Inverse problems for aerospace applications
From experience feedback to a generic approach

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Summary

1. Why is a generic approach valuable?
2. Illustration on some aerospace applications
   i. Adjoint computation
   ii. Noisy measurements
   iii. Uncertainties
   iv. Surrogate models
3. Conclusion & perspectives
Inverse problem in industry

- **Inverse problem**: search for the causes of observed or desired effects

- Only 3 ingredients…
  - Governing equations?
  - Discretization?
  - Implementation?
  - Criterion?
  - Ill-posedness?
  - Linear or non linear IP?
  - Observed data accuracy?
  - Regularization?
  - Solution interpretation?
  - ….

- But, many questions addressing many underlying research or industrial topics
  - Governing equations?
  - Discretization?
  - Implementation?
  - Criterion?
  - Ill-posedness?
  - Linear or non linear IP?
  - Observed data accuracy?
  - Regularization?
  - Solution interpretation?
  - ….

- Measurement devices
- Software availability
- Computing resources
- Uncertainties
- Numerical analysis
- HPC
- Optimization
- Surrogates
- Physics

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Towards a generic approach

• **A structuring approach is valuable**
  - **Step 1**: industrial context / parameter / observables / sources of uncertainties
  - **Step 2**: governing equations / criterion / forward model
  - **Step 3**: implementation (forward model / optimization)
  - **Step 4**: sensitivity & robustness analysis

• **Should help to deal with the real difficulties**
  - Define the problem with BUs
  - Establish dialog between experts and field engineers
  - Assess models/data availability
  - Solution analysis
Inverse problem & Adjoint computation by Lagrangian approach (1/2)

- **Industrial context**:
  - Identification of shock levels along cutting line from acceleration measurements

- **Forward model**: equations of mechanics (ODE) - M, B, K computed by FEM
  \[ M\ddot{q}(t) + B\dot{q}(t) + Kq(t) = g(t) \]

- **Criterion**: least-squares problem subject to the ODE constraint
  \[
  \text{Min } J(g) = J(g, \ddot{q}) = \frac{1}{2} \int_0^T \| \ddot{q}(g(t)) - \ddot{q}_{\text{mes}}(t) \|^2 \, dt
  \]
  \[ s.t. \quad F(g, \ddot{q}) = 0 \]

  \[
  \Rightarrow \text{ solved by a Lagrangian approach}
  \]

- **Lagrangian function**:
  \[ L(g, \ddot{q}, \lambda) = J(g, \ddot{q}) + \int_0^T \lambda(t) < F(g, \ddot{q}) > dt \]

- **Optimality conditions on L**: Adjoint equations
  \[
  \frac{\partial L}{\partial \ddot{q}} = 0 \Rightarrow M\ddot{\lambda}(t) + B\dot{\lambda}(t) + K\lambda(t) = -[\ddot{q}(t) - \ddot{q}_{\text{mes}}(t)]
  \]

- **Gradient of J deduced from adjoint state \( \lambda \)**
  \[
  \nabla_g J = -\ddot{\lambda}
  \]

  \[
  \Rightarrow \text{Gradient-based descent algorithm (Quasi-newton)}
  \]
Inverse problem & Adjoint computation by Lagrangian approach (2/2)

- Good results with discrete adjoint approach
- But requires to create the adjoint program: not straightforward
- Promising technology: Automatic Differentiation
Inverse problem & Adjoint computation by Automatic differentiation (1/2)

- Industrial context:
  - identification of aerodynamic coefficients from in-flight measurement
  - improvement of existing tables

- Forward model: equations of 3D dynamics (ODE)
  \[
  \begin{align*}
  m \frac{dV}{dt} &= \sum F_{ext} \\
  \frac{dH}{dt} &= \sum M_{applied}
  \end{align*}
  \]
  Drag, Lift: \( D = C_D \cdot Q \cdot S \)
  \( L = C_L \cdot Q \cdot S \)

- Criterion: least-squares problem subject to an ODE constraint
  \[
  \min_{p \in \mathbb{P}} J(p) = \int_0^T \left\| O(p,t) - O^{met} \right\|^2 dt \\
  \text{s.t. } F(O(p,t), p) = 0
  \]

- Optimization & industrial constraints
  - Existing discretized forward model (FORTRAN 90)

⇒ Gradient computed by AD (TAPENADE) / 2 modes: Tangent & Reverse

⇒ Quasi-Newton algorithm
Inverse problem & Adjoint computation by Automatic differentiation (2/2)

- Good results with Automatic Differentiation (Tangent mode)
- Reverse mode more relevant for adjoint computation: generates automatically the adjoint of the discretized problem
- Many potential applications: aerodynamics, structural optimization, control, …
Inverse problem & Combinatorial optimization (1/2)

- **Industrial context**: optimal positioning of microphones for robust identification of sources

- **Forward model**: acoustic waves equation (Helmholtz)

  \[
  \alpha \rightarrow \text{ACTIPOLE} \rightarrow u
  \]

  Parameter
  modal sources

  Observables
  pressure at microphones

- **Criterion**: least-squares problem - Robustness = minimization of the relative error on source reconstruction

  \[
  J(\alpha) = \frac{1}{2} \| T \cdot \alpha - u^{\text{mes}} \|^2
  \]

  \[
  \frac{\| \Delta \alpha \|}{\| \alpha^{\text{exact}} \|} \leq \text{cond}(T) \cdot \frac{\| \Delta u \|}{\| u^{\text{exact}} \|}
  \]

- **Optimization problem**: optimal positioning \( p \) searched on a discrete grid

  \[
  p^* = \arg \min_p \text{cond } T(p)
  \]

  - Global optimization, many minima
  - Gradient methods not adapted
  - Use of Operational Research technics (TABU)
Inverse problem & Combinatorial optimization (2/2)

8 microphones / 8 sources / 100 possible locations on semi-sphere

- How to handle with measurement errors on microphones?
- How to handle situations where \( N_{src} \gg N_{mic} \)?

Some questions remain:

- How to handle with measurement errors on microphones?
- How to handle situations where \( N_{src} \gg N_{mic} \)?
Inverse Problems with Noisy Observations (1/2)

- **Probabilistic Modelling:**
  
  \[ U = U_{obs} \text{ is noised} \quad \Rightarrow \quad U_{obs} = U^* + \varepsilon \quad \text{with} \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{Nmic} \end{pmatrix} \]

  \[ U^* = T \cdot \alpha \]

  \[ U_{obs} = T \cdot \alpha + \varepsilon \]

- **Probabilistic Inverse Problem:**
  
  Noise minimization

  \[ \hat{\alpha} = \arg \min_{\alpha} \frac{1}{2} \| T \cdot \alpha - u_{obs} \|^2 \]

  Computationally, **YES** ... **BUT**... the solution \( \hat{\alpha} = \hat{\alpha}(\varepsilon) \) is **random**!

  How to quantify the random error ?...
Inverse Problems with Noisy Observations (2/2)

Error Quantification:

\[ \text{Var}(\hat{\alpha}(\mathbf{e})) = \sigma^2 (TT)^{-1} \]

How to reduce?

(At fixed \( \sigma^2 \)) Find microphone positions \( \hat{p} \) satisfying

\[ \hat{p} = \text{Argmin}_p \text{Det Var}(\hat{\alpha}(\mathbf{e})) = \text{Argmin}_p \text{Det} [(TT)^{-1}] \]

Overdetermined Problem -- \( N_{src} >> N_{mic} \):

Penalized procedures

\[ \hat{\alpha} = \arg\min_\alpha \frac{1}{2} \| T \cdot \alpha - u \|^2 + \lambda \text{pen}(\alpha) \]

To be tuned …
(cross validation etc.)

\[ \text{Combinatorial methods: } \text{TABU search etc.} \]
Inverse problems with Uncertainties

Gas Turbine Performance application

- **Objective:**
  
  To guarantee the engine performances along its service

- **Data:**
  
  Fuel mass consumptions in the cruise phase at time $T$, for a given line between 2 fixed countries

- **Engine Software:**
  
  - $X = $ Uncertain variables (cruise speed, L/D)
  - $SFC = $ to be identified!

<table>
<thead>
<tr>
<th>Reference Fuel Masses [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7918</td>
</tr>
<tr>
<td>7872</td>
</tr>
<tr>
<td>7755</td>
</tr>
<tr>
<td>8058</td>
</tr>
</tbody>
</table>
Inverse Problems with Uncertainties

Inverse Problem: From experimental fuel mass consumptions, identify the Specific Fuel Consumption (SFC) = $\hat{\theta}$ of the engines.

Given by observations

$$\hat{\theta} = \underset{\theta=SFC}{\text{Argmin}} D(\text{distrib.}(\text{mass\_fuel}), \text{distrib.}(\text{Soft}(X, \theta)))$$

where $D$ is some «measure of distance» between density distributions.

Optimisation algorithm

Highly Non-Linear problem

Computed with Quasi-Newton (BFGS) method

Computation of the distribution of SFC

Error of reconstruction due to

- Limited data M_fuel (=32)
- Model error of Software
- Possibly noised observations

N. Rachdi, J-C. Fort and T. Klein, *Stochastic Inverse Problem with Noisy Simulator*, AFST (accepted)
Inverse Problems using Surrogate Models

Cabin Comfort Application

- **Objective**: Maximize the *comfort* in the cabin through a « comfort function » $C_f$ (to be minimized).

$$C_f = C_f \circ H(\theta)$$

$\theta$ = Control parameters (flow, temperature…)

- **Inverse problem**: Compute $\hat{\theta}$ such that

$$\hat{\theta} = \text{Argmin}_{\theta} C_f(H(\theta))$$

Optimisation bottleneck

1 run of $H$ = ~ 5 hours!
Inverse problems using surrogate models

- Alternative:
  
  Replace $H$ by a surrogate model $H_{surr}$

  
  - Radial Basis Functions
  - Polynomial Regression
  - Kriging
  - Neural Network
  - Etc.

- Intermediate Inverse Problem:
  
  Construction of $H_{surr}$ leads to an inverse problem = parameters fitting

- Approximated Inverse Problem:
  
  $\hat{\theta}_{surr} = \text{Argmin}_\theta C_f(H_{surr}(\theta))$

  Robustness Study of $\hat{\theta}_{surr}$

  Not deterministic! …
Conclusion & Perspectives

- A structuring methodology: more than valuable in our industrial context
- Filling the gap between engineers and experts: fruitful collaboration on long-term
- Classifying by research topics: synergy, new highlights on problem of interest
- Some challenging perspectives (among many others) !!!!!
  - Automatic differentiation
    - Tools implementing efficiently the reverse mode
  - Optimization
    - Large scale linear and non linear programming
    - Noisy objective function & constraints
  - Uncertainties
    - Work on “noise” assumptions (distribution, correlation, ...) and impact
  - Surrogate modelling
    - Adapt surrogate models for optimisation problems
THANK YOU FOR YOUR ATTENTION !!!