SHAPE OPTIMIZATION WITH CONVEXITY CONSTRAINT

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ABSTRACT
We consider the following general shape optimization problem:

\[ J(K_0) = \min \{ J(K), K \text{ convex } \subset \mathbb{R}^d \}, \]

where \( J \) is a shape functional. Many open problems, from Functional Analysis, Convex geometry or PDE can be formulated in this setting; the most famous ones are probably the Mahler conjecture and the Polyà-Szegö conjecture. They concern the research of a minimizer, with \( J(K) = |K||K^*| \) the product of the volumes of \( K \) and of its dual body \( K^* \) for the first one, and with \( J(K) = \text{Cap}(K)^2/P(K) \) the ratio between the electrostatic capacity and the surface area \( (K \subset \mathbb{R}^3 \text{ here}) \), for the second one. We focus on the way to analyze the convexity constraint on the shapes, using methods from Calculus of Variations, and to deduce some informations on optimal shapes. In dimension 2, we show a large class of functionals \( J \) leading to polygonal optimal shapes. In higher dimension, we give a similar weaker result (which applies to both conjecture).

1. REFERENCES