ABSTRACT

We consider the problem of detecting bounded inhomogeneous obstacles in an infinite tubular waveguide. We have in mind the application of acoustic techniques to inspect underground pipes such as sewers: In this application a loud-speaker and microphone are lowered into a man-hole. Sound pulses are created in the pipe, and the acoustic field reflected by obstructions in the pipe is measured. From these data one searches the size and position of the blockages. In this work, we analyze, both theoretically and numerically, the use of two qualitative algorithms: The Linear Sampling Method (LSM), first introduced by Colton and Kirsch [6]; and the Reciprocity Gap Method (RGM), due to Colton and Haddar [5]. Generally, most applications of the LSM and RGM have been to the detection of bounded scatterers in an infinite background medium. However, Bourgeois and Luneville [2] have considered the use of the LSM for detecting sound-hard obstacles in infinite sound hard tubular pipes. The assumption of a sound hard pipe is in accordance with engineering practice for hard plastic and clay pipes [7]. But the computational examples in [2] are all of scatterers (e.g. balls) away from the boundary of the pipe, whereas in the application we have in mind, the scatterers are perturbations to the boundary of the pipe. Nevertheless this paper is strongly motivated by [2], specially by the highly successful numerical results therein. Indeed, we seek to extend [2] in two ways:

- To start with, porous sediments can support acoustic waves, so we will analyze the RGM and LSM for detecting penetrable scatterers in the pipe allowing these scatterers to touch the wall of the pipe. This involves the analysis of a new interior transmission problem in which the usual interior transmission conditions are present on part of the obstacle, while the sound hard boundary condition applies on other parts where the blockage touches the pipe walls.

- Even more importantly, inverse scattering algorithms for this problem face the difficulty that the manhole is a significant perturbation of the pipe. In particular, the application of the LSM to a realistic sewer would require the calculation of the fundamental solution for the sewer with its manhole (as proposed for the technique in [13]). On the contrary, in order to apply the RGM to the pipe away from the manhole, we need the fundamental solution for the pipe alone, although in principle it requires to measure more data (the field and its derivative along the pipe). Thus, for the RGM, no modeling of the manhole is required, fact the could perhaps outweigh its usual disadvantage of requiring many measurements and many sources (i.e. multistatic data).

In addition, let us underline that the RGM and LSM in a waveguide maintain their usual strong points: speed of reconstruction and independence of the nature of the blockage (in fact, the same RGM or LSM is applied independently of whether there is a hard or penetrable blockage in the pipe, since only the mathematical justification changes).

Other work on detecting objects in a tubular waveguide includes the time-reversal technique [9] which is generally analyzed for small obstacles (whereas we may be wanting to detect substantial blockages). In addition, there has been much more work on inverse problems for layered waveguides (i.e. infinite in two directions, rather than infinite in just one direction as we consider). Examples of such works include qualitative methods [14] and time-reversal [12, 13, 11].

In this work, we analyze a particular RGM method (and related LSM) using the single layer ansatz for the near field inverse waveguide problem. We extend the analysis of [3, 8] to the waveguide problem. Moreover, as in [8], we prove the theoretical equivalence of this RGM and a generalized LSM in which the source and
measurement domains are possibly disjoint. We can thus apply the RGM or the LSM depending on the data available, although, as we have said, the RGM has the advantage that background scatterers (e.g. the manhole) outside the region enclosed by the source curve do not need to be explicitly modeled. We also see that the proposed RGM and LSM possess the properties needed for regularization, and prove standard theorems about the methods, now in the waveguide context. In addition, as part of the analysis, we prove that the single layer operator for waveguides can be defined on open arcs or surfaces, and it is an injective and surjective operator on suitable function spaces.

We have implemented both the RGM and LSM in MatLab codes, that confirm the expected behaviour of these methods in some academic examples. Nevertheless, the numerical results contain a surprising example: the LSM fails to detect a complete blockage of the pipe, that to our knowledge is the first case in which the LSM completely fails to detect an obstacle; so that this example, which is not covered by our theory neither that in [2], is obviously troubling and requires future analysis.

1. REFERENCES