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Addendum to “An approximation result for special functions with bounded deformation”: the N–dimensional case.

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Abstract

We explain in this note how to adapt the proofs in our previous work [5] “An approximation result for special functions with bounded deformation” (to appear in J. Math. Pures Appl., 2004), to dimension higher than two.

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1 Introduction

In a previous work, we have shown the following theorem, only in dimension $N = 2$ (*cf* [5],Theorem 3 and Remark 5.3):

Theorem 1 *Let $\Omega \subset \mathbb{R}^N$ a bounded open subset, and assume it satisfies the regularity assumption* (H) *below. Let $u \in SBD(\Omega) \cap L^2(\Omega; \mathbb{R}^N)$, such that*

$$\int_\Omega |e(u)|^2 \, dx \, + \, \mathcal{H}^{N-1}(J_u) \; < \; +\infty \, .$$

Then, there exists a sequence $(u_n)_{n \geq 1}$ of displacements in $SBD(\Omega) \cap L^2(\Omega; \mathbb{R}^N)$, with $\|u_n - u\|_{L^2(\Omega;\mathbb{R}^N)} \overset{n\to\infty}{\longrightarrow} 0$, such that each J_{u_n} is closed in Ω, contained in a finite union J_n of closed connected pieces of C^1 hypersurfaces, $u_n \in H^1(\Omega \setminus J_n; \mathbb{R}^2)$, and

(i) $e(u_n) \to e(u)$ *strongly in* $L^2(\Omega; \mathcal{S}^{N\times N})$,

(ii) $\lim_{n\to\infty} \mathcal{H}^{N-1}(J_{u_n}) = \lim_{n\to\infty} \mathcal{H}^{N-1}(\overline{J}_{u_n}) = \lim_{n\to\infty} \mathcal{H}^{N-1}(J_n) = \mathcal{H}^{N-1}(J_u)$.

Moreover, if $\|u\|_{L^\infty} < +\infty$, one can ensure that $\|u_n\|_{L^\infty} \leq \|u\|_{L^\infty}$ for all n.

Here, $\mathcal{S}^{N\times N}$ is the $(N(N+1)/2)$–dimensional space of symmetric $N \times N$ matrices, and assumption (H) states that Ω has a boundary which is locally a subgraph:

$$\text{(H)} \quad \begin{cases} \text{At every boundary point } x \in \partial\Omega, \text{ there exist coordinates} \\ (\xi_1, \dots, \xi_N) \text{ and a continuous function } f : \mathbb{R}^{N-1} \to \mathbb{R} \text{ such that} \\ \text{near } x,\ \Omega \text{ coincides with the subgraph } \{\xi_N < f(\xi_1, \dots, \xi_{N-1})\}. \end{cases}$$

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The scope of this note is to show how the proofs in [5] can be adapted to the N–dimensional case, with $N \geq 3$. We refer to the original paper for more details on the space SBD (introduced in [2, 3]), and the main motivations for Theorem 1. In particular, the consequence of Theorem 1 pointed out in [5, Sec. 6], that is, Theorem 4 of Γ–convergence, is valid in any dimension.

The proof of this result is based on a discretization argument which is adapted from [6], and has been used in a similar setting in [4] (in the scalar case) and [1] (in the vectorial case). Then, a re-interpolation technique allows to rebuild an approximating function with almost the desired property. Due to the anisotropy inherent to the discretization step, it is impossible with this technique to approximate correctly the surface of the jump set of the displacement. A further localization method based on the rectifiability of this jump set (that is, the fact that up to a small set, it is almost a finite union of C^1 hypersurfaces) can handle this problem.

The fact that the original paper [5] is written only in dimension 2 is due to the misleading belief that the discretization–interpolation trick on which the result is based (Section 4), and the subsequent localization step (Section 5) would work only with totally isotropic bulk energies. However, if the same orthonormal basis $(e_1, \ldots, e_N)$ of $\mathbb{R}^N$ is used during the whole process (and in all subdomains where the operation is performed) one realizes that it is not true. If one drops this requirement, one realizes that it is not too difficult to find an *anisotropic* positive-definite quadratic form on the space of $N \times N$ symmetric matrices $W_N : \mathcal{S}^{N\times N} \to \mathbb{R}$, for which the constructions in Sections 4 and 5 of [5] can be performed, and an equivalent of Lemma A.1 [5, Appendix A] can be shown. In what follow, we will merely stress on the adaptions that have to be done to the statements and proofs in [5] to deduce Theorem 1. Let us observe that Section 3 in [5] is valid in arbitrary dimension, and consider the adaption of Section 4.

2 The N–dimensional construction

In dimension 2, the following bulk energy is introduced in [5, Eq. (3)]:

$$W_2(A) \;=\; \mathrm{Tr}\,(AA^T) + \frac{1}{2}(\mathrm{Tr}\,(A))^2\,, \tag{1}$$

$A \in \mathcal{S}^{2\times 2}$. For $A = (a_{i,j})_{1\leq i,j\leq N} \in \mathcal{S}^{N\times N}$ and $1 \leq i < j \leq N$, we will denote by $A^{i,j}$ the 2×2 symmetric matrix

$$A^{i,j} \;=\; \begin{pmatrix} a_{i,i} & a_{i,j} \\ a_{i,j} & a_{j,j} \end{pmatrix}. \tag{2}$$

We then introduce the following quadratic form

$$W_N(A) \;=\; \sum_{1\leq i<j\leq N} W_2(A^{i,j})\,, \tag{3}$$

$A \in \mathcal{S}^{N\times N}$. One has

$$W_N(A) \;=\; \sum_{1\leq i<j\leq N} a_{i,i}^2 + a_{j,j}^2 + 2a_{i,j}^2 + \frac{1}{2}(a_{i,i}^2 + a_{j,j}^2) + a_{i,i}a_{j,j}\,,$$

and since for any $(x_1, \dots, x_N)$,

$$\sum_{1 \le i < j \le N} x_i + x_j = (N-1) \sum_{i=1}^{N} x_i \,, \tag{4}$$

we find that

$$W_N(A) = \frac{3(N-2)}{2} \sum_{i=1}^{N} a_{i,i}^2 + \mathrm{Tr}\,(AA^T) + \frac{1}{2}(\mathrm{Tr}\,(A))^2 \,.$$

In particular, we see that it is a positive definite quadratic form on $\mathcal{S}^{N\times N}$, which is anisotropic, in the sense that it is not invariant with respect to an orthonormal change of coordinates.

As in [5], we fix $u \in SBD(\Omega) \cap L^2(\Omega;\mathbb{R}^N)$, and given $\varepsilon > 0$, we find by [5, Lemma 3.2] a set $\Omega' \supset\supset \Omega$ and $u' \in SBD(\Omega') \cap L^2(\Omega';\mathbb{R}^N)$ with $\|u - u'\|_{L^2(\Omega)} \le \varepsilon$ and

$$\int_{\Omega'} |e(u')|^2 \, dx \le \int_{\Omega} |e(u)|^2 \, dx + \varepsilon \quad \text{and} \quad \mathcal{H}^{N-1}(J_{u'}) \le \mathcal{H}^{N-1}(J_u) + \varepsilon. \tag{5}$$

We then choose a system of coordinates $(e_1, \dots, e_N)$ such that for all $e \in \{e_i, i = 1, \dots, N\,, e_i + e_j, e_i - e_j, 1 \le i < j \le N\}$, one has $\mathcal{H}^{N-1}(\{x \in J_{u'} \,:\, [u'(x)] \cdot e = 0\}) = 0$ (almost any orthonormal basis of $\mathbb{R}^N$ will do, *cf* [2, Eq. (4.5)]). We fix a small discretization step $h > 0$. Given $y \in [0,1)^N$ we denote by $u_h^y(\xi)$ the discretization of u' given by $u_h^y(\xi) = u'(hy + \xi)$, $\xi \in h\mathbb{Z}^N \cap (\Omega' - hy)$. We still denote by J^τ the set $\bigcup_{x \in J_{u'}} [x, x - \tau]$ for any $\tau \in \mathbb{R}^N$. Let us observe that a similar construction is found in [1]. The set of directions of interactions is now $D = \{e_i \,:\, i = 1, \dots, N\} \cup \{e_i + e_j, e_i - e_j \,:\, 1 \le i < j \le N\}$. For $e \in D$ we denote, again, $l_{e,h}^y = \chi_{J^{he}}(hy + \xi) \in \{0, 1\}$ for any $\xi \in h\mathbb{Z}^N \cap (\Omega' - hy)$. For a fixed y, the discrete energy $E_h^y(u_h^y, l_h^y)$, with $l_h^y = (l_{e,h}^y)_{e \in D}$ has a definition slightly different from [5, Eq. (5)]: we introduce a parameter $\alpha(e)$ which is $(N-1)$ whenever $e = e_i$, $i = 1, \dots, N$, and $1/4$ for $e = e_i \pm e_j$, $1 \le i < j \le N$, and let

$$E_h^y(u_h^y, l_h^y) = h^N \sum_{e \in D} \sum_{\xi} \alpha(e) \frac{\left((u_h^y(\xi + he) - u_h^y(\xi)) \cdot e\right)^2}{h^2} (1 - l_{e,h}^y(\xi)) + \beta \frac{l_{e,h}^y(\xi)}{|e| h} \tag{6}$$

where the sum on the ξ runs on all the points $\xi \in h\mathbb{Z}^N$ such that both $hy + \xi$ and $hy + \xi + he$ are in Ω', and the parameter $\beta > 0$ is fixed later on. For $N = 2$, one has $\alpha(e) = 1/|e|^4$ and the energy is the same as [5, Eq. (5)]. On the other hand, for $N \ge 3$, one checks that the discrete bulk part of energy (6) is a sum on all pairs (i, j), $i < j$, of the 2–dimensional discrete bulk part of [5, Eq. (5)] with directions $e \in \{e_i, e_j, e_i + e_j, e_i - e_j\}$, which makes it coherent with definition (3) of W_N. Proceeding as in [5], we find (following the slicing technique of Gobbino [6] also used, in a discrete setting, in [4, 1]) that

$$\int_{[0,1)^N} E_h^y(u_h^y, l_h^y) \, dy$$
$$\le \int_{\Omega'} \left(\sum_{e \in D} \alpha(e) [(e(u)(x) e) \cdot e]^2 \right) dx + \beta \int_{J_{u'}} \left(\sum_{e \in D} \left| \nu_{u'}(x) \cdot \frac{e}{|e|} \right| \right) d\mathcal{H}^{N-1}(x) \,. \tag{7}$$

Then, we check that given any matrix $A = (a_{i,j})_{i,j=1}^N \in \mathcal{S}^{N\times N}$, we have (using (4))

$$
\begin{aligned}
&\sum_{e\in D}\alpha(e)[(Ae)\cdot e]^2\\
&= (N-1)\sum_{i=1}^N a_{i,i}^2 + \frac{1}{4}\sum_{1\le i<j\le N}[(A(e_i+e_j))\cdot(e_i+e_j)]^2 + [(A(e_i-e_j))\cdot(e_i-e_j)]^2\\
&\qquad = \sum_{1\le i<j\le N} a_{i,i}^2 + a_{j,j}^2 + \frac{1}{4}\left((a_{i,i}+a_{j,j}+2a_{i,j})^2 + (a_{i,i}+a_{j,j}-2a_{i,j})^2\right)\\
&\qquad\qquad\qquad\qquad = \sum_{1\le i<j\le N} W_2(A^{i,j}) \;=\; W_N(A)\,.
\end{aligned}
$$

If we let, for any $\nu \in \mathbb{S}^{N-1}$, $h(\nu) = \sum_{e\in D}|\nu\cdot e|/|e|$, and $\beta' = (\max_{|\nu|=1} h(\nu))\beta$, we deduce from (7) that

$$
\begin{aligned}
\int_{[0,1)^N} E_h^y(u_h^y, l_h^y)\,dy \;&\le\; \int_{\Omega'} W_N(e(u'))\,dx \,+\, \beta\int_{J_{u'}} h(\nu_{u'}(x))\,d\mathcal{H}^{N-1}(x)\\
&\le\; \int_{\Omega'} W_N(e(u'))\,dx \,+\, \beta'\mathcal{H}^{N-1}(J_{u'})\,. \qquad (8)
\end{aligned}
$$

We now introduce the function, for $x = (x_1,\dots,x_N)\in\mathbb{R}^N$,

$$
\Delta(x) \;=\; \prod_{i=1}^N (1-|x_i|)^+
$$

(where $t^+ = \max(t,0)$ is the positive part), and let, for $y\in[0,1)^N$ and any $x\in\Omega$,

$$
w_h^y(x) \;=\; \sum_{\xi\in h\mathbb{Z}^N\cap\Omega'} u_h^y(\xi)\Delta\left(\frac{x-\xi}{h} - y\right)\,.
$$

The function w_h^y is a continuous interpolation of u_h^y in Ω, and, the same argument as in [5] shows that there exists a subsequence $(h_k)_{k\ge1}$, $h_k\downarrow 0$ as $k\to\infty$, and $y\in A$, such that

$$
\begin{cases}
\lim_{k\to\infty}\|u' - w_{h_k}^y\|_{L^2(\Omega;\mathbb{R}^N)} = 0 \quad\text{and}\\
\lim_{k\to\infty} E_{h_k}^y(u_{h_k}^y, l_{h_k}^y) \le \int_{\Omega'} W_N(e(u'))\,dx + \beta'\mathcal{H}^{N-1}(J_{u'})\,.
\end{cases} \qquad (9)
$$

We now fix y to this value and drop the corresponding superscript, a well, we denote simply by $(h)_{h>0}$ $(h\to0)$ the subsequence $(h_k)_{k\ge1}$. As in [5], we define a new function v_h as follows: we let $v_h = 0$ in the hypercube $C = hy+\xi+[0,h)^N$ whenever $J_{u'}$ crosses either one edge $[hy+\xi+h\eta, hy+\xi+h\eta+he_i]$, $i=1,\dots,N$, $\eta\in\{0,1\}^N, \eta_i = 0$, that is, when the corresponding $l_{e_i,h}(\xi+h\eta) = 1$, or a diagonal of a 2–dimensional facet: $[hy+\xi+h\eta, hy+\xi+h\eta+h(e_i+e_j)]$, $i<j$, $\eta\in\{0,1\}^N, \eta_i=\eta_j=0$ ($l_{e_i+e_j,h}(\xi+h\eta)=1$) or $[hy+\xi+h\eta+he_j, hy+\xi+h\eta+he_i)]$, $i<j$, $\eta\in\{0,1\}^N, \eta_i=\eta_j=0$ ($l_{e_i-e_j,h}(\xi+h\eta+he_j) = 1$). In the other case, we let $v_h = w_h$ in C.

The function v_h is in $SBD(\Omega)$, and J_{v_h} is contained in a union of ($(N-1)$–dimensional) facets of hypercubes. We claim that the total surface of these facets can be bounded by $c\times h^N\sum_{e\in D}\sum_\xi \frac{l_{e,h}(\xi)}{|e|h}$, for some constant c, indeed, if v has been set to 0 in $C =$

$hy+\xi+[0,h)^N$, the measure $\mathcal{H}^{N-1}(\partial C)$ is $2Nh^{N-1}$, on the other hand, the contribution of C to the term $h^N\sum_{e\in D}\sum_{\xi'}\frac{l_{e,h}(\xi')}{|e|h}$ in the discrete energy, which is

$$h^{N-1}\left(\frac{1}{2^{N-1}}\sum_{i=1}^N\sum_{\substack{\eta\in\{0,1\}^N\\ \eta_i=0}} l_{e_i,h}(\xi+h\eta)\right.$$
$$\left.+\frac{1}{2^{N-2}}\sum_{1\le i<j\le N}\sum_{\substack{\eta\in\{0,1\}^N\\ \eta_i=\eta_j=0}}\frac{l_{e_i+e_j,h}(\xi+h\eta)+l_{e_i-e_j,h}(\xi+h\eta+he_j)}{\sqrt{2}}\right)$$

(since each edge is common to 2^{N-1} hypercubes, while a diagonal of a 2–dimensional facet is common to 2^{N-2} hypercubes), is at least $h^{N-1}/2^{N-1}$ (since at least one of the above $l_{e,h}$'s is equal to 1). Hence, taking $c=2^{N-2}/N$ proves the claim.

On the other hand, if $v_h=w_h$ in the hypercube C, it means all the corresponding $l_{e,h}$'s are 0, and the contribution of C to the energy (6) is

$$I=\frac{(N-1)h^N}{2^{N-1}}\sum_{i=1}^N\sum_{\substack{\eta\in\{0,1\}^N\\ \eta_i=0}}\frac{((u_h(\xi+h\eta+he_i)-u_h(\xi+h\eta))\cdot e_i)^2}{h^2}$$
$$+\frac{h^N}{2^{N-2}}\sum_{1\le i<j\le N}\sum_{\substack{\eta\in\{0,1\}^N\\ \eta_i=\eta_j=0}}\left(\frac{((u_h(\xi+h\eta+h(e_i+e_j))-u_h(\xi+h\eta))\cdot(e_i+e_j))^2}{4h^2}\right.$$
$$\left.+\frac{((u_h(\xi+h\eta+he_j)-u_h(\xi+h\eta+he_i))\cdot(e_i-e_j))^2}{4h^2}\right)$$

Let us show that this is larger than $\int_C W_N(e(v_h)(x))\,dx$. First of all, using again (4), we see that I can be written as a sum on all pairs (i,j) $i<j$,[1] of

$$I_{i,j}=\frac{h^N}{2^{N-2}}\sum_{\substack{\eta\in\{0,1\}^N\\ \eta_i=\eta_j=0}}\left(\frac{((u_h(\xi+h\eta+he_i)-u_h(\xi+h\eta))\cdot e_i)^2}{2h^2}\right.$$
$$+\frac{((u_h(\xi+h\eta+h(e_i+e_j))-u_h(\xi+h\eta+he_j))\cdot e_i)^2}{2h^2}$$
$$+\frac{((u_h(\xi+h\eta+he_j)-u_h(\xi+h\eta))\cdot e_j)^2}{2h^2}$$
$$+\frac{((u_h(\xi+h\eta+h(e_j+e_i))-u_h(\xi+h\eta+he_i))\cdot e_j)^2}{2h^2}$$
$$+\frac{((u_h(\xi+h\eta+h(e_i+e_j))-u_h(\xi+h\eta))\cdot(e_i+e_j))^2}{4h^2}$$
$$\left.+\frac{((u_h(\xi+h\eta+he_j))-u_h(\xi+h\eta+he_i))\cdot(e_i-e_j))^2}{4h^2}\right).$$

By [5, Lemma A.1], it turns out that the term in the sum bounds the integral

$$h^{-2}\int_{(x_i,x_j)\in(0,h)^2}W_2\left([e(v_h)]^{i,j}(hy+\xi+h\eta+(x_i,x_j))\right)\,dx_idx_j$$

[1] Notice that the pairs (i,j), $1\le i<j\le N$ and the points $\eta\in\{0,1\}^N$, $\eta_i=\eta_j=0$, label all the $2^{N-2}N(N-1)/2$ 2–dimensional facets of a hypercube.

(with the notation introduced in (2)), so that

$$I_{i,j} \geq \left(\frac{h}{2}\right)^{N-2} \sum_{\substack{\eta\in\{0,1\}^N \\ \eta_i=\eta_j=0}} \int_{(x_i,x_j)\in(0,h)^2} W_2\left([e(v_h)]^{i,j}(hy+\xi+h\eta+(x_i,x_j))\right)\, dx_i dx_j\,. \quad (10)$$

Now, we check that if $x \in (0,h)^N$, for any $i<j$,

$$[e(v_h)]^{i,j}(hy+\xi+x)) = \sum_{\substack{\eta\in\{0,1\}^N \\ \eta_i=\eta_j=0}} \left([e(v_h)]^{i,j}(hy+\xi+h\eta+(x_i,x_j)) \prod_{\substack{k=1 \\ k\neq i,j}}^{N} \left|\frac{x_k}{h}-(1-\eta_k)\right|\right) \quad (11)$$

To check that, it is enough to observe that if $hy+\xi+x\in C$ (that is, $x\in(0,h)^N$), one has

$$\begin{aligned} v_h(hy+\xi+x) &= \sum_{\eta\in(0,1)^N} u_h(\xi+h\eta)\Delta\left(\frac{x}{h}-\eta\right) \\ &= \sum_{\substack{\eta\in\{0,1\}^N \\ \eta_i=\eta_j=0}} \prod_{\substack{k=1 \\ k\neq i,j}}^{N} \left|\frac{x_k}{h}-(1-\eta_k)\right| \Big(u_h(\xi+h\eta)\left(1-\frac{x_i}{h}\right)\left(1-\frac{x_j}{h}\right) \\ &\quad + u_h(\xi+h\eta+he_i)\left(\frac{x_i}{h}\right)\left(1-\frac{x_j}{h}\right) + u_h(\xi+h\eta+he_j)\left(1-\frac{x_i}{h}\right)\left(\frac{x_j}{h}\right) \\ &\quad + u_h(\xi+h\eta+h(e_i+e_j))\left(\frac{x_i}{h}\right)\left(\frac{x_j}{h}\right)\Big) \end{aligned}$$

from which it is clear that taking derivatives with respect only to x_i and x_j yields the expression (11). By convexity of W_2, we deduce from (11) and (10) that

$$\begin{aligned} \int_C W_2\left([e(v_h)]^{i,j}(x)\right) dx &= \int_{(0,h)^N} W_2\left([e(v_h)]^{i,j}(hy+\xi+x)\right) dx \\ \leq \sum_{\substack{\eta\in\{0,1\}^N \\ \eta_i=\eta_j=0}} &\int_{(0,h)^N} W_2\left([e(v_h)]^{i,j}(hy+\xi+h\eta+(x_i,x_j))\right) \prod_{\substack{k=1 \\ k\neq i,j}}^{N} \left|\frac{x_k}{h}-(1-\eta_k)\right| dx \\ = \left(\frac{h}{2}\right)^{N-2} &\sum_{\substack{\eta\in\{0,1\}^N \\ \eta_i=\eta_j=0}} \int_{(0,h)^2} W_2\left([e(v_h)]^{i,j}(hy+\xi+h\eta+(x_i,x_j))\right) dx_i dx_j \leq I_{i,j}\,, \end{aligned}$$

from which we deduce

$$I \geq \int_C W_N(e(v_h)(x))\, dx\,.$$

If h is small enough, we see that we get the existence of a function v and a closed set J made of a finite union of facets of hypercubes such that $v\in H^1(\Omega\setminus J;\mathbb{R}^N)$, $\|v-u\|_{L^2(\Omega)}\leq 2\varepsilon$,

$$\int_\Omega W_N(e(v)(x))\, dx + \mathcal{H}^{N-1}(J) \leq \int_\Omega W_N(e(u)(x))\, dx + c_0\mathcal{H}^{N-1}(J_u) + c\varepsilon$$

for some constant c_0 depending only on N (and c a constant). This yields in particular the N–dimensional version of Theorem 1 in [5]. In particular, we have $v\in SBD(\Omega)$ and $J_v\subset J$ (and an infinitesimal perturbation of v will ensure that $J_v = J\cap\Omega$ up to a $\mathcal{H}^{N-1}$–negligible set).

3 Conclusion

Theorem 1 is now easily deduced from the construction in the previous section and [5, Section 5]. Indeed, the construction in the proof of [5, Thm. 2], based on the rectifiability of the set J_u for $u \in SBD(\Omega)$, although written only in 2D, is valid in any dimension (the Γ_i are now $(N-1)$–dimensional C^1 hypersurface, and ρ has to be replaced with ρ^{N-1} in the density ratios). An important detail is that when the construction of Section 2 is invoked in the sets $B_j \setminus \Gamma_i$ and A_t, then the *same* orthonormal basis $(e_1, \dots, e_N)$ of $\mathbb{R}^N$ must be used in each of these sets, on order to find an energy estimate involving the same bulk energy $W_N(e(u))$ everywhere (this bulk energy is indeed not invariant with respect to a change of basis).

Using then, as previously, [3, Thm. 1.1] (Lemma 5.1 in [5]) we can deduce Theorem 1. Notice also that nothing in the proof of Theorem 4 of [5] is strictly bidimensional.

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References

[1] R. Alicandro, M. Focardi, and M. S. Gelli, *Finite-difference approximation of energies in fracture mechanics*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **29** (2000), no. 3, 671–709. MR 2001k:49068

[2] L. Ambrosio, A. Coscia, and G. Dal Maso, *Fine properties of functions with bounded deformation*, Arch. Rational Mech. Anal. **139** (1997), no. 3, 201–238. MR 98m:49058

[3] G. Bellettini, A. Coscia, and G. Dal Maso, *Compactness and lower semicontinuity properties in* SBD(Ω), Math. Z. **228** (1998), no. 2, 337–351. MR 99e:49011

[4] A. Chambolle, *Finite-differences discretizations of the Mumford-Shah functional*, M2AN Math. Model. Numer. Anal. **33** (1999), no. 2, 261–288. MR 2000d:49049

[5] ———, *An approximation result for special functions with bounded deformation*, J. Math. Pures Appl. (2004), to appear.

[6] M. Gobbino, *Finite difference approximation of the Mumford–Shah functional*, Comm. Pure Appl. Math. **51** (1998), no. 2, 197–228.