Designing piezoelectric modal sensors/actuators

J.C. Bellido

PICOF 2012, École Polytechnique, April 2012

UNIVERSIDAD DE CASTILLA-LA MANCHA

Departamento de Matemáticas (ETSII-Ciudad Real)
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Joint work with A. Donoso
Piezoelectricity

ability of some materials (notably crystals and certain ceramics) to generate a voltage in response to applied mechanical stress and vice versa.
Introduction to piezoelectricity

Piezoelectricity

ability of some materials (notably crystals and certain ceramics) to generate a voltage in response to applied mechanical stress and vice versa.

- **Sensors**: they produce an electric signal proportional to their deformation.

- **Actuators**: they strain under an applied voltage.

These transducers can appear surface bonded to structures or embedded in laminated composites, uniformly distributed or like patches.

Applications lighters, quartz clocks, ultrasonic transducers, bio-sensors, modal control, etc.
Introduction to piezoelectricity

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**Applications**

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What can piezoelectric actuators do?

- Poling voltage
- PZT
- BEAM
- Compression and tension

- Voltage addition
- Bending up and down

- Designing piezoelectric modal sensors/actuators

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Modal sensors/actuators (MSAs)

MSAs

- Those which measure/excite a particular mode of a structure and remain insensitive to the rest (⇒ behave as ideal filters).

\[ F_j(x) \propto \phi''_j(x) \]
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![Normalized Surface Electrode Width](image1)

![Tip-Response, [dB]](image2)

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Modelling

SIDE VIEW

ACTUATOR
PLATE
SENSOR

TOP VIEW

PIEZOELECTRIC MATERIAL

material variable $\chi_m = \{0, 1\}$

poling variable $\chi_p = \{-1, 1\}$

Aim: systematic design of distributed piezoelectric MSAs for plates.

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Modelling

**Aim**: systematic design of distributed piezoelectric MSAs for plates.
Modelling

The signal response (electrical charge) of the piezoelectric sensor layer (Lee-Moon 1990, J. Appl. Mech.):

\[
q(t) = -\frac{(h_p + h_s)}{2} \int_0^{L_x} \int_0^{L_y} \chi_m \chi_p \left( e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2} + 2e_{36} \frac{\partial^2 w}{\partial x \partial y} \right) \, dy \, dx,
\]

where:

- \( h_p, h_s \) thickness of the plate and sensor layer
- \( e_{31} = e_{32} = e \) (piezo’s charge the same in both directions), \( e_{36} = 0 \) (piezo’s axes the same as the plate) piezo stress/charge constants
- \( w \) out-of-plane displacement of the plate
- piezoelectric layers negligable stiffness and mass compared to the plate
Modal-Fourier expansion of $w$:

$$w(x, y, t) = \sum_{j=1}^{\infty} \phi_j(x, y) \eta_j(t),$$

$\phi_j$ mode shape, $\eta_j$ modal coordinate.
Modelling

Modal-Fourier expansion of $w$:

$$w(x, y, t) = \sum_{j=1}^{\infty} \phi_j(x, y) \eta_j(t),$$

$\phi_j$ mode shape, $\eta_j$ modal coordinate. Inserting the expansion of $w$ into the expression of $q$ we get into

$$q(t) = -e^{\frac{(h_p + h_s)}{2}} \sum_{j=1}^{\infty} B_j \eta_j(t),$$

with

$$B_j = \int_0^{L_x} \int_0^{L_y} \chi_m(x, y) \chi_p(x, y) \Delta \phi_j(x, y) \, dy \, dx$$
Taking \( \chi(x, y) = \chi_m(x, y)\chi_p(x, y) \), the optimization problem is given by

Maximize \( \chi(x, y) \in \{-1, 0, 1\} : \)

\[
B_k(\chi)
\]

subject to:

\[
B_j(\chi) = 0, \quad \text{for } j = 1, \ldots, M, \text{ and } j \neq k,
\]

where

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B_j(\chi) = \int_0^{L_x} \int_0^{L_y} \chi(x, y) \Delta \phi_j(x, y) \, dy \, dx.
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Taking $\chi(x, y) = \chi_m(x, y)\chi_p(x, y)$, the optimization problem is given by

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Key point

We are looking for an ideal sensor that best observes the $k$-th mode and filters the rest of the first $M$ modes.
Modelling

Likewise, such optimal profiles let the actuator layer control both the magnitude and location of the forces induced by the electric field $\epsilon(t)$ to the plate through the actuator equation and therefore to excite the mode at interest (Lee-Moon 1990):

\[
\frac{Eh_p^3}{12(1 - \nu^2)} \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho h_p \frac{\partial^2 w}{\partial t^2} = -h_p h_a (h_p + h_a) e \epsilon(t) \Delta(\chi_m \chi_p(x, y))
\]
Taking $\chi(x, y) = \chi_m(x, y)\chi_p(x, y)$, the optimization problem is given by

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The set of functions where we optimize is not compact, so, in principle, we cannot guarantee the existence of optimal solutions for $(P)$. 
Optimization problem

Taking $\chi(x, y) = \chi_m(x, y)\chi_p(x, y)$, the optimization problem is given by

Maximize $\chi(x,y) \in \{-1,0,1\}$:

$$B_k(\chi)$$

subject to:

$$B_j(\chi) = 0, \quad \text{for } j = 1, \cdots, M, \text{ and } j \neq k,$$

where

$$B_j(\chi) = \int_{0}^{L_x} \int_{0}^{L_y} \chi(x, y) \Delta \phi_j(x, y) \, dy \, dx.$$

- The set of functions where we optimize is not compact, so, in principle, we cannot guarantee the existence of optimal solutions for $(P)$.
- A relaxed formulation is required ($\Longrightarrow$ just replace $\chi(x, y)$ by $\rho(x, y) \in [-1, 1]$).
Analysis of the relaxed formulation

The relaxed problem is given by

\[
\text{Maximize}_{\rho(x,y) \in [-1,1]}: \quad B_k(\rho)
\]
subject to:

\[B_j(\rho) = 0, \quad \text{for } j = 1, \cdots, M, \text{ and } j \neq k,\]

- Both objective function and constraints are linear.
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- Both objective function and constraints are linear.
- By using the Lemma I in [Artstein 1980], it is analytically proved that optimal solutions for (RP) just take either -1 or 1.
The relaxed problem is given by

\[
\begin{align*}
\text{Maximize} & \quad \rho(x,y) \in [-1,1]: \\
& \quad B_k(\rho) \\
\text{subject to:} & \quad B_j(\rho) = 0, \quad \text{for } j = 1, \ldots, M, \text{ and } j \neq k,
\end{align*}
\]

- Both objective function and constraints are linear.
- By using the Lemma I in [Artstein 1980], it is analytically proved that optimal solutions for \((RP)\) just take either -1 or 1.
- Optimal solutions for \((P)\) correspond to taking: \(\chi_m \equiv 1\) and \(\chi_p \in \{-1, 1\}\).
Analysis of the relaxed formulation

The relaxed problem is given by

Maximize $\rho(x,y) \in [-1,1]$: $B_k(\rho)$

subject to:

$B_j(\rho) = 0$, for $j = 1, \cdots, M$, and $j \neq k$.

- Both objective function and constraints are linear.
- By using the Lemma 1 in [Artstein 1980], it is analytically proved that optimal solutions for $(RP)$ just take either -1 or 1.
- Optimal solutions for $(P)$ correspond to taking: $\chi_m \equiv 1$ and $\chi_p \in \{-1, 1\}$.
- The discrete problem can be easily solved by the simplex method.
Numerical simulations

Example 1: Isolate 1st flexural mode in a plate simply-supported at all four sides

(i) $M = 5$

(j) $M = 10$

(k) $M = 15$

(l) $M = 26$
Numerical simulations

Example 2: Isolate 6th extensional mode in a plate cantilevered in its left side.

$M = 12$
Numerical simulations

**Example 3:** Isolate 1st flexural mode in a plate cantilevered in its left side.

(\( n \)) \( M = 20 \) (flexural)

(\( \tilde{n} \)) \( M = 20 + 12 \) (flexural + extensional)
**Example 4**: Isolate 2nd mode in a half cylindrical shell cantilevered in its left curved side.
Manufacturing and Experimental validation

Joint work with the **Microsystems, actuators and sensors Group** led by J.L. Sanchez-Rojas in ETSII-UCLM.
Manufacturing and Experimental validation

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**Example**: Isolate 1st flexural mode in a microbridge clamped in both sides.
Sensitivity analysis - gap

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modo
respuesta norm.
gap = 10 gap = 5 gap = 2 gap = 0
Future work

Simultaneous optimization of both the supporting structure and the polarization profile of the piezoelectric sensor/actuator.
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Simultaneous optimization of both the supporting structure and the polarization profile of the piezoelectric sensor/actuator.

Doctoral student David Gracia working on that.


