Preconditioning for electromagnetic scattering via localization

Jutta Steiner

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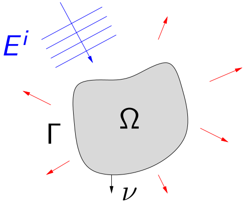
Joint work with
F. Alouges (CMAP) and D. Levadoux (Onera)

Summary

- Time-harmonic Maxwell equations
- General ideas of Boundary Integral Methods
- A well-conditioned scheme for electromagnetic scattering
- Numerical results and future directions

Electromagnetic scattering

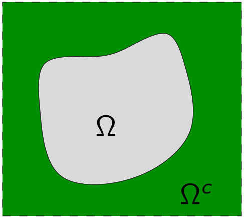
Scattering of time-harmonic waves $E^i(x)e^{-\omega t}$



$$\nabla \times \nabla E - k^2 E = 0 \text{ in } \Omega^c$$
$$\gamma_D E = -\nu \times E_0 \text{ on } \Gamma$$
$$\lim_{|x|\to\infty} ((\nabla \times E) \times x - ikE|x||) = 0$$

"Dirichlet" trace $\gamma_D = \nu\times$

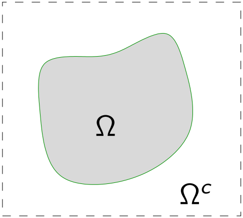
Boundary integral method for PDE-problems



Direct discretization of

$$Lu = 0 \text{ in } \Omega^c$$
$$\gamma_D u = u_0 \text{ on } \Gamma$$

$\rightsquigarrow$ sparse matrix equation for n-dim domain (unbounded!)



Parametrization $u = V(g)$ with

$$LV = 0 \text{ in } \Omega^c$$
$$\gamma_D V(g) = u_0 \text{ on } \Gamma$$

$\rightsquigarrow$ dense matrix equation for $n-1$-dim boundary

Natural choices for the potential

$$\left.\begin{array}{r}\text{Single-layer potential } \mathcal{T} = \frac{1}{ik}\nabla \times (\nabla(\mathcal{G}_k u)) \\ \text{Double-layer potential } \mathcal{K} = \nabla \times (\mathcal{G}_k)\end{array}\right\}$$

where $\mathcal{G}_k u = \int g_k(x-y)u(y)dy$ and $g_k = -\frac{1}{4\pi}\frac{e^{ik|x|}}{|x|}$

Suitable coupling

$$E = (\mathcal{T}\,\mathrm{R}_1 - \mathcal{K}\,\mathrm{R}_2)g$$

Classical choices for the coupling

	R_1	R_2
EFIE	1	0
MFIE	0	1
CFIE	λ	$1-\lambda$

lead to ill-conditioned and dense systems

$\rightsquigarrow$ complexity $\sim \mathcal{O}(n \times N^2)$

An intrinsically well-conditioned formulation

...using representation formula

$$E = \mathcal{T}\gamma_N E - \mathcal{K}\gamma_D E$$

...and Dirichlet-to-Neumann operator $\mathsf{Y} : \gamma_D E \to \gamma_N E$

$$E = \mathcal{T}\,\mathsf{Y}\,\gamma_D E - \mathcal{K}\,\gamma_D E$$

...boundary-integral operator reduces to identity

$$\gamma_D E = \underbrace{\gamma_D(\mathcal{T}\,\mathsf{Y} - \mathcal{K})}_{=\mathrm{id}}\,\gamma_D E$$

Y not explicitely known $\rightsquigarrow$ construct suitable approximations $\tilde{\mathsf{Y}}$

Crucial observation [Nédélec]

Define

$$\mathsf{T} := \gamma_D \mathcal{T}$$
$$\mathsf{K} - \frac{1}{2} := \gamma_D \mathcal{K}$$

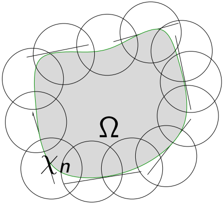
Calderon formula

$$\underbrace{\mathsf{T}^2}_{\text{order } 1} = \underbrace{\mathsf{K}^2}_{\text{order } -1} - \frac{1}{4}$$
$$= \text{compact perturbation of identity}$$

Formal expansion

$$\mathsf{Y} = -2\,\mathsf{T}(1 - 2\,\mathsf{K})^{-1}$$
$$\approx -2\,\mathsf{T} + h.o.t.$$

Localization for high frequencies [Borel]



Dirichlet-to-Neumann operator on a plane

$$\mathrm{Y} = -2\,\mathrm{T}$$

Local approximation using partition of unity $\sum \chi_n^2 = 1$

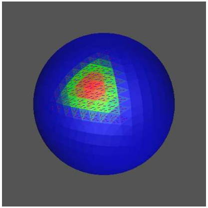
$$\tilde{\mathrm{Y}} = \sum_n \chi_n \, \mathrm{Y_n} \, \chi_n$$

Library at Onera

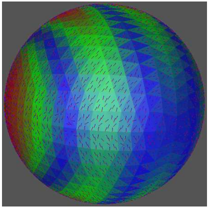
So far: Implementation of

- Classical methods (EFIE,MFIE)
- Helmholtz decomposition (smooth surface)
- Fast Multipole Method (FMM)

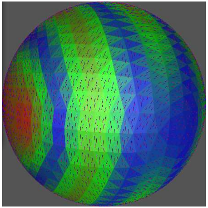
Results using localization



χ_n



Incident wave



Solution

Observe

$$\text{diam supp } \chi_n \text{ and } \lambda \text{ are comparable}$$

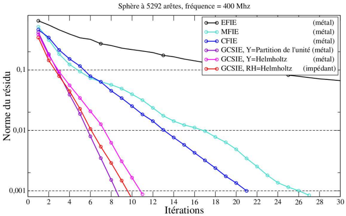


FIGURE 2.5 – Résultats numériques pour une fréquence de 400 MHz, sur une sphère maillée avec 5292 arêtes.

Thesis J. Bourguignon

Future directions

- Geometric singularities
- Consider higher order expansion of $Y = -2\,\mathsf{T}(1 - 2\,\mathsf{K})^{-1}$
- Open surfaces