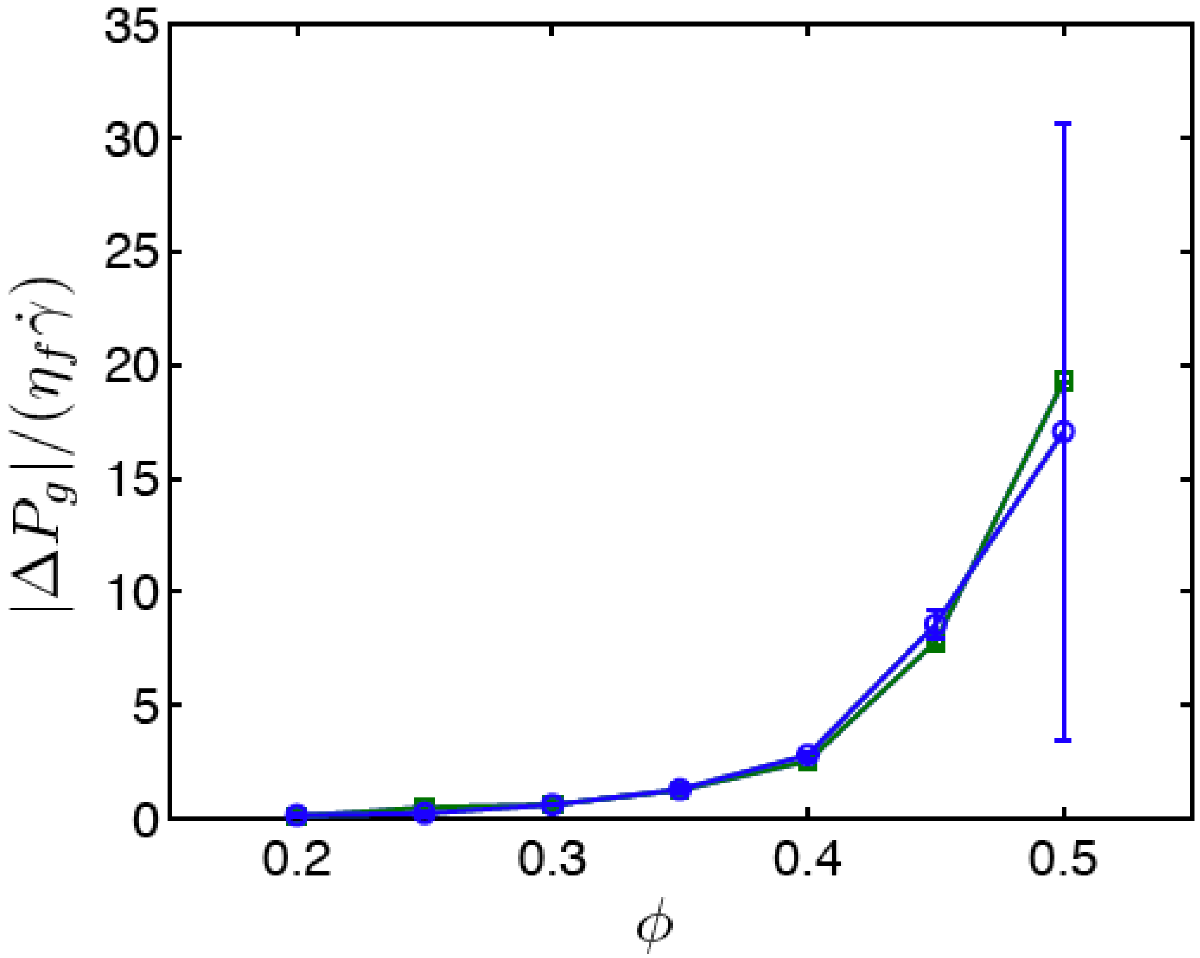
a = 40μm; Φ=45%

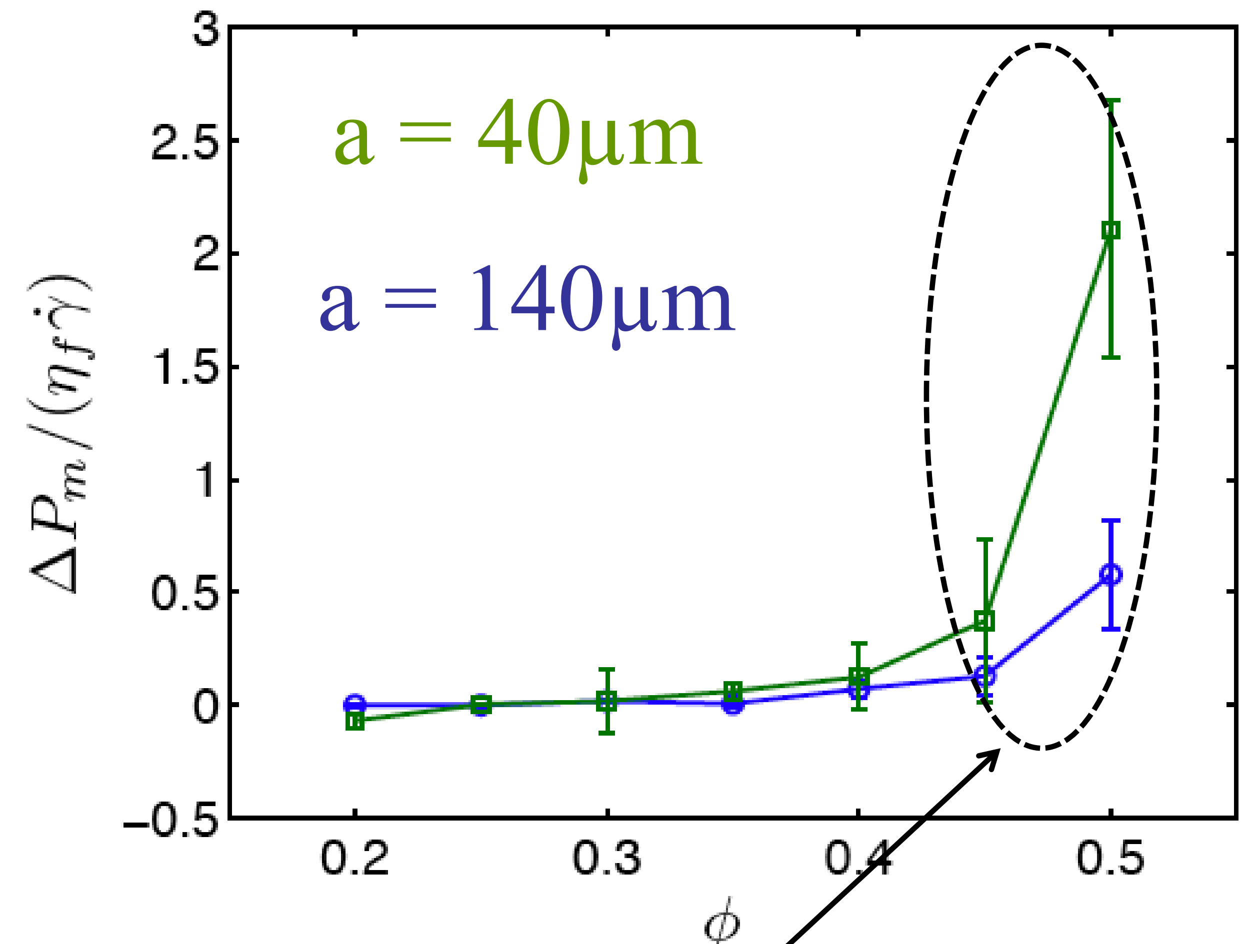




slope ⟶ Normalized induced pressures

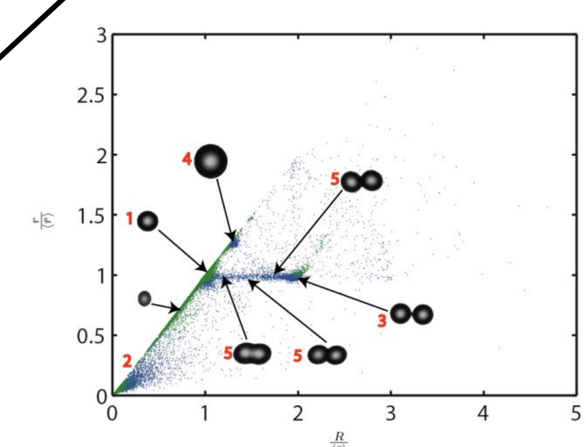
Normalized grid and membrane measurements: Variations with volume fraction





$\Delta P_m << \Delta P_g$

Effect of particles anisotropy?

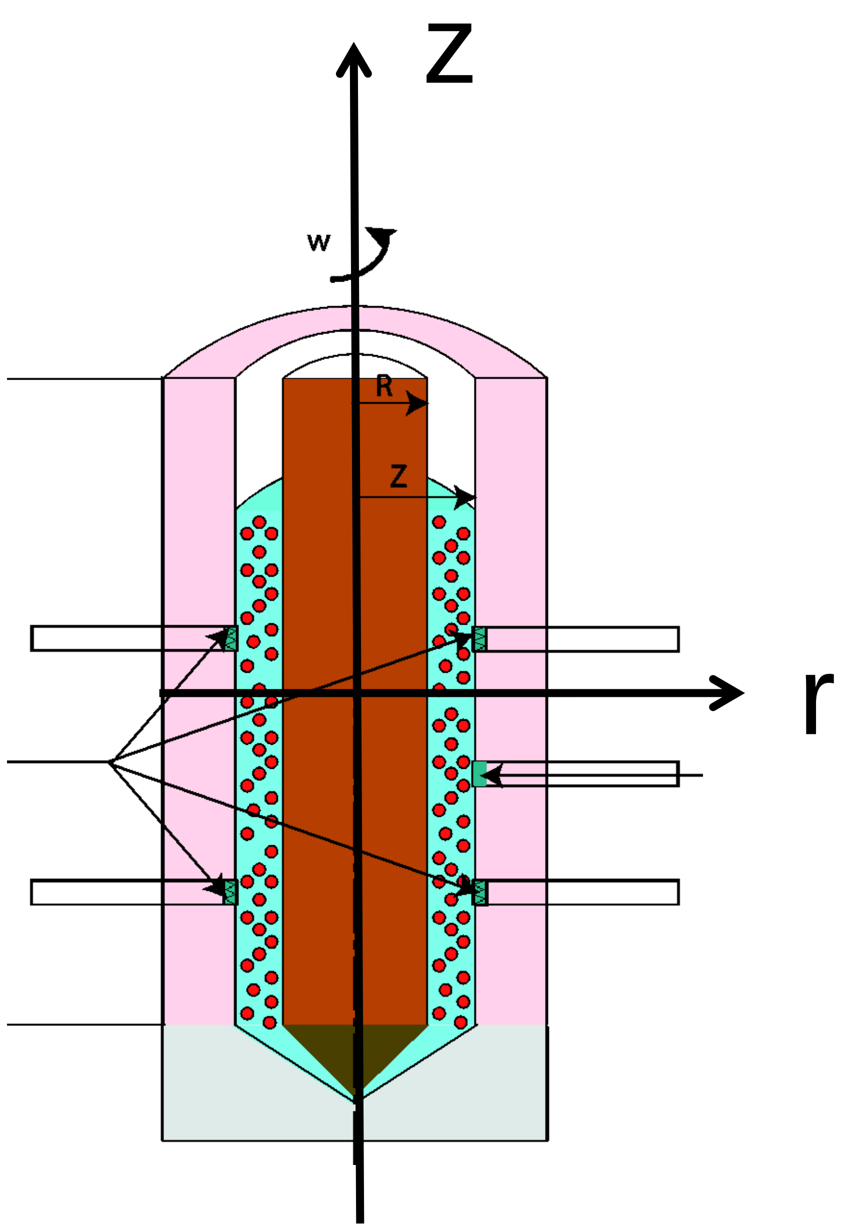


Which normal stress?

$$\boxed{\Delta P_g = -\Delta\Sigma^f_{rr}}$$

$$\boxed{\Delta P_m = -\Delta\Sigma^{susp}_{rr}}(= -\Delta\Sigma^p_{rr} - \Delta\Sigma^f_{rr})$$

$$\Rightarrow \quad \boxed{\Delta\Sigma^p_{rr} = \Delta P_g - \Delta P_m}$$



No vertical friction at the wall

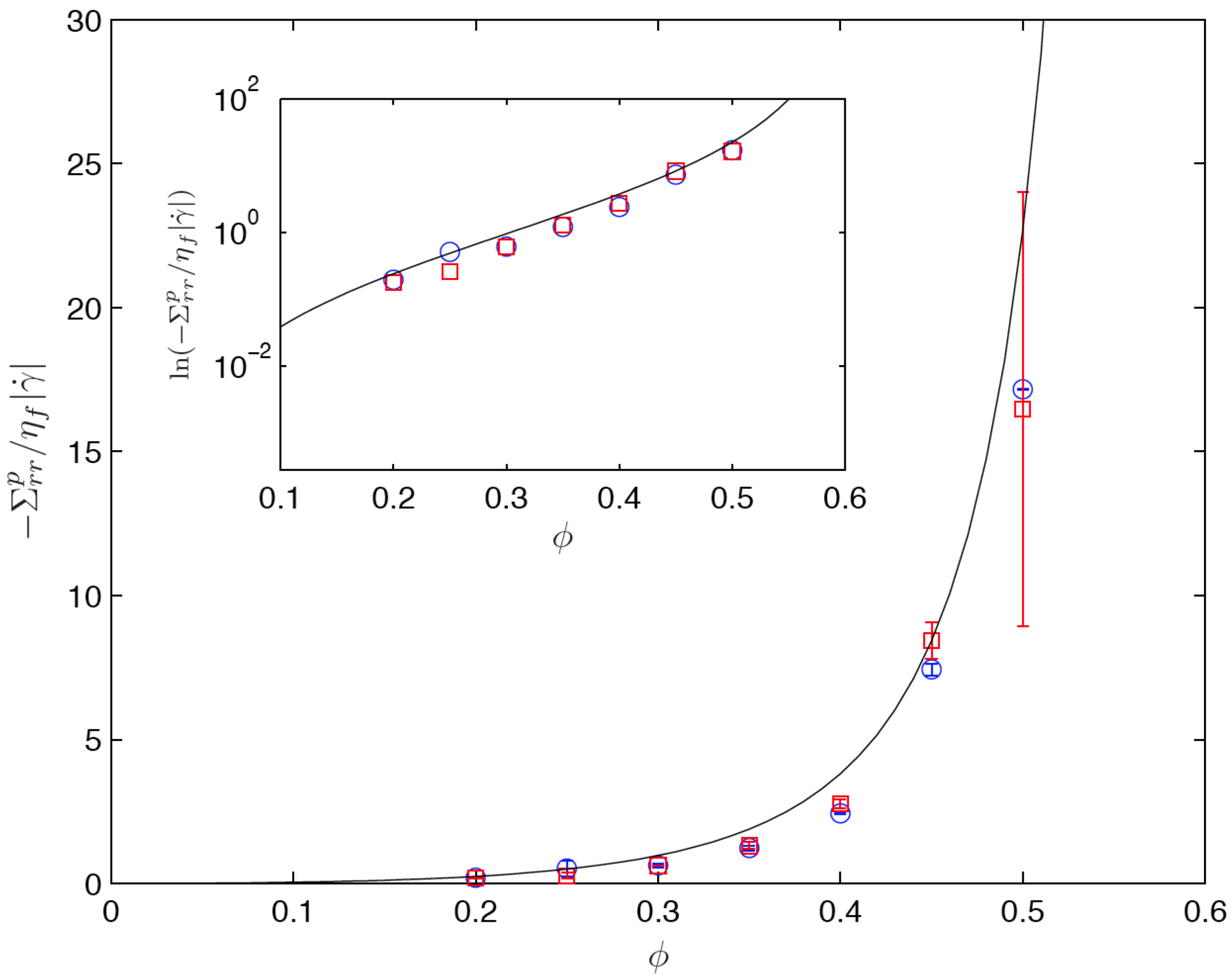
$$\Sigma^{susp}_{zr} = \Sigma^{susp}_{z\theta} = 0 \qquad \Sigma^{susp}_{zz} = -P_{hydrostatic}$$

$$\Delta\Sigma^{susp}_{zz} = 0$$

$$\Delta\Sigma^{susp}_{rr} - \Delta\Sigma^{susp}_{zz} = \boxed{N_2^{susp} = -\Delta P_m}$$

$$\Rightarrow \quad \boxed{N_2^{susp} < 0 \quad \text{and} \quad |\, N_2^{susp} \,| \ll \Delta\Sigma^p_{22}}$$

Radial particle stress vs particle pressure (Mills)

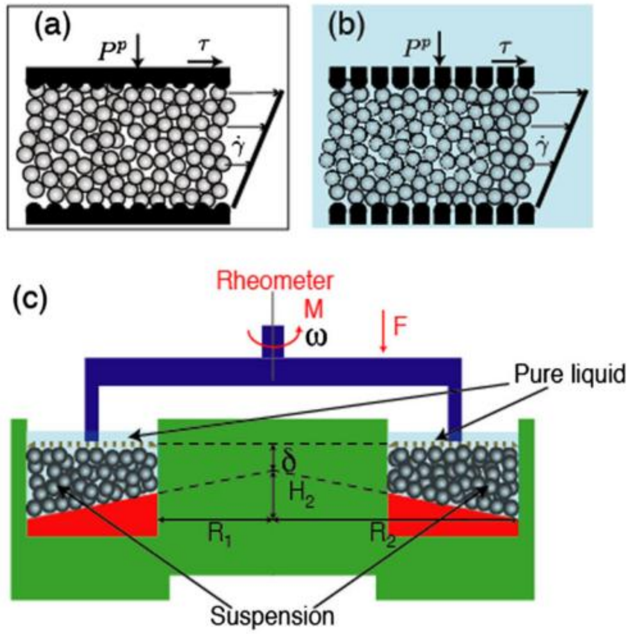


(Mills & Snabre 2009, Boyer et al. 2011)

$$p(\chi) = \frac{\Pi^p}{\eta_f\dot\gamma} == \frac{tr(\Sigma^p)}{3\eta_f\dot\gamma} = \frac{\chi^2}{(1-\chi)^2}$$

$$\chi = \frac{\Phi}{\Phi^\star}$$

$$\boxed{\Delta P_g \approx \Delta\Sigma^p_{22} \approx p(\chi)}$$



(*Boyer et al. 2011*)

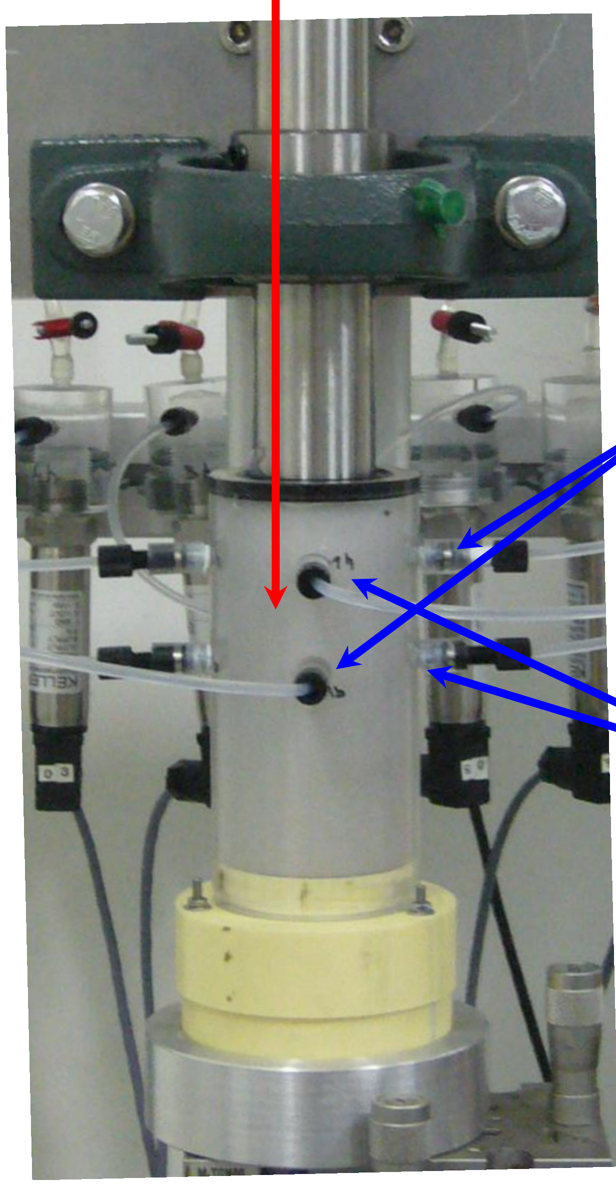
At low Φ:

(Brady &Morris 97) $\Sigma^p_{11} \propto \eta_f\dot\gamma\phi^2$

(Ingber *et al.* 2008) $D \propto a^2\dot\gamma\phi$

Set up used with bidisperse suspensions

Bidisperse suspension



$$\begin{cases} a_s = 40\mu m\ , & \Phi_s \\ a_l = 140\mu m\ , & \Phi_l \end{cases}$$

$$\Phi_{tot} = \Phi_s + \Phi_l = 0.4;\ 0.45;\ 0.5$$

$$\Phi_s / \Phi_{tot} \in [0\ ,\ 1]$$

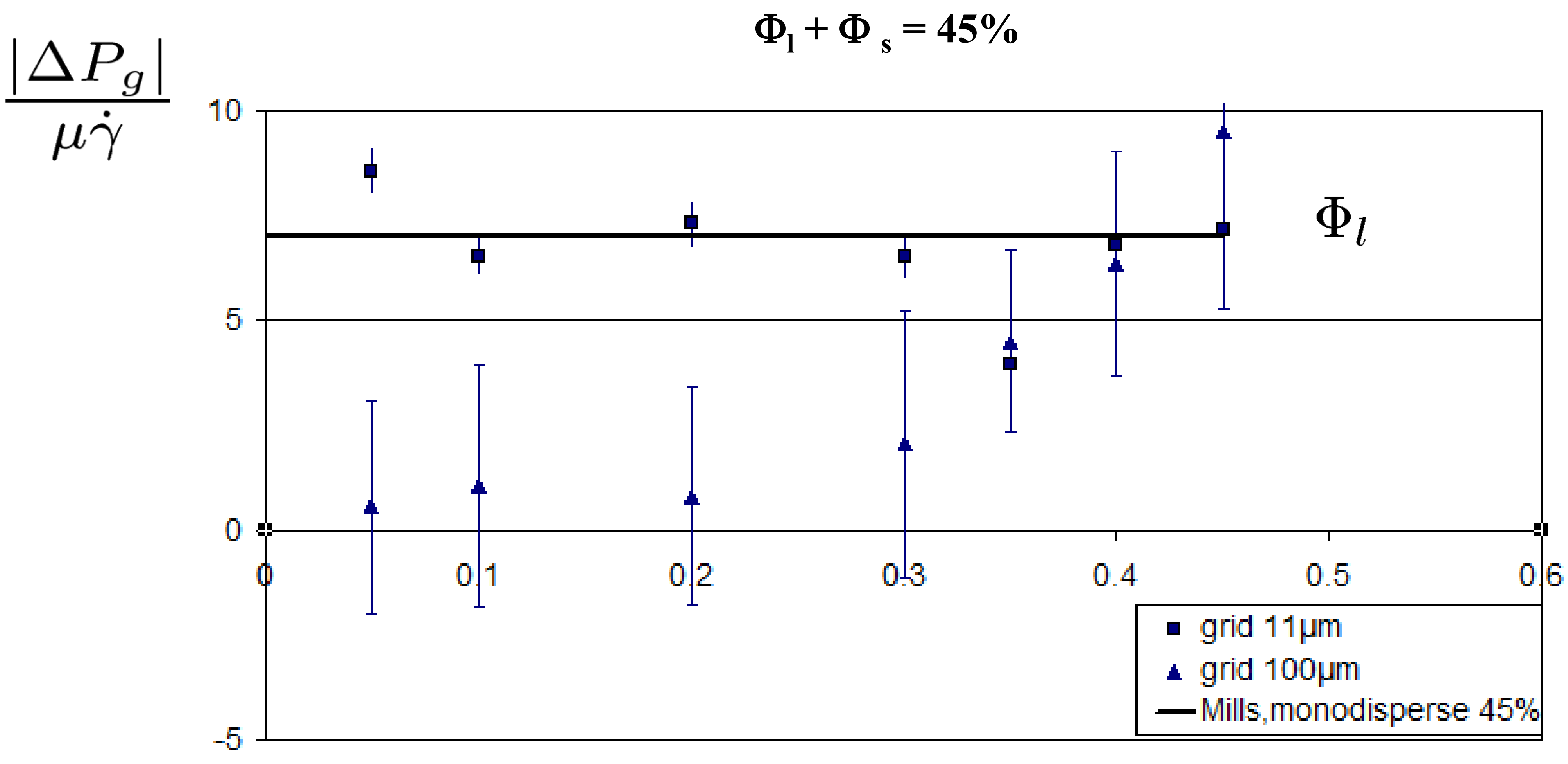
4 grids: Ø 11µm ⟹ Pressure in the fluid

⟹ **Shear-induced pressure of both particles ?**

4 grids: Ø 100µm ⟹ Pressure in an effective fluid ? (=suspension of small particles?)

⟹ **Shear-induced pressure of large particles ?**

Normalized grid measurements:

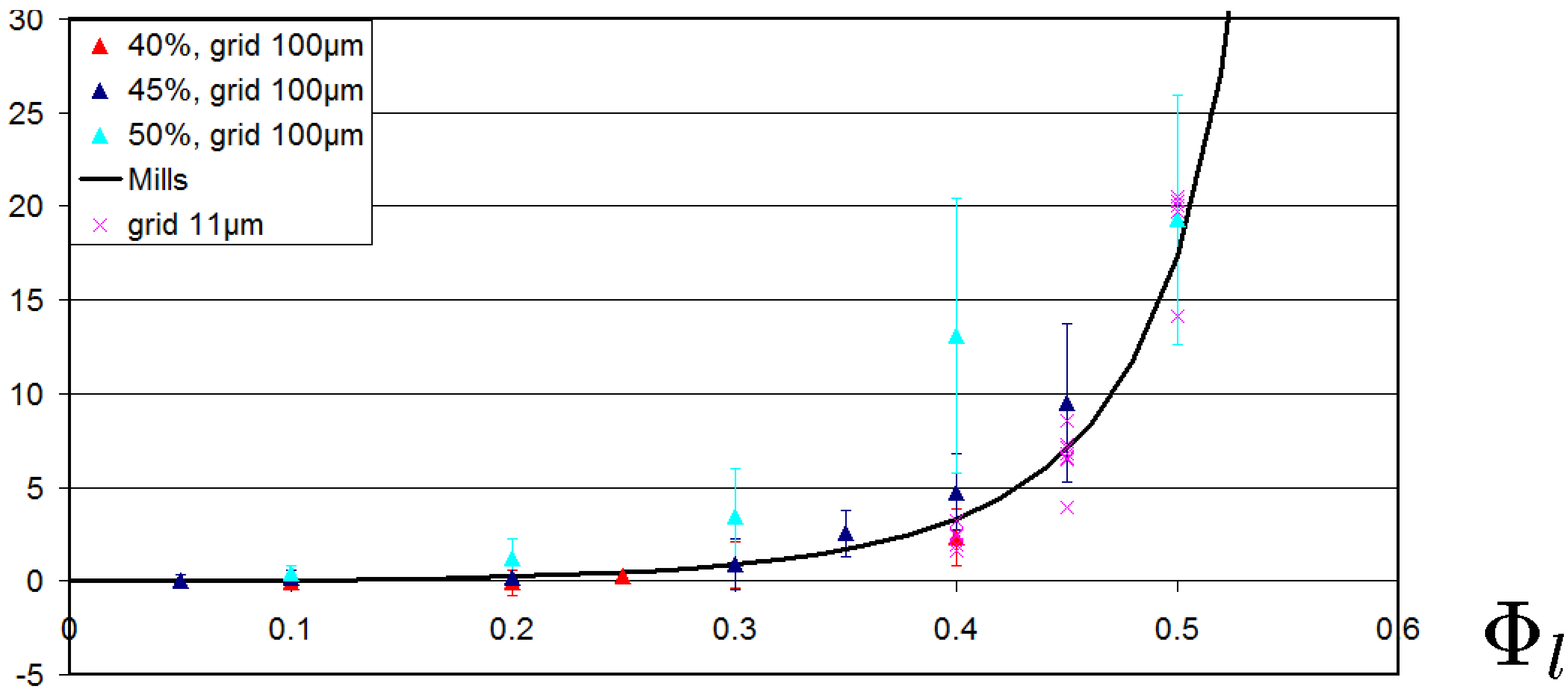


$$\frac{|\Delta P_g^{100\mu m}|}{\mu\dot{\gamma}}(\Phi_l;\Phi) \leq \frac{|\Delta P_g^{11\mu m}|}{\mu\dot{\gamma}}(\Phi)$$

Partial pressure (of large particles)? $= f(\Phi_l)$ **?**

Normalized grid pressure measurements in bidisperse suspensions:

$$\frac{|\Delta P_g^{11\mu m}|}{\mu\dot{\gamma}}(\Phi) \quad \text{and} \quad \frac{|\Delta P_g^{100\mu m}|}{\mu_{eff}\dot{\gamma}}(\Phi_l) \text{ with } \mu_{eff} = \mu_{susp}\left(\frac{\Phi_s}{1-\Phi_l}\right)$$



- **Partial pressure** of large particles, in an effective suspending fluid, i.e. the suspension of small particles at: $\Phi_{susp} = \left(\frac{\Phi_s}{1-\Phi_l}\right)$
- **Grid pressures** give total or partial « shear-induced osmotic pressure »,

Conclusions and open issues:

- **2nd normal particle stress in monodisperse and bidisperse suspensions**
- **contribution of the large particles to the 2nd normal particle stress**
- **all described by:**

$$\frac{\Pi^p}{\mu\dot{\gamma}} == \frac{tr(\Sigma^p)}{3\Sigma^f_{12}} = \frac{\chi^2}{(1-\chi)^2}$$ **(*Mills & Snabre 2009, Boyer et al. 2011)***

Has to be tested with migration experiments

$$\overrightarrow{j_i} = \frac{-2a_i^2}{9\eta} f_i(\phi_i;\phi_j)\overrightarrow{\nabla}.\Pi_i^p = \phi_i(\overrightarrow{u_i^p} - \overrightarrow{U})$$ **with** $f_i(\phi_i;\phi_j)$ **?**

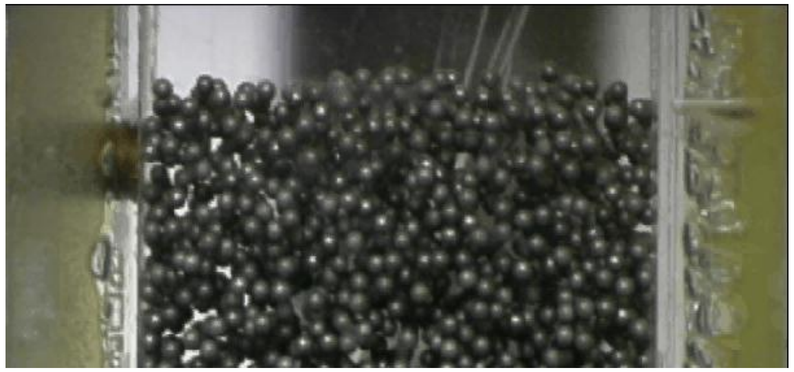
- **Different evaluations of particle stresses componants due to?**
- **Particle size (and shape) distributions?**
- **Rugosity?**
- **Structurations under shear, effect of large scale clusters?**
- **Differences in stresses in the suspension bulk and on a wall ?**

(cf swapping trajectories, *Zurita-Gotor et al. 2007, Zarraga Leighton 2002*)

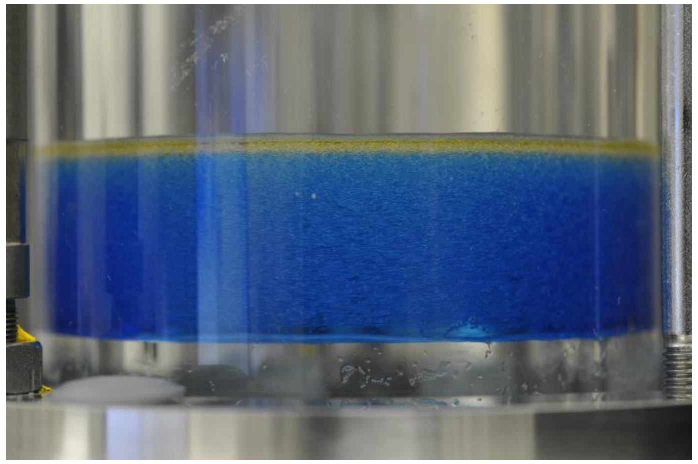
Needs of Simulations!!!

And also…

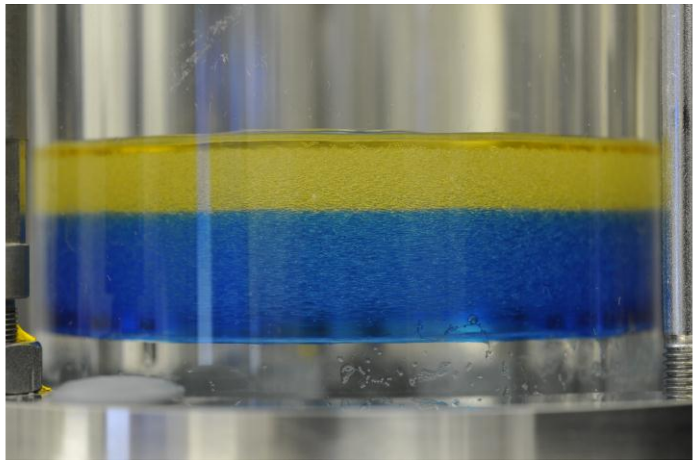
Migration of buoyant particles



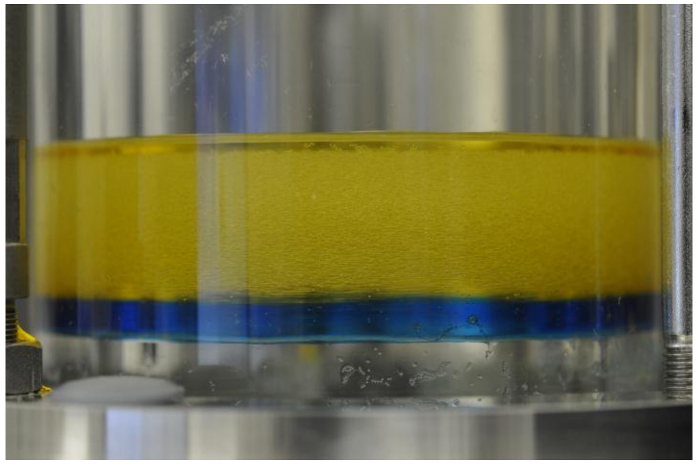
Mixing (self diffusion coefficient) inside one phase



$\dot{\gamma} = 2s^{-1}$



$\dot{\gamma} = 0s^{-1}$



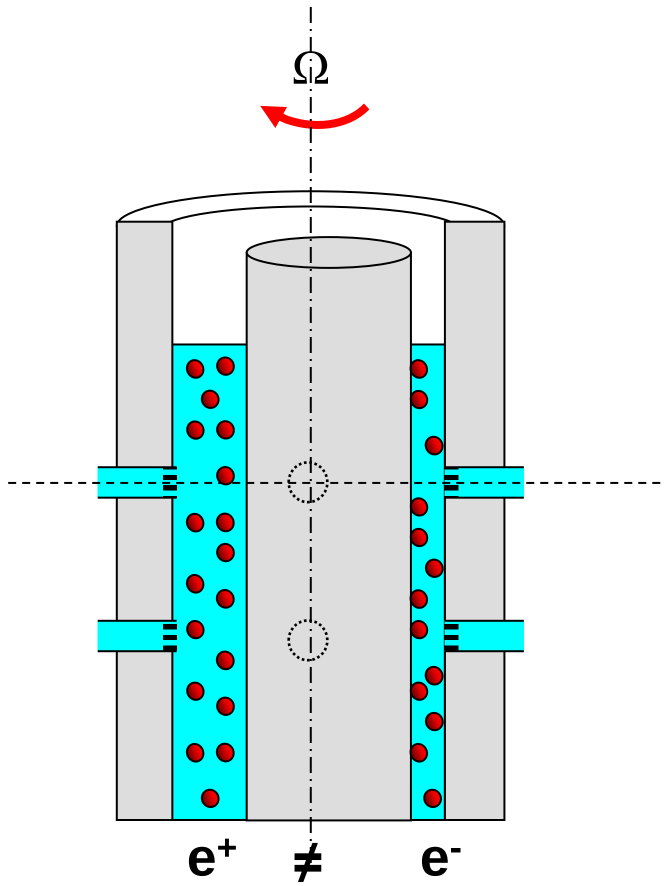
$\dot{\gamma} = 5s^{-1}$

(movie accelerated by a factor 70)

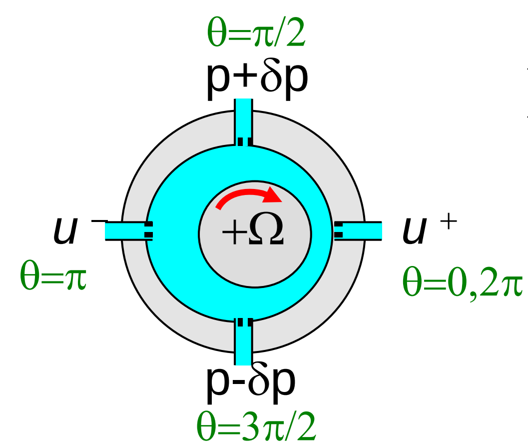
Eccentricity drawbacks

$e \neq$ cst ⟹ **$\dot{\gamma} \neq$ cst**

⟹ **P(θ) ≠ cst + Particle migration**



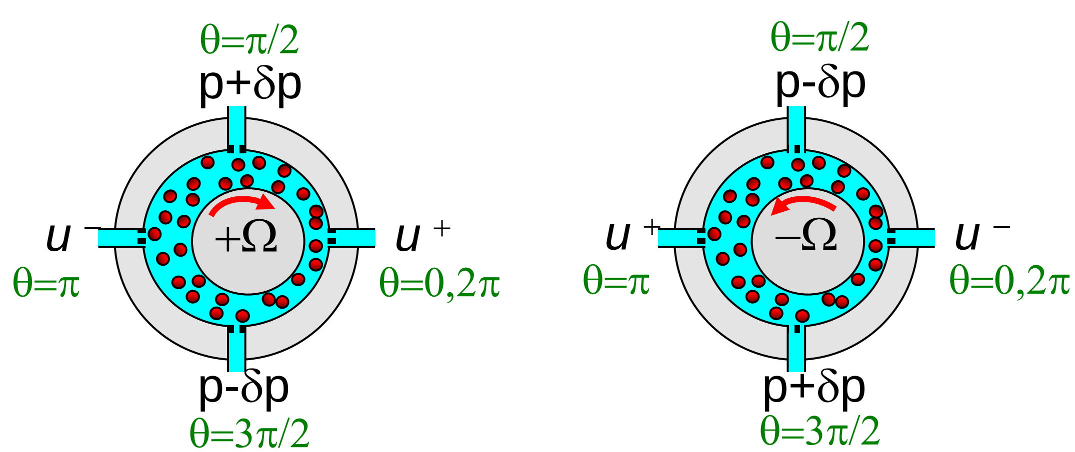
- With the fluid alone:



Matching of the axes of the cylinders, by minimising δp

δp ≈ cst ⟹ e ≈ cst

- With the suspension:



Averaging pressures at +Ω and -Ω

$$2\,P_g(\Omega) = P(+\Omega)+P(-\Omega)$$

What can we measure in a bidisperse suspension?

$$\boxed{\Delta P_g^{11\mu m} = -\Delta\Sigma_{rr}^{f} \qquad\qquad \Delta P_g^{100\mu m} = -\Delta\Sigma_{rr}^{susp(s)}}$$

$$\Delta\Sigma_{rr}^{susp(s+l)} = \Delta\Sigma_{rr}^{f} + \Delta\Sigma_{rr}^{p(s)} + \Delta\Sigma_{rr}^{p(l)}$$

susp(s) p(s+l)

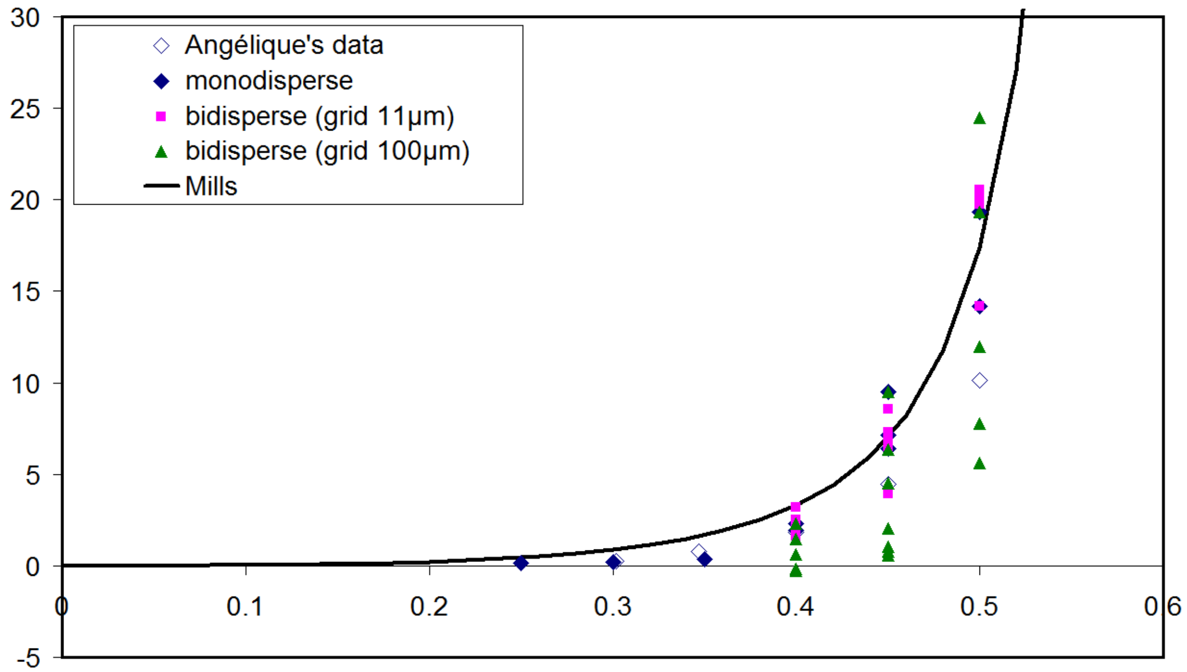
$$\Delta\Sigma_{rr}^{susp(s+l)} = \Delta\Sigma_{rr}^{f} + \Delta\Sigma_{rr}^{p(s+l)} \approx 0 \quad \Rightarrow \quad \boxed{\Delta\Sigma_{rr}^{p(s+l)} \approx \Delta P_g^{11\mu m}}$$

$$\Delta\Sigma_{rr}^{susp(s+l)} = \Delta\Sigma_{rr}^{susp(s)} + \Delta\Sigma_{rr}^{p(l)} \approx 0 \quad \Rightarrow \quad \boxed{\Delta\Sigma_{rr}^{p(l)} \approx \Delta P_g^{100\mu m}}$$

and $$\boxed{\Delta\Sigma_{rr}^{p(s)} = \Delta P_g^{11\mu m} - \Delta P_g^{100\mu m}}$$

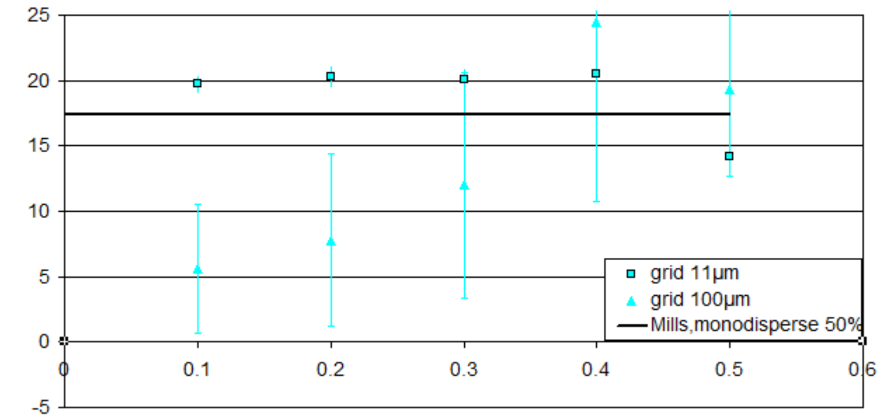
Normalized grid measurements:

$$\frac{|\Delta P_g|}{\mu \dot{\gamma}}$$



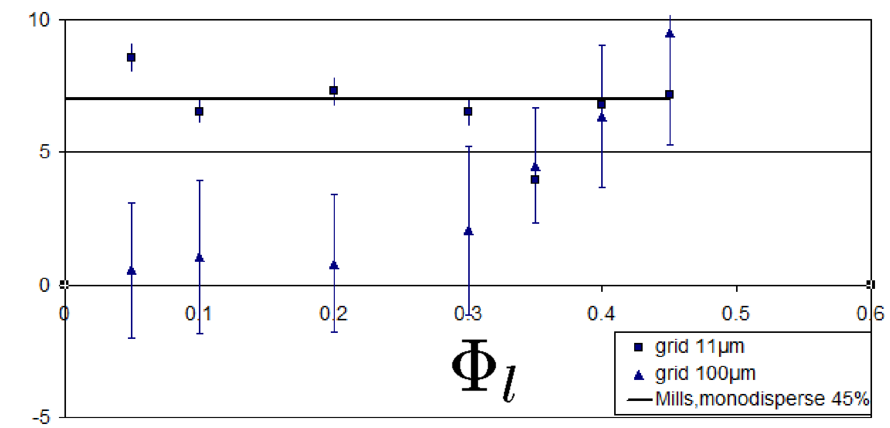
$$\Phi = \Phi_s + \Phi_l$$

$\Phi_l + \Phi_s = 50\%$

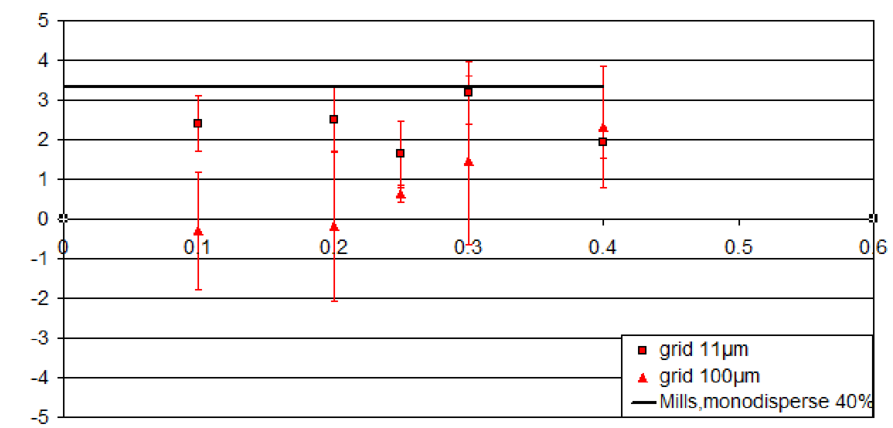


$$\frac{|\Delta P_g|}{\mu \dot{\gamma}}$$

$\Phi_l + \Phi_s = 45\%$



$\Phi_l + \Phi_s = 40\%$



$$\Rightarrow \frac{|\Delta P_g^{100\mu m}|}{\mu \dot{\gamma}}(\Phi_l;\Phi) \leq \frac{|\Delta P_g^{11\mu m}|}{\mu \dot{\gamma}}(\Phi)$$

$\Rightarrow$ **Partial pressure (of large particles)?** $= f(\Phi_l)$ **?**