RECONSTRUCTION OF A PERFECTLY CONDUCTING OBSTACLE COATED WITH A THIN DIELECTRIC LAYER

Nicolas Chaulet

(joint work with Laurent Bourgeois and Houssem Haddar)

Project team DEFI, INRIA Saclay, Palaiseau, France

nicolas.chaulet@inria.fr

ABSTRACT

We consider the inverse scattering problem consisting in the identification of a perfectly conducting obstacle coated with a thin layer of a dielectric material from few far field measurements at a fixed frequency. We propose in this talk an iterative procedure to retrieve the scatterer and the coating. Solving the forward problem is computationally expensive in this case and we shall use an approximate model which involves a generalized impedance boundary condition (GIBC) during the reconstruction process.

1. AN APPROXIMATE MODEL FOR THIN LAYERS

For the forward problem we consider the scattering of an incident plane wave of transverse electric polarization with a perfectly conducting obstacle coated by a dielectric material of constant non-zero permittivity $\epsilon$ and permeability $\mu$ immersed in a reference medium ($\epsilon = 1$ and $\mu = 1$). Moreover let us assume that the geometry and the physical coefficients are invariant in one direction. In the harmonic regime, the full electromagnetic problem can be reduced to the following 2D

\[ \begin{cases} 
\text{div}(\epsilon^{-1}\nabla u_\delta) + \mu k^2 u_\delta = 0 & \text{in } \Omega_\delta \\
\Delta u_\delta + k^2 u_\delta = 0 & \text{on } \Omega \\
[\epsilon^{-1}\partial_n u_\delta] = 0, \quad [u_\delta] = 0 & \text{on } \Gamma \\
\epsilon^{-1}\partial_n u_\delta = 0 & \text{on } \Gamma_\delta,
\end{cases} \]

(1)

where $k$ is the wave number and $\delta \in L^\infty(\Gamma)$ is the width of the coating which can be non-constant (see figure 1 for the details of the geometry). The scattered field also satisfies the Sommerfeld radiation condition

\[ \lim_{R \to \infty} \int_{|x|=R} |\partial_r u_\delta^s - iku_\delta^s|^2 ds = 0. \]

Following [1] we can derive an approximate model of order 1 (i.e. $||u_\delta - u_\delta^s|| \leq C\delta^2$): find $\tilde{u}_\delta = \tilde{u}_\delta^s + u^i \in \tilde{V} := \{v \in D'(\Omega), \varphi v \in H^1(\Omega) \forall \varphi \in D(\mathbb{R}^2)\}$ such that

\[ \begin{cases} 
\Delta \tilde{u}_\delta + k^2 \tilde{u}_\delta = 0 & \text{in } \Omega \\
\partial_n \tilde{u}_\delta + \partial_s (\epsilon^{-1} \delta \partial_s \tilde{u}_\delta) + \delta k^2 \mu \tilde{u}_\delta = 0 & \text{on } \Gamma \\
\lim_{R \to \infty} \int_{|x|=R} |\partial_r \tilde{u}_\delta^s - ik\tilde{u}_\delta^s|^2 ds,
\end{cases} \]

(2)

where $s$ is the curvilinear abscissa along $\Gamma$. In the following we will assume that $\Re(\epsilon) > 0$, $\Im(\epsilon) \geq 0$ and $\Im(\mu) \geq 0$ so that problems [1] and [2] have a unique solution (see [2]).

The idea is to use such an approximate model to retrieve the geometry and the parameters of the coating.
2. SHAPE OPTIMIZATION TECHNIQUE TO RETRIEVE A COATED OBSTACLE

2.1. Setting of the inverse problem

From now on we only consider scattering of incident plane waves \( u^i(\hat{x}, \hat{d}) = e^{ikr} \) of incident direction \( \hat{d} \) and we recall that the radiating part of any solution to (1) or (2) has the following asymptotic behaviour
\[
u^s(x, \hat{d}) \sim \frac{e^{ikr}}{r} \left( u^\infty(\hat{x}, \hat{d}) + O\left(\frac{1}{r}\right)\right) \quad r \rightarrow +\infty
\]
uniformly for every \( \hat{x} = x/r \in S^1 \) where \( S^1 \) denotes the unit circle. The inverse problem consists in finding the scatterer \( D \) and the parameters of the layer \( (\mu, \delta) \) (\( \epsilon \) is assumed to be fixed and known for uniqueness reasons) from the knowledge of \( u^\infty_{\delta, \text{obs}}(\cdot, \hat{d}_j) \) for a finite number of incident directions \( (\hat{d}_j)_{j=1}^{\cdots J} \). Here \( u^\infty_{\delta, \text{obs}} \) is the far field associated with the scattered field solution to the exact problem (1). The use of iterative techniques based on the resolution of the exact model would be too computationally expensive, that is why we introduced the approximate model (2). We define the far field map for the approximate model by
\[
T: \quad (\Gamma, \delta, \mu, \hat{d}) \rightarrow u^\infty_{\delta, \text{obs}}(\cdot, \hat{d}),
\]
where \( u^\infty_{\delta, \text{obs}} = \tilde{u}^\infty - u^s \) for \( \tilde{u}^\infty \) the unique solution of (2).

We propose to solve the inverse problem by minimizing
\[
F(\Gamma, \delta, \mu) := \frac{1}{2} \sum_{j=1}^{J} \| T(\Gamma, \delta, \mu, \hat{d}_j) - u^\infty_{\delta, \text{obs}}(\cdot, \hat{d}_j) \|_{L^2(S^1)}^2
\]
with respect to the geometry \( \Gamma \) and the physical parameters \( \mu \) and \( \delta \). For this purpose, we have to compute the derivative of the far field with respect to the parameters and the shape which can be done with a slight adaptation of [3]. As mentioned in such article, due to the fact that \( \delta \) is a function only defined on the shape \( \Gamma \), the definition of an appropriate shape derivative is needed.

2.2. Shape derivative of the scattered field

Take \( h \in C^{1,\infty}(\mathbb{R}^2, \mathbb{R}^2) \) with \( C^{1,\infty} := C^1 \cap W^{1,\infty} \) equipped with the norm \( \| h \| := \| h \|_{W^{1,\infty}(\mathbb{R}^2, \mathbb{R}^2)} \) such that \( \| h \| < 1 \), then the mapping \( f_h := Id + h \) is a \( C^1 \)-diffeomorphism of \( \mathbb{R}^2 \). The perturbed shape \( \Gamma_h \) is defined by \( \Gamma_h = \{ x + h(x), \ x \in \Gamma \} \), while \( \delta_h \) is given by \( \delta_h = \delta \circ f_h^{-1} \). Then the definition of the shape derivative follows.

**Definition 1** We say that the far field operator \( T \) is differentiable with respect to \( \Gamma \) if there exists a continuous linear operator \( T'_{\delta, \mu}(\Gamma) : C^{1,\infty}(\mathbb{R}^2, \mathbb{R}^2) \rightarrow L^2(S^1) \) such that
\[
T(\Gamma, \delta, \mu, \hat{d}) - T(\Gamma, \delta, \mu, \hat{d}_j) \approx T'_{\delta, \mu}(\Gamma) f_h - T'_{\delta, \mu}(\Gamma) f_{\hat{d}_j}
\]
for \( h \in C^{1,\infty}(\mathbb{R}^2, \mathbb{R}^2) \) small enough.

![Figure 2: Numerical reconstruction of the shape of a coated perfectly conducting body and of the width of the coating with 8 incident waves.](image)

Then, following the work in [3] on shape derivative for GIBC scattering problems, one obtains the shape derivative of the cost function \( F \).

2.3. Applicability of the method

We implemented the minimization process (the derivative with respect to \( \mu \) and \( \delta \) is quite straightforward, see [4] for details) using a \( H^1(\Gamma) \) regularization for the computation of the descent directions. In the case where \( \epsilon = 0.1 \) and \( \mu \) is unknown, the procedure gives a quite accurate reconstruction of the shape, of the thickness of the layer (see figure 2) and of the permeability \( \mu \) (exact \( \mu = 2.5 \), initial \( \mu = 1 \), recovered \( \mu = 2.3 \)).

3. REFERENCES


