Infrared aspects of the Nelson model on a static space-time

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joint work with C.Gérard, F.Hiroshima, A.Suzuki

Quantization

QM=n degrees of freedom

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M phase space, $\quad M = \mathbb{R}^{2n}$

$p_i, q_i : M \to \mathbb{R}$
symplectic coordinates

$H = H(p, q)$

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$\mathcal{H}$ Hilbert space

$p_i, q_i : D \subset \mathcal{H} \to \mathcal{H}$
$[p_i, iq_j] = \delta_{ij}\mathbb{1}, \quad [p_i, p_j] = [q_i, q_j] = 0$
$\Leftrightarrow a_j := \frac{(p_j + iq_j)}{\sqrt{2}}, \quad a_j^* := \frac{(p_j - iq_j)}{\sqrt{2}}$
$[a_i, a_j^*] = \delta_{ij}\mathbb{1}, \quad [a_i, a_j] = [a_i^*, a_j^*] = 0$

H "= " $H(p, q) = H(a^*, a)$

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H well defined as a self-adjoint operator

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Definition of H as a self-adjoint operator:
introduction of cutoff functions

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$$(|\bar{g}|^{-\frac{1}{2}}\partial_\mu \bar{g}^{\mu\nu}|\bar{g}|^{\frac{1}{2}}\partial_\nu + m^2 + \theta R(x))\varphi(t,x) + V(x,\varphi(x,t)) = 0$$

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$$\partial_t^2\varphi(t,x) + (g(x)(D_j a_{ij}(x)D_i)g(x) + c(x))\varphi(t,x) + V_1(x,\varphi(x,t)) = 0.$$

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$$\omega^2 := g(x)(D_j a_{ij}(x)D_i)g(x) + c(x) \quad g(x), a(x) \to 1, c(x) \to m^2$$

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Nelson-type models (standard)

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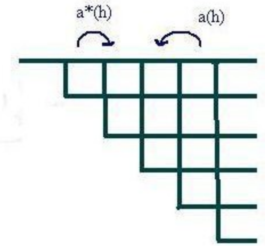

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$$a^*(h)u := \sqrt{n+1}\, u \otimes_S h$$

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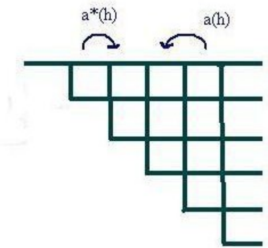

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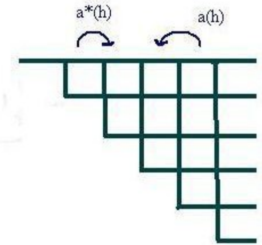

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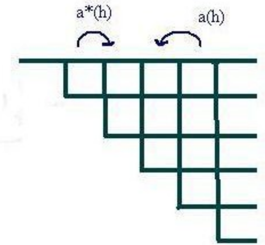

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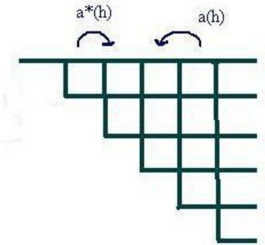

$$\phi(h) = \frac{1}{\sqrt{2}}(a^*(h) + a(h)) \quad h \in \mathfrak{h}$$

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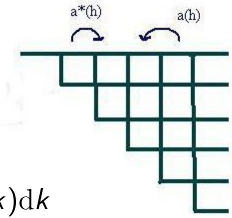
$H = K \otimes 1\!\!1 + 1\!\!1 \otimes \mathrm{d}\Gamma(\omega) + \lambda\Phi(v)$

$\mathrm{d}\Gamma(\omega)|_{\bigotimes_{\mathrm{s}}^{n}\mathfrak{h}} := \sum_{i=1}^{n} \omega_i$

$\omega_N^2 = -\triangle + m^2 \quad m \geq 0$



$\Phi(v) = \int v(k) \otimes a^*(k) + v^*(k) \otimes a(k)\mathrm{d}k$

$v_N(k) := \frac{e^{-\mathrm{i}k\mathrm{x}}}{\omega(k)^{1/2}} \hat{\rho}(k)$

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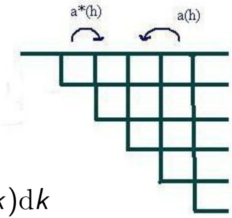

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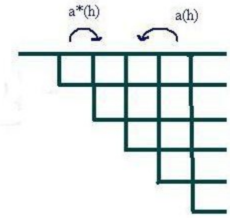

Nelson-type models (standard)

General situation

$\sigma(K)$

Nelson-type models (standard)

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si $m > 0$

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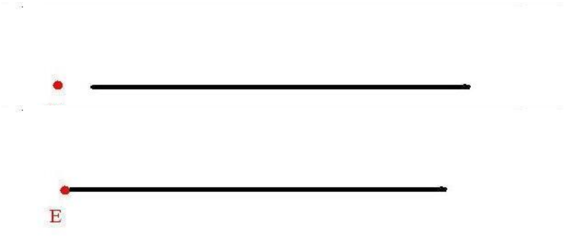

if $\|\omega^{-1} v\|_{B(\mathcal{K},\mathcal{K}\otimes\mathfrak{h})} < \infty$ (infrared regular condition)
E is an eigenvalue in the Fock representation [DeGe,BruDe]

otherwise (Nelson)
E is NOT an eigenvalue in the Fock representation [LMS,DeGe]
E is an eignevalue in another representation[Arai]

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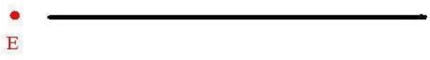
$\Phi(v) = a^*(v) + a(v)$
$v = \omega^{-1/2}\rho(x - \mathrm{x})$

$K = -\partial_j A_{ij}(x)\partial_i) + V(x)$
$0 < C_0 < A_{ij}(x) \leq C_1$

$\omega^2 = g(x)(-\partial_j a_{ij}(x)\partial_i)g(x) + m^2(x) = h_0 + m^2(x)$
$0 < c_0 < g(x), a_{ij}(x) \leq c_1, \quad m^2(x) \to m_\infty \geq 0$

$(K + 1)^{-1}$ compact
$\inf \sigma(\omega) = m \geq 0$



Theorem

Let $m = \inf \sigma(\omega)$, *then*

$$\inf \sigma_{\text{ess}}(H) \subset [\inf \sigma(H) + m, +\infty[.$$

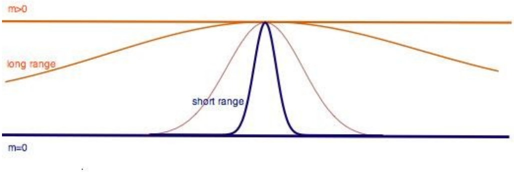
In particular, if $m > 0$, H *has a ground state.*

Theorem

If $\omega^2 = h_0 + m^2(x)$, $m^2(x) \geq a\langle x\rangle^{-2}$ for some $a > 0$, then H has no ground state.

Theorem

If $\omega^2 = h_0 + m^2(x)$, $m^2(x) \leq C\langle x\rangle^{-2-\varepsilon}$, for some $C, \epsilon > 0$, then H has a ground state.



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If $m^2(x) \geq a\langle x\rangle^{-2}$ *then*

$$\omega^{-\beta} \leq \langle x\rangle^{\beta+\varepsilon}, \ \textit{for all } 0 < \beta \leq d, \ \textit{where } d \geq 3.$$

In particular this implies $\|\omega^{-1}v\|_{B(\mathcal{K},\mathcal{K}\otimes\mathfrak{h})} < \infty$ *(and* $\omega^{-1}v(K+1)^{-1}$ *compact).*

obtained using [Porper-Eidel'man]

Theorem (existence abstract version)

Let $\mathcal{H} := \mathcal{K} \otimes \Gamma(\mathfrak{h})$ *and*

$$H = K \otimes \mathbb{1} + \mathbb{1} \otimes \mathrm{d}\Gamma(\omega) + \lambda \Phi(v),$$

with $\omega : \mathfrak{h} \to \mathfrak{h}$ *self-adjoint* , $\omega \geq 0$, $0 \notin \sigma_{\rm pp}(\omega), v \in B(\mathcal{K}, \mathcal{K} \otimes \mathfrak{h})$.
Assume there exists a self-adjoint operator $d \geq 0$ *on* $\mathfrak{h}$ *such that*

$$[F(\frac{d}{R}), \omega_\mu] = O(R^{-1}), \quad \omega^{-\frac{1}{2}} F(\frac{d}{R}) v_\mu (K+1)^{-\frac{1}{2}} = O(R^0)$$

and $(K+1)^{-1}$, $\omega^{-1} v (K+1)^{-\frac{1}{2}}$ *are compact.*
Then H *has a ground state.*

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If $\mathfrak{h} = L^2(\mathbb{R}^3, dx)$, one can replace $(K+1)^{-1}$ compact with a binding condition (follow [Griesimer] for exponential decay).

Proof (idea)

H_μ has a ground state ψ_μ

$H_\mu \to H$ in the norm resovent sense

Theorem (General lemma, Arai)

If

$$\psi_\mu \to_w \psi \neq 0,$$

Then ψ is a ground state for H.

Key bound:

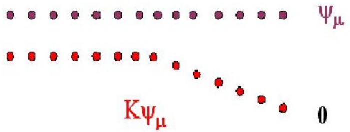
$(\psi_\mu, N\psi_\mu) < C$ *uniformely in* $\mu \Longrightarrow$ *existence*

$(\psi_\mu, N\psi_\mu) < C$ *uniformly in* $\mu \Longrightarrow$ *existence*

proof:

$K := \chi(N \leq \lambda)\chi(H_0 \leq \lambda)\Gamma(F_R)$ compact

$\lim_{\mu\to\infty} K\psi_\mu = 0$



Consider first the FLAT CASE:

$$N = \int a^*(k)a(k)dk$$

$$\begin{aligned}(\psi, N\psi) &= \int \|a(k)\psi\|^2 dk \\ &= \int \|(H - E + \omega(k))^{-1} v(k)\psi\|^2 dk \\ &\leq \int \|\omega(k)^{-1} v(k)(K+1)^{-1}\|^2 dk\end{aligned}$$

With a metrics: you have to avoid to decompose in k

$$N = A^* A$$

A abstract pullthrough operator [Bruneau,Derezinski]

$$A : \Gamma(\mathfrak{h}) \to \Gamma(\mathfrak{h}) \otimes \mathfrak{h}$$

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$$A : \Gamma(\mathfrak{h}) \to \Gamma(\mathfrak{h}) \otimes \mathfrak{h}$$

$$\begin{aligned}(\psi, N\psi) &= \|A\psi\| \\ &= \|(H \otimes \mathbb{1} - E + \mathbb{1} \otimes \omega)^{-1} v\psi\| \\ &\leq C\|\omega^{-1} v(K+1)^{-\frac{1}{2}}\|\end{aligned}$$

$$\begin{array}{llll} U: & \mathcal{K}\otimes\Gamma(\mathfrak{h}) & \to & L^2(M,m) \text{ m probablilty measure} \\ & \psi\otimes\Omega & \mapsto & 1 \end{array}$$

Set

$$\gamma(t) = \frac{(1, e^{-tH}1)}{\|e^{-tH}1\|}$$

Lemma

Set $E := \inf \sigma(H)$. *Then*

$$\lim_{t\to\infty} \gamma(t) = \|\mathbb{1}_E(H)1\|^2$$

In particular if $\lim_{t\to\infty}\gamma(t) = 0$ *then* H *has no ground state.*

One can compute this quantity explicitly, and it depends on $(\rho_{\mathrm{x}}, \frac{e^{-t\omega}}{2\omega}\rho_y)$ to be estimated using [Semenov]

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Theorem

There exists a family of unitary operators U_κ on $\mathcal{H}$ and a self-adjoint operator H_∞ such that:

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