# TIME-HARMONIC ELECTROMAGNETIC FIELDS IN THIN CHIRAL CURVED LAYERS

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**Abstract.** In this paper, we prove existence and uniqueness of the solution to the diffraction problem of a plane electromagnetic field by a chiral curved layer covering a perfectly conducting object. Approximative impedance conditions are given for thin chiral curved layers and optimal error estimates are obtained.

**Key words.** Integral Equations, Chiral Media, Approximative Impedance Conditions, Drude-Born-Fedorov Equations. Error Estimates.

**AMS subject classifications.** 35C15, 35Q60, 45B05, 45L05.

**1. Introduction.** The interaction of electromagnetic fields at microwave frequencies with the optically active chiral media has attracted the attention of the electromagnetic community due to its potential applications in the field of antennas, microwave devices, waveguide propagation, scattering, etc. The original investigations of optical activity, or the effect of chirality on the polarization of light, also date back to the nineteenth century. Arago [5], Biot [7], Cauchy [12], Pasteur [23] and Fresnel [15] all examined optical activity in solid and liquid chiral medium. The concept of chirality has also played an increasingly important role in chemistry (see Prelog [24]). Chiral media can be characterized by a generalized set of constitutive relations in which the electric and magnetic fields are coupled. The coupling strength is given by the magnitude of a quantity known as the chirality admittance which determines the bulk electromagnetic properties of these materials. In scattering applications it may be possible to use chiral material to coat a scatterer and thus control its scattering properties more efficiently than a dielectric coating due to the extra degree of freedom offered by the presence of chiral parameter. Antennas coated with chiral materials may have significantly interesting radiation characteristics and surface-relief gratings constructed using chiral material may find application in integrated optics and holography, for instance.

In this paper, we prove uniqueness and existence of the solution to the diffraction problem by a layer of chiral material covering a perfectly conducting object. Assuming that the thickness of the layer is sufficiently small compared to the wavelength of the incident radiation, we derive impedance boundary conditions which can be used, along with the radiation condition, to compute the scattered field without requiring detailed modeling of the field quantities inside the chiral coating. This procedure permits accurate numerical calculations of the scattered wave for chiral coatings as in the achiral case [13]. The starting points of the present paper are the Maxwell equations and the constitutive relations for the chiral layer. Different expressions exist for the constitutive relations. The Drude-Born-Fedorov constitutive equations are used here. For interesting explanation of these equations and various physical aspects of the propagation inside chiral media, we refer to [14], [8], [9], [10], [6], [17], [18], [20], [21], [22], [25], [16] and the references therein. It is noted that this list of papers is by no means complete, but it is representative of the work that is presently being conducted in this area. All these papers have come from physicists and engineering

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community. To our knowledge, the existence and the uniqueness of the solution to the diffraction problem by a chiral layer has never been established. The present paper is devoted to present some results and approaches dealing with the rigorous scattering by a chiral medium.

**2. Chirality and Reduction of Scattering.** Let $\delta$ be a strictly positive parameter. Let $\Omega^i \subset \mathbb{R}^3$ be an open bounded set of class $\mathcal{C}^\infty$, $\Gamma = \partial\Omega^i$ its boundary, $n$ the outward normal. Let $\Omega_1^\delta$ be the chiral layer, $\Omega_2^\delta = \Omega^e \setminus \overline{\Omega_1^\delta}$, $\Gamma^\delta$ the interface between the chiral material $\Omega_1^\delta$ and the exterior domain $\Omega_2^\delta$. We assume that $\Gamma^\delta$ is parallel to the curve $\Gamma$. Let $\Omega_{2,R}^\delta = \Omega_2^\delta \cap \left\{ r < R \right\}$ and $\Sigma_R = \left\{ r = R \right\}$. For $x \in \Omega^e = \mathbb{R}^3 \setminus \overline{\Omega^i}$ sufficiently close to $\Gamma^\delta$, we denote by $x_{\Gamma^\delta}$ the orthogonal projection of $x$ on $\Gamma^\delta$, $s = -(x - x_{\Gamma^\delta}) \cdot n(x_{\Gamma^\delta})$, where $n(x_{\Gamma^\delta})$ is the outward normal to $\Gamma^\delta$. $(x_{\Gamma^\delta}, s)$ is a parameterization of the neighborhood of $\Gamma^\delta$ :

$$x(x_{\Gamma^\delta}, s) = x_{\Gamma^\delta} + s\, n(x_{\Gamma^\delta})$$

The curve $\Gamma$ is such that

$$\Gamma = \left\{ x_{\Gamma^\delta} - \delta n(x_{\Gamma^\delta}), x_{\Gamma^\delta} \in \Gamma^\delta \right\}.$$

We shall need some differential operators on the curve $\Gamma^\delta$ that we will introduce. The normal vector on $\Gamma^\delta$ is the opposite of the gradient of the function $s$. We use this property to define

$$n(x) = -\overrightarrow{\text{grad}}\, s(x) = n(x_{\Gamma^\delta}).$$

The mean curvature is defined as $2\, c(x_{\Gamma^\delta}) = \text{div}\, n$. We also introduce the family of parallel curves $\Gamma_s^\delta$ :

$$\Gamma_s^\delta = \left\{ x \in \Omega^e \text{ such that } x = x_{\Gamma^\delta} + s\, n(x_{\Gamma^\delta}) \,, x_{\Gamma^\delta} \in \Gamma^\delta \right\}.$$

The vector field $n$ is the field of the normal vectors to the curves $\Gamma_s^\delta$. To any function $f$ defined on the curve $\Gamma^\delta$, we associate the function $\tilde{f}$ defined for $x$ near $\Gamma^\delta$ by

$$\tilde{f}(x) = f(x_{\Gamma^\delta}).$$

We also define for any smooth tangential vector field $u$ on $\Gamma^\delta$, $\tilde{u}$ as

$$\tilde{u}(x_{\Gamma^\delta} + sn(x_{\Gamma^\delta})) = u(x_{\Gamma^\delta}) - s\,\mathcal{R}\,u(x_{\Gamma^\delta}),$$

where $\mathcal{R}$ is the tensor of curvature at the point $x_{\Gamma^\delta}$. It is defined as $\mathcal{R} = \overrightarrow{\mathrm{grad}}\, n$. The tangential gradient of the tangential field $u$ is :

$$\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\, u = \overrightarrow{\mathrm{grad}}\, \tilde{u}\,|_{\Gamma^\delta}\ .$$

This vector belongs to the tangent to the curve $\Gamma^\delta$. For any smooth tangential vector field $u$ on $\Gamma^\delta$, let us define $\mathrm{div}_{\Gamma^\delta}\, u$ as

$$\mathrm{div}_{\Gamma^\delta}\, u = \mathrm{div}\,\tilde{u}\,|_{\Gamma^\delta}\ .$$

Let $\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}$ denote the tangential rotational operator on $\Gamma^\delta$. It is such that :

$$\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\, u = \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\, u \wedge n,$$

for any function $u$ defined on the curve $\Gamma^\delta$. The scalar tangential rotational operator is defined as

$$\mathrm{curl}_{\Gamma^\delta}\, u = \mathrm{curl}\,\tilde{u}\,.\,n\,|_{\Gamma^\delta},$$

where $u$ is a tangential vector to $\Gamma^\delta$ and $\tilde{u}(x) = u(x_{\Gamma^\delta})$. It is well-known that

$$\mathrm{curl}_{\Gamma^\delta}\, u = \mathrm{div}_{\Gamma^\delta}\,(u \wedge n). \tag{1}$$

Finally, we recall the following formulae :

LEMMA 2.1. *Let $u$ be a smooth vectorial function defined in $\Omega$, we have :*

$$\tau_0(\mathrm{div}\, u) = \mathrm{div}_{\Gamma^\delta} u_{\Gamma^\delta} + 2\,c(x_{\Gamma^\delta})\; u\,.\,n + \tau_1(u\,.\,n), \tag{2}$$

*and*

$$\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}(u\,.\,n) = \tau_1(u_{\Gamma^\delta}) + n \wedge \mathrm{curl}\, u + \mathcal{R}(u_{\Gamma^\delta}), \tag{3}$$

*where $\tau_0$ and $\tau_1$ are respectively the first and the second trace and $u_{\Gamma^\delta}$ the tangential component of $u$.*

Optical activity of the chiral layer $\Omega_1^\delta$ of thickness $\delta$ can be explained by the direct substitution of the Drude-Born-Fedorov constitutive equations :

$$\begin{cases} D^\delta &= \varepsilon^\delta \left( E^\delta + \beta^\delta \overrightarrow{\mathrm{curl}}\, E^\delta \right), \\ B^\delta &= \mu^\delta \left( H^\delta + \beta^\delta \overrightarrow{\mathrm{curl}}\, H^\delta \right), \end{cases} \tag{4}$$

into Maxwell's equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\, E^\delta &= i\omega\, B^\delta, \\ \overrightarrow{\mathrm{curl}}\, H^\delta &= -i\omega\, D^\delta. \end{cases} \tag{5}$$

It follows the equivalent monochromatic constitutive relations :

$$\begin{cases} \left(1 - (k^\delta \beta^\delta)^2\right) D^\delta &= \varepsilon^\delta E^\delta + \dfrac{i\beta^\delta}{\omega} (k^\delta)^2\, H^\delta, \\ \left(1 - (k^\delta \beta^\delta)^2\right) B^\delta &= \mu^\delta H^\delta - \dfrac{i\beta^\delta}{\omega} (k^\delta)^2\, E^\delta, \end{cases} \tag{6}$$

where the function $\varepsilon^\delta$ is the electric permittivity, $\mu^\delta$ the magnetic permeability and $\beta^\delta$ the chirality admittance and

$$k^\delta \;=\; \omega\sqrt{\varepsilon^\delta\,\mu^\delta} \tag{7}$$

is simply a shorthand notation and does not represent any wavenumber inside the chiral medium. Throughout what follows we assume that $|k^\delta\beta^\delta| \neq 1$. We also assume that $\varepsilon^\delta = \varepsilon_2, \mu^\delta = \mu_2$ and $\beta^\delta = 0$ outside the chiral layer $\Omega_1^\delta$ where $\varepsilon_2$ and $\mu_2$ are two strictly positive constants. Now, combining (5) to (6) gives

$$\left\{\begin{array}{rcl} \overrightarrow{\mathrm{curl}}\,E^\delta &=& (\gamma^\delta)^2\,\beta^\delta E^\delta + i\omega\mu^\delta(\dfrac{\gamma^\delta}{k^\delta})^2\,H^\delta, \\ \overrightarrow{\mathrm{curl}}\,H^\delta &=& (\gamma^\delta)^2\,\beta^\delta\,H^\delta - i\omega\varepsilon^\delta(\dfrac{\gamma^\delta}{k^\delta})^2\,E^\delta. \end{array}\right. \tag{8}$$

In these equations, the parameter $\gamma^\delta$ is defined as :

$$(\gamma^\delta)^2 \;=\; \frac{(k^\delta)^2}{1-(\,k^\delta\,\beta^\delta\,)^2}. \tag{9}$$

We are interested in the scattering problem for a plane incident field $E^{in}$ defined as : $E^{in}(x) \;=\; p\,e^{i\,\omega\;q\,.\,x}$, where the vectors $p, q \in \mathbb{R}^3$ must satisfy $q\,.\,q \;=\; \mu_2\varepsilon_2$ and $q\,.\,p \;=\; 0$. Then the pair $(E^{in}, \overrightarrow{\mathrm{curl}}E^{in}/i\omega\mu_2)$ satisfies the Maxwell equations in $\Omega_2^\delta$ and our scattering problem can be formulated as follows.

$$\left\{\begin{array}{l} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,E^\delta - \omega^2\,\beta^\delta\varepsilon^\delta\overrightarrow{\mathrm{curl}}\,E^\delta - \omega^2\,\overrightarrow{\mathrm{curl}}\,(\beta^\delta\varepsilon^\delta)\,E^\delta \\ \quad -\omega^2\,\varepsilon^\delta\,E^\delta \;=\; 0 \text{ in } \Omega^e, \\ n\,\wedge\,E^\delta \;=\; 0 \quad \text{on } \Gamma, \\ E^\delta - E^{in} \text{satisfies the classical radiation condition.} \end{array}\right. \tag{10}$$

**3. Uniqueness and Existence Results for the Diffraction Problem by the Chiral Layer.** This section is devoted to prove the existence and uniqueness of a field $E^\delta$ satisfying the equations (10). We use the technique of boundary integral equations. We shall reduce the Drude-Born-Fedorov equations (10) to a boundary integral equation on $\Gamma^{\delta,h} \;=\Big\{x \;=\; (x_{\Gamma^\delta}, s \;=\; h)\Big\}$. First, we consider the following boundary-value problems :

$$\left\{\begin{array}{l} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta\,(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,e^\delta - \omega^2\,\beta^\delta\varepsilon^\delta\overrightarrow{\mathrm{curl}}\,e^\delta - \omega^2\,\overrightarrow{\mathrm{curl}}\,\beta^\delta\varepsilon^\delta\,e^\delta \\ \quad -\omega^2\,\varepsilon^\delta\,e^\delta \;=\; 0 \text{ in } \Omega_1^{\delta,h}, \\ n\,\wedge\,e^\delta \;=\; 0 \quad \text{on } \Gamma, \\ n\,\wedge\,e^\delta \;=\; g^\delta \quad \text{on } \Gamma^{\delta,h} \;=\Big\{x = (x_{\Gamma^\delta}, s = h)\Big\}, \end{array}\right. \tag{11}$$

and

$$\left\{\begin{array}{l} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}v^\delta - \omega^2\,\varepsilon_2\mu_2\,v^\delta \;=\; 0 \quad \text{in } \Omega_2^{\delta,h}, \\ n\,\wedge\,v^\delta \;=\; g^\delta \quad \text{on } \Gamma^{\delta,h} (=\,\partial\Omega_2^{\delta,h}), \\ v^\delta \text{ satisfies the outgoing radiation condition at infinity,} \end{array}\right. \tag{12}$$

where

$$g^\delta \in TH^{1/2}(\mathrm{div},\Gamma^{\delta,h}) = \Big\{\ c \in (H^{1/2}(\Gamma^{\delta,h}))^3, n\,.\,c = 0 \text{ on } \Gamma^{\delta,h}, \mathrm{div}_{\Gamma^{\delta,h}}\, c \in H^{1/2}(\Gamma^{\delta,h})\ \Big\}$$

is a known vector field. $h$ is an arbitrarily strictly positive constant which must be small enough. $\Omega_1^{\delta,h}$ and $\Omega_2^{\delta,h}$ are given by $\Omega_1^{\delta,h} = \Big\{(x_{\Gamma^\delta}, s), 0 < s < h\Big\}$, and $\Omega_2^{\delta,h} = \mathbb{R}^3 \setminus \overline{\Omega_1^{\delta,h} \cup \Omega^i}$.

The weak form of the Drude-Born-Fedorov equations (11) in a neighborhood of the interface $\Gamma^\delta$ gives the following jump relations on $\Gamma^\delta$:

$$\left\{\begin{array}{l} \Big(e_1^\delta - e_2^\delta\Big) \wedge n = 0, \\ \Big(\overrightarrow{\mathrm{curl}}\, e_1^\delta - \dfrac{\mu_1}{\mu_2}\overrightarrow{\mathrm{curl}}\, e_2^\delta\Big) \wedge n = 0, \end{array}\right. \tag{13}$$

where $e_1^\delta = e^\delta\,|_{\Omega_1^\delta}$ and $e_2^\delta = e^\delta\,|_{\Omega_1^{\delta,h}\setminus\overline{\Omega_1^\delta}}$, if we assume that the jump of $\beta^\delta$ across the interface $\Gamma^\delta$ is null.

Let

$$X^\delta = \Big\{\ H(\overrightarrow{\mathrm{curl}}, \Omega_1^{\delta,h}), w^\delta \wedge n = 0 \text{ on } \partial\Omega_1^{\delta,h} = \Gamma \cup \Gamma^{\delta,h}\ \Big\},$$

where

$$H(\overrightarrow{\mathrm{curl}}, \Omega_1^{\delta,h}) = \Big\{\ w^\delta \in (L^2(\Omega_1^{\delta,h}))^3, \overrightarrow{\mathrm{curl}}\, w^\delta \in (L^2(\Omega_1^{\delta,h})^3\ \Big\}.$$

Let $\tilde{g}^\delta$ be in $(H^1(\Omega_1^{\delta,h}))^3$ such that $n \wedge \tilde{g}^\delta = g^\delta$ on $\Gamma^{\delta,h}$ and $n \wedge \tilde{g}^\delta = 0$ on $\Gamma$. We define $\tilde{e}^\delta$ by

$$\tilde{e}^\delta = e^\delta - \tilde{g}^\delta.$$

It is easily seen that

$$\begin{array}{l} \overrightarrow{\mathrm{curl}}\, \dfrac{1}{\mu^\delta}\,(1 - \omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,\tilde{e}^\delta - \omega^2\,\beta^\delta\varepsilon^\delta\overrightarrow{\mathrm{curl}}\,\tilde{e}^\delta - \omega^2\,\overrightarrow{\mathrm{curl}}\,\beta^\delta\varepsilon^\delta\,\tilde{e}^\delta - \omega^2\,\varepsilon^\delta\,\tilde{e}^\delta \\ = -\overrightarrow{\mathrm{curl}}\, \dfrac{1}{\mu^\delta}\,(1 - \omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,\tilde{g}^\delta + \omega^2\,\beta^\delta\varepsilon^\delta\overrightarrow{\mathrm{curl}}\,\tilde{g}^\delta + \omega^2\,\overrightarrow{\mathrm{curl}}\,\beta^\delta\varepsilon^\delta\,\tilde{g}^\delta \\ +\omega^2\,\varepsilon^\delta\,\tilde{g}^\delta \text{ in } (\mathcal{D}'(\Omega_1^{\delta,h}))^3, \end{array} \tag{14}$$

where $\mathcal{D}'(\Omega_1^{\delta,h})$ is the space of all distributions in $\Omega_1^{\delta,h}$. If we multiply (14) by a test function $v^\delta \in X^\delta$, integrate over $\Omega_1^{\delta,h}$, and use integration by parts and the jump relations (13), we obtain

$$
\begin{aligned}
&\int_{\Omega_1^{\delta,h}} \frac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,\tilde{e}^\delta\,.\,\overrightarrow{\mathrm{curl}}\,v^\delta \;+\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,\tilde{e}^\delta\,.\,v^\delta\\
&-\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\tilde{e}^\delta\,.\,\overrightarrow{\mathrm{curl}}\,v^\delta-\omega^2\int_{\Omega_1^{\delta,h}}\varepsilon^\delta\,\tilde{e}^\delta\,.\,v^\delta\\
&=-\int_{\Omega_1^{\delta,h}}\frac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,\tilde{g}^\delta\,.\,\overrightarrow{\mathrm{curl}}\,v^\delta-\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,\tilde{g}^\delta\,.\,v^\delta\\
&+\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\tilde{g}^\delta\,.\,\overrightarrow{\mathrm{curl}}\,v^\delta+\omega^2\int_{\Omega_1^{\delta,h}}\varepsilon^\delta\,\tilde{g}^\delta\,.\,v^\delta.
\end{aligned}
\tag{15}
$$

Our variational problem is then to find the vector field $e^\delta \in H(\overrightarrow{\mathrm{curl}},\Omega_1^{\delta,h})$ such that (15) holds for all $v^\delta \in X^\delta$. Because the injection $X^\delta \hookrightarrow (L^2(\Omega^{\delta,h}))^3$ is not compact, we can not apply the Fredholm theory to (15) to show the existence and uniqueness of a solution of this problem in $X^\delta$. We shall prove Lemma 3.1 by using the generalized Lax-Milgram lemma of Babuška and Brezzi [11]. We discuss the unique solvability of (15) and (12). We may state and prove the following results.

LEMMA 3.1. *Assume that* (11) *has at most one solution. Then the variational problem* (15) *has a unique solution in* $H(\overrightarrow{\mathrm{curl}},\Omega_1^{\delta,h})$.

LEMMA 3.2. *The boundary value problem* (12) *has a unique solution in* $H^{1,loc}(\Omega_2^{\delta,h})$.

It is well known that (12) has exactly one solution (*cf.* [2] for instance). Now, our aim is to sketch a proof of lemma 3.1. By essentially the same arguments as in Abboud [1], we can establish the Fredholm alternative for the variational problem (15). We seek for $e^\delta \in H(\overrightarrow{\mathrm{curl}},\Omega_1^{\delta,h})$ and we decompose it into a field and a gradient :

$$
e^\delta \;=\; u^\delta+\overrightarrow{\mathrm{grad}}p^\delta+\tilde{g}^\delta,
\tag{16}
$$

where $u^\delta \in H(\overrightarrow{\mathrm{curl}},\Omega_1^{\delta,h})$ satisfies

$$
\mathrm{div}\,(\,\varepsilon^\delta\,u^\delta-\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,u^\delta)\;=\;0\quad\text{in }\Omega_1^{\delta,h},
\tag{17}
$$

$u^\delta \wedge n = 0$ on $\Gamma\cup\Gamma^{\delta,h}$ and $p^\delta \in H_0^1(\Omega_1^{\delta,h})$. (15) will lead to a saddle point problem involving the form $a^\delta$ defined as

$$
\begin{aligned}
a^\delta(u^\delta,w^\delta) \;=\;&\int_{\Omega_1^{\delta,h}}\frac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,u^\delta\,.\,\overrightarrow{\mathrm{curl}}\,\overline{w}^\delta+\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,u^\delta\,.\,\overrightarrow{\mathrm{curl}}\,\overline{w}^\delta\\
&-\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,u^\delta\,.\,\overline{w}^\delta-\omega^2\int_{\Omega_1^{\delta,h}}\varepsilon^\delta\,u^\delta\,.\,\overline{w}^\delta,
\end{aligned}
$$

for all $w^\delta \in X^\delta$, and the form $b^\delta$ given as

$$
\begin{aligned}
b^\delta(w^\delta,q^\delta) \;&=\; a^\delta(\overrightarrow{\mathrm{grad}}q^\delta,w^\delta)\\
&=\;\omega^2\int_{\Omega_1^{\delta,h}}\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{grad}}q^\delta\,.\,\overrightarrow{\mathrm{curl}}\,\overline{w}^\delta-\omega^2\int_{\Omega_1^{\delta,h}}\varepsilon^\delta\,\overrightarrow{\mathrm{grad}}q^\delta\,.\,\overline{w}^\delta,
\end{aligned}
$$

for all $w^\delta \in X^\delta$ and $q^\delta \in H_0^1(\Omega_1^{\delta,h})$. Now we show how to split the variational problem (15) into two distinct equations. First, by multiplying (17) by $q \in H_0^1(\Omega_1^{\delta,h})$ and integrating by parts over $\Omega_1^{\delta,h}$, we obtain

$$b^\delta(u^\delta, q^\delta) \;=\; 0, \quad \text{for all } q^\delta \in H_0^1(\Omega_1^{\delta,h}).$$

Second, multiplying (14) by $w \in X^\delta$ and integrating by parts over $\Omega_1^{\delta,h}$, give by using the decomposition (16) of the electric field $e^\delta$

$$a^\delta(u^\delta, w^\delta) + b^\delta(w^\delta, p^\delta) \;=\; -a^\delta(\tilde{g}^\delta, w^\delta), \quad \text{for all } w^\delta \in X^\delta.$$

Then, we obtain

$$\text{(18)} \qquad \begin{cases} a^\delta(u^\delta, w^\delta) + b^\delta(w^\delta, p^\delta) \;=\; -a^\delta(\tilde{g}^\delta, w^\delta), \quad \text{for all } w^\delta \in X^\delta, \\ b^\delta(u^\delta, q^\delta) \;=\; 0, \quad \text{for all } q^\delta \in H_0^1(\Omega_1^{\delta,h}). \end{cases}$$

It is clear that there exist two strictly positive constants $C_1$ and $C_2$ such that :

$$|a^\delta(u^\delta, u^\delta)| \;\geq\; C_1 \,||\overrightarrow{\text{curl}}\, u^\delta||^2_{(L^2(\Omega_1^{\delta,h}))^3} - C_2 \,||u^\delta||^2_{(L^2(\Omega_1^{\delta,h}))^3}.$$

It is easily shown that the form $b^\delta$ satisfies the inf-sup condition : for each $q^\delta$ in $H_0^1(\Omega_1^{\delta,h})$, it is clear that $\overrightarrow{\text{grad}}\, q^\delta$ satisfies $\overrightarrow{\text{grad}} q^\delta \wedge n \;=\; 0$ on $\Gamma \cup \Gamma^\delta$ and there exists a strictly positive constant $C_3$ such that

$$|b^\delta(\overrightarrow{\text{grad}} q^\delta, q^\delta)| \;\geq\; C_3 \;\; ||q^\delta||^2_{H_0^1(\Omega_1^{\delta,h})}.$$

On the other hand, if we assume that $\overrightarrow{\text{grad}}(\beta^\delta \varepsilon^\delta) \in (L^\infty(\Omega_1^{\delta,h}))^3$ then the imbedding $N^\delta \hookrightarrow (L^2(\Omega_1^{\delta,h}))^3$ is compact, where

$$N^\delta \;= \Big\{ u^\delta \in H(\overrightarrow{\text{curl}}, \Omega_1^{\delta,h}), \text{div}\,(\,\varepsilon^\delta\, u^\delta - \beta^\delta\, \varepsilon^\delta\, \overrightarrow{\text{curl}}\, u^\delta) \;=\; 0, u^\delta \wedge n \;=\; 0 \text{ on } \partial\Omega_1^{\delta,h} \;\Big\}$$

is the kernel of the form $b^\delta$. Therefore, $a^\delta$ is a compact perturbation of a coercive form on the kernel $N^\delta$ of $b^\delta$. It follows that the Fredholm alternative holds and then, uniqueness implies existence of the solution of (18). In order to complete the proof of lemma 3.1 we should prove the following decomposition lemma which may be considered as a generalization of the Hodge decomposition.

LEMMA 3.3. *Let $\tilde{e}^\delta$ be in*

$$X^\delta \;= \Big\{ v^\delta \in H(\overrightarrow{\text{curl}}, \Omega_1^{\delta,h}), v^\delta \wedge n = 0 \text{ on } \partial\Omega_1^{\delta,h} = \Gamma^{\delta,h} \cup \Gamma \Big\}.$$

*Then, there exist $u^\delta \in N^\delta$ and $p^\delta \in H_0^1(\Omega_1^{\delta,h})$ such that*

$$\text{(19)} \qquad \tilde{e}^\delta \;=\; u^\delta + \nabla p^\delta \quad \text{in } \Omega_1^{\delta,h}.$$

*Proof.* We define $f^\delta$ by

$$f^\delta \;=\; \varepsilon^\delta \beta^\delta\, \overrightarrow{\text{curl}}\, \tilde{e}^\delta - \varepsilon^\delta\, \tilde{e}^\delta.$$

$\tilde{e}^\delta \in H(\overrightarrow{\mathrm{curl}}, \Omega^{\delta,h})$ yields $f^\delta \in (L^2(\Omega^{\delta,h}))^3$. It is well-known that the transmission problem

$$\begin{cases} \operatorname{div} \varepsilon^\delta \nabla \varphi^\delta_{c_1,c_2} = \operatorname{div} f^\delta & \text{in } \Omega_1^{\delta,h}, \\ \varphi^\delta_{c_1,c_2} = c_1 & \text{on } \Gamma^{\delta,h}, \\ \varphi^\delta_{c_1,c_2} = c_2 & \text{on } \Gamma, \end{cases}$$

has a unique solution in $H^1(\Omega^{\delta,h})$ for any $(c_1, c_2) \in \mathbb{C} \times \mathbb{C}$. Let $p^\delta$ be defined by

$$p^\delta = \varphi^\delta_{0,0} \in H^1_0(\Omega_1^{\delta,h}),$$

and $u^\delta$ given by

$$u^\delta = \tilde{e}^\delta - \nabla \varphi^\delta_{0,0}.$$

It is clear that $u^\delta$ is in $N^\delta$ and so, the decomposition (19) holds. □ Now, using Lemma 3.3, the uniqueness of a solution in $H(\overrightarrow{\mathrm{curl}}, \Omega_1^{\delta,h})$ to (11) gives the uniqueness of a solution to (18). The proof of Lemma 3.1 is then over.

Throughout what follows we assume that there exists $h > 0$ such that 0 is not an eigenvalue of the Drude-Born-Fedorov operator

$$\overrightarrow{\mathrm{curl}} \frac{1}{\mu^\delta} (1 - \omega^2 \varepsilon^\delta \mu^\delta (\beta^\delta)^2) \overrightarrow{\mathrm{curl}} \,. - \omega^2 \beta^\delta \varepsilon^\delta \overrightarrow{\mathrm{curl}} \,. - \omega^2 \overrightarrow{\mathrm{curl}} \beta^\delta \varepsilon^\delta \,. - \omega^2 \varepsilon^\delta \,. \quad \text{in } \Omega_1^{\delta,h}$$

with the boundary condition $n \wedge . = 0$ on $\Gamma^{\delta,h} \cup \Gamma$. The existence of such $h$ can be verified but it is not our purpose here.

Now, in order to reduce the diffraction problem by a chiral layer covering a perfectly conducting object to a boundary integral equation on $\Gamma^{\delta,h}$, we consider the two pseudo-differential operators on $\Gamma^{\delta,h}$ defined as :

$$(20) \qquad T_1(\beta, \omega) : g^\delta \in TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h}) \longmapsto \overrightarrow{\mathrm{curl}}\, e^\delta \wedge n \in TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h}),$$

where $e^\delta$ is the unique solution of (11) and

$$(21) \qquad T_2(\omega) : g^\delta \in TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h}) \longmapsto \overrightarrow{\mathrm{curl}}\, v^\delta \wedge n \in TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h}),$$

where $v^\delta$ is the unique solution of (12) satisfying the radiation condition. We have to show that $T_1(\beta, \omega)\,(g^\delta)$ and $T_2(\omega)\,(g^\delta)$ are in $TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h})$ for all $g^\delta$ in $TH^{1/2}(\operatorname{div}, \Gamma^{\delta,h})$. Let us define $h^\delta$ in $\Omega_1^{\delta,h}$ by

$$\overrightarrow{\mathrm{curl}}\, e^\delta = (\gamma^\delta)^2 \beta^\delta e^\delta + i\omega \mu^\delta (\frac{\gamma^\delta}{k^\delta})^2 h^\delta.$$

(11) yields a magnetic-boundary value problem for $h^\delta$, and a solution can be found through a variational method similar to (15). $e^\delta$ and $h^\delta$ are in $\left(H^1(\Omega_1^{\delta,h} \cap \Omega_2^{\delta,h'})\right)^3$ for all $0 < h' < h$. By the trace theorem, we find that

$$\overrightarrow{\mathrm{curl}}\, e^\delta \wedge n \in TH^{1/2}(\Gamma^{\delta,h}),$$

and furthermore

$$\begin{aligned} \operatorname{div}_{\Gamma^{\delta,h}} \left(\overrightarrow{\mathrm{curl}}\, e^\delta \wedge n\right) &= \overrightarrow{\mathrm{curl}}\, \overrightarrow{\mathrm{curl}}\, e^\delta \,. n \\ &= \omega^2 \varepsilon_2 \mu_2 \, e^\delta \,. n \,|_{\Gamma^{\delta,h}} \in H^{1/2}(\Gamma^{\delta,h}). \end{aligned}$$

Similarly, we define $w^\delta$ by

$$\overrightarrow{\mathrm{curl}}\, v^\delta \;=\; i\omega\mu_2\, w^\delta.$$

$w^\delta$ is the unique outgoing solution of the Maxwell equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\, w^\delta - \omega^2\varepsilon_2\mu_2\, w^\delta \;=\; 0 \quad \text{in } \Omega_2^{\delta,h}, \\ i\omega\mu_2 w^\delta \,.\, n \;=\; \mathrm{div}_{\Gamma^{\delta,h}}\, g^\delta \;\in\; H^{1/2}(\Gamma^{\delta,h}), \quad \text{on } \Gamma^{\delta,h}. \end{cases}$$

It is well known that, under the regularity assumption $g^\delta \in H^{1/2}(\Gamma^{\delta,h})$, $v^\delta$ and $w^\delta$ are in $(H^{1,loc}(\Omega_2^{\delta,h}))^3$. Thus we have

$$\overrightarrow{\mathrm{curl}}\, v^\delta \wedge n \;=\; i\omega\mu_2\, w^\delta \wedge n \in TH^{1/2}(\Gamma^{\delta,h}),$$

and

$$\mathrm{div}_{\Gamma^{\delta,h}}\left(\overrightarrow{\mathrm{curl}}\, v^\delta \wedge n\right) \;=\; \omega^2\varepsilon_2\mu_2\, v^\delta \,.\, n\,|_{\Gamma^{\delta,h}} \;\in\; H^{1/2}(\Gamma^{\delta,h}).$$

The jump relations lead to the boundary integral equation

$$T_1(\beta,\omega)(g^\delta) - T_2(\omega)(g^\delta) \;=\; g^{in}, \quad \text{on } \Gamma^{\delta,h}, \tag{22}$$

where

$$g^{in} \;=\; T_2(\omega)(n \wedge E^{in}) - \overrightarrow{\mathrm{curl}}\, E^{in} \wedge n,$$

and

$$g^\delta \;=\; n \wedge E^\delta,$$

$E^\delta$ satisfies (10). Using lemma 3.1 and lemma 3.2, it is clear that the unique solvability of (8) is equivalent to the unique solvability of (22). To discuss existence and uniqueness of solutions of (22) let us introduce the following operators

$$L_0 \;=\; T_1(\beta=0,\omega) - T_2(\omega),$$

$$L_1 \;=\; T_1(\beta,\omega) - T_2(\omega),$$

and

$$L_t \;=\; t\,L_1 + (1-t)\,L_0, \qquad \text{for all } 0 \le t \le 1.$$

The diffraction problem of Maxwell's equations by a layer of dielectric achiral material has a unique solution (*cf.* [2]). Therefore, we have the following result.

LEMMA 3.4. *The operator* $L_0 : TH^{1/2}(\mathrm{div},\Gamma^{\delta,h}) \mapsto TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$ *is an isomorphism.*

*Proof.* First we prove the injectivity of the operator $L_0$. Let $g \in TH^{1/2}(\Gamma^{\delta,h})$ be a solution to

$$L_0 g \;=\; 0. \tag{23}$$

Let $E_1$ be the unique solution of the Maxwell equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,\overrightarrow{\mathrm{curl}}\, E_1 - \omega^2\varepsilon^\delta\, E_1 \;=\; 0 \quad \text{in } \Omega_1^{\delta,h}, \\ n \wedge E_1 \;=\; 0 \quad \text{on } \Gamma, \\ n \wedge E_1 \;=\; g \quad \text{on } \Gamma^{\delta,h}. \end{cases}$$

Let $E_2$ be given as the unique solution to the following problem.

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\,E_2 - \omega^2\varepsilon_2\mu_2\,E_2 \;=\; 0 \quad \text{in } \Omega_2^{\delta,h}, \\ n \wedge E_2 \;=\; g \quad \text{on } \Gamma^{\delta,h}, \\ E_2 \text{ satisfies the outgoing radiation condition at infinity.} \end{cases}$$

The field $E_2$ is in $(H^{1,loc}(\Omega_2^\delta))^3$. We define the field $E$ by

$$E \;=\; \begin{cases} E_1 \quad \text{in } \Omega_1^{\delta,h}, \\ E_2 \quad \text{in } \Omega_2^{\delta,h}. \end{cases}$$

$L_0 g = 0$ implies that the jump of $\overrightarrow{\mathrm{curl}}\,E \wedge n$ across $\Gamma^{\delta,h}$ is null. It follows that the field $E$ satisfies the Maxwell equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,\overrightarrow{\mathrm{curl}}\,E - \omega^2\varepsilon^\delta\,E \;=\; 0 \quad \text{in } \Omega^e, \\ n \wedge E \;=\; 0 \quad \text{on } \Gamma, \\ E \text{ satisfies the outgoing radiation condition at infinity,} \end{cases}$$

in a weak sense. We deduce from [2] that $E = 0$. Then, it follows that

$$g \;=\; n \wedge E|_{\Gamma^{\delta,h}} \;=\; 0.$$

Second, we verify the surjectivity of the operator $L_0$. Let $g^{in}$ be in $TH^{1/2}(\mathrm{div}, \Gamma^{\delta,h})$. Let $E^{in} \in (H^{1,loc}(\Omega_2^{\delta,h}))^3$ be defined as the unique solution of the following problem

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\,E^{in} - \omega^2\varepsilon_2\mu_2\,E^{in} \;=\; 0 \quad \text{in } \Omega_2^{\delta,h}, \\ T_2(\omega)(n \wedge E^{in}) - \overrightarrow{\mathrm{curl}}\,E^{in} \wedge n \;=\; g^{in} \quad \text{on } \Gamma^{\delta,h}, \\ E^{in} \text{ satisfies the downgoing radiation condition at infinity.} \end{cases}$$

The uniqueness of $E^{in}$ follows from the fact that the boundary condition

$$T_2(\omega)(n \wedge E^{in}) - \overrightarrow{\mathrm{curl}}\,E^{in} \wedge n \;=\; 0 \quad \text{on } \Gamma^{\delta,h},$$

implies (from the definition of the operator $T_2(\omega)$) that $E^{in}$ satisfies the outgoing radiation condition. But it also satisfies the downgoing radiation condition and then, it is a trivial solution. To show the existence of a solution, we make use of the boundary integral equation method. If we make an ansatz for $E^{in}$ in the form

$$E^{in}(x) = -\omega\sqrt{\varepsilon_2\mu_2}\overrightarrow{\mathrm{curl}}\int_{\Gamma^{\delta,h}} a(y)\frac{e^{-i\omega\sqrt{\varepsilon_2\,\mu_2}|x-y|}}{4\pi|x-y|}ds(y) \quad \text{in } \Omega_2^{\delta,h},$$

where $a$ is a vector in $TH^{1/2}(\mathrm{div}, \Gamma^{\delta,h})$, we obtain that $a$ satisfies the following integral equation on $\Gamma^{\delta,h}$

$$-i\sqrt{\frac{\varepsilon_2}{\mu_2}}n \wedge \overrightarrow{\mathrm{curl}}\overrightarrow{\mathrm{curl}}\int_{\Gamma^{\delta,h}} a(y)\Big(\frac{e^{-i\omega\sqrt{\varepsilon_2\,\mu_2}|x-y|}}{4\pi|x-y|} + \frac{e^{i\omega\sqrt{\varepsilon_2\,\mu_2}|x-y|}}{4\pi|x-y|}\Big)ds(y) = \frac{1}{i\omega\mu_2}\,g^{in},$$

which is of Fredholm type. The existence of $E^{in}$ follows then from the uniqueness. From [2], the Maxwell system

$$\begin{cases} \overrightarrow{\text{curl}}\, E \;=\; i\omega\mu^{\delta}\, H \quad \text{in } \Omega^{e}, \\ \overrightarrow{\text{curl}}\, H \;=\; -i\omega\varepsilon^{\delta}\, E \quad \text{in } \Omega^{e}, \\ E - E^{in} \text{ satisfies the outgoing radiation condition at infinity.} \end{cases}$$

with the boundary condition $E \wedge n = 0$ on $\Gamma$ has a unique solution

$$(E,H) \in \Big(H^{1,loc}(O)\Big)^{3} \times \Big(H^{1,loc}(O)\Big)^{3}, \quad \forall\, \overline{O} \subset \Omega_{2}^{\delta,h}.$$

By construction, the tangential field $n \wedge E \in TH^{1/2}(\Gamma^{\delta,h})$ satisfies

$$L_0(n \wedge E) \;=\; -g^{in}.$$

In order to complete the proof of this lemma, we should verify that $\text{div}_{\Gamma^{\delta,h}}\,(n \wedge E)$ is in $H^{1/2}(\Gamma^{\delta,h})$. Using the identity

$$\text{div}_{\Gamma^{\delta,h}}\,(n \wedge E) \;=\; -n\,.\,\overrightarrow{\text{curl}}\, E|_{\Gamma^{\delta,h}},$$

we obtain

$$\text{div}_{\Gamma^{\delta,h}}\,(n \wedge E) \;=\; -i\omega\mu_2\, n\,.\,H \in H^{1/2}(\Gamma^{\delta,h}),$$

and then the proof is complete.□

Our next result plays an important role in the proof of the existence and uniqueness results for the Drude-Born-Fedorov equations (8).

LEMMA 3.5.

$$\Im m\,(L_t\, g, n \wedge g) \;=\; 0 \Longrightarrow g \;=\; 0, \quad \text{for all } t \in\ [0,1\,]. \tag{24}$$

*Proof.* Multiplying (11) by $\overline{e}^{\delta}$ and integrating by parts, we obtain

$$\Im m\,(T_1(\beta,\omega)\, g^{\delta}, n \wedge g^{\delta}) \;=\; 0.$$

Now, multiplying (12) by $\overline{v}^{\delta}$ and integrating by parts over $\Omega_{2,R}^{\delta,h} \;=\; \Omega_{2}^{\delta,h} \cap \Big\{r < R\Big\}$ yield

$$\Im m\,(T_2(\omega)\, g^{\delta}, n \wedge g^{\delta}) \;=\; \Im m\,\Big(\, T_R(\omega)(n \wedge (n \wedge v^{\delta})\,), n \wedge (n \wedge v^{\delta})\,\Big),$$

where $T_R(\omega)$ is the pseudo-differential operator on $\Sigma_R$ given by

$$-n \wedge (n \wedge v^{\delta})|_{\Sigma_R} \longmapsto \overrightarrow{\text{curl}}\, v^{\delta} \wedge n|_{\Sigma_R}.$$

Here $v^{\delta}$ is the solution to the Maxwell equations on $\Omega_{2}^{\delta,h}$ satisfying the radiation condition. From the following property (*cf.* Appendix A)

$$\Im m\,(T_R(\omega)(n \wedge (n \wedge v^{\delta})), n \wedge (n \wedge v^{\delta})) \;=\; 0 \Longrightarrow n \wedge v^{\delta} \;=\; 0,$$

and by the uniqueness theorem of Cauchy-Kowaleska, (24) holds for all $t$ in [ 0, 1 ]. □

We write the operator $L_1$ as

$$L_1 \;=\; L_0 + \Big( T_1(\beta,\omega) - T_1(\beta=0,\omega) \Big).$$

We show that equation (22) is uniquely solvable in $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$.

LEMMA 3.6. *Assume that the Maxwell equations*

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\,u - \omega^2\,\varepsilon_2\,\mu_2\,u \;=\; 0 \quad \text{in } \Omega_1^{\delta,h}, \\ n \wedge u \;=\; 0 \quad \text{on } \Gamma, \\ n \wedge u \;=\; 0 \quad \text{on } \Gamma^{\delta,h}, \end{cases}$$

*have only the trivial solution in* $(H^1(\Omega_1^{\delta,h}))^3$. *Then the operator* $T_1(\beta=0,\omega) - T_1(\beta,\omega)$ *is compact in* $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$.

*Proof.* Let $g$ be in $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$. We define $v_0$ as the unique solution of Maxwell's equations :

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\,v_0 - \omega^2\,\varepsilon_2\,\mu_2\,v_0 \;=\; 0 \quad \text{in } \Omega_1^{\delta,h}, \\ n \wedge v_0 \;=\; 0 \quad \text{on } \Gamma, \\ n \wedge v_0 \;=\; g \quad \text{on } \Gamma^{\delta,h}, \end{cases}$$

and $v_1$ as the unique solution of the following boundary value problem :

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,(1-\omega^2\varepsilon^\delta\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,v_1 - \omega^2\,\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,v_1 \\ -\omega^2\,\overrightarrow{\mathrm{curl}}\,\beta^\delta\,\varepsilon^\delta\,v_1 \;-\omega^2\,\varepsilon^\delta\,v_1 \;=\; 0 \quad \text{in } \Omega_1^{\delta,h}, \\ n \wedge v_1 \;=\; 0 \quad \text{on } \Gamma, \\ n \wedge v_1 \;=\; g \quad \text{on } \Gamma^{\delta,h}. \end{cases}$$

Therefore,

$$\Big( T_1(\beta=0,\omega) - T_1(\beta,\omega) \Big)\,g \;=\; \overrightarrow{\mathrm{curl}}\,v \wedge n,$$

where $v = v_0 - v_1$. $v$ satisfies

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\,v - \omega^2\,\varepsilon_2\,\mu_2\,v \;=\; w \quad \text{in } \Omega_1^{\delta,h}, \\ n \wedge v \;=\; 0 \quad \text{on } \Gamma, \\ n \wedge v \;=\; 0 \quad \text{on } \Gamma^{\delta,h}, \end{cases}$$

where

$$w \;=\; \begin{cases} \omega^2\varepsilon_2\mu_2 v_1 - 2(\gamma_1)^2\beta\overrightarrow{\mathrm{curl}}\,v_1 - (\gamma_1)^2 v_1 \quad \text{in } \Omega_1^\delta, \\ 0 \quad \text{in } \Omega_1^{\delta,h} \setminus \overline{\Omega_1^\delta}. \end{cases}$$

Since $w \;=\; 0$ in a neighborhood of $\Gamma^{\delta,h}$, it follows under the regularity assumption of $\Gamma^{\delta,h}$ that

$$T_1(\beta=0,\omega) \;=\; T_1(\beta,\omega) \bmod OPS^{-\infty}(\Gamma^{\delta,h}),$$

(*cf.* [26]) and so, the proof of the lemma is over. □ Since the operator $L_0$ is invertible, the operator $L_1$ from $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$ into itself satisfies then the Fredholm

alternative. From the uniqueness lemma 3.5, it follows that the mapping $L_1$ maps $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$ onto $TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$. Thus, we have the following theorem.

THEOREM 3.7. *The boundary value problem* (10) *has a unique solution.*

*Proof.* If $E$ is a solution of (10) then $n\wedge E\in TH^{1/2}(\mathrm{div},\Gamma^{\delta,h})$ is a solution of the integral equation $L_1\,u\,=\,g^{in}$. Conversely, let $g$ be the unique solution of the above integral equation. Let $E$ be defined by

$$E\,=\,\begin{cases} E_1 \quad \text{in } \Omega_1^{\delta,h},\\ E_2+E^{in} \quad \text{in } \Omega_2^{\delta,h},\end{cases}$$

where $E_1$ is the unique solution of

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\dfrac{1}{\mu^\delta}\,(1-\omega^2\varepsilon\mu^\delta(\beta^\delta)^2)\,\overrightarrow{\mathrm{curl}}\,E_1-\omega^2\,\beta^\delta\,\varepsilon^\delta\,\overrightarrow{\mathrm{curl}}\,E_1\\ -\omega^2\,\overrightarrow{\mathrm{curl}}\,\beta^\delta\,\varepsilon^\delta\,E_1\,-\omega^2\,\varepsilon^\delta\,E_1\,=\,0 \quad \text{in } \Omega_1^{\delta,h},\\ n\wedge E_1\,=\,0 \quad \text{on } \Gamma,\\ n\wedge E_1\,=\,g \quad \text{on } \Gamma^{\delta,h},\end{cases}$$

and $E_2$ is the unique solution of Maxwell's equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}v^\delta-\omega^2\,\varepsilon_2\mu_2\,E_2\,=\,0 \quad \text{in } \Omega_2^{\delta,h},\\ n\wedge E_2\,=\,g \quad \text{on } \Gamma^{\delta,h},\\ E_2 \text{ satisfies the outgoing radiation condition at infinity.}\end{cases}$$

It follows that $E$ satisfies the equations (10) and the proof of the theorem is over.□

**4. Generalized Impedance Boundary Conditions.** In this section, we assume that $\Omega_1^\delta$ and $\Omega_2^\delta$ are filled with materials in such a way that the magnetic permeability $\mu^\delta$ satisfies

$$\mu^\delta=\begin{cases}\mu_1 \quad \text{in } \Omega_1^\delta,\\ \mu_2 \quad \text{in } \Omega_2^\delta,\end{cases} \tag{25}$$

the dielectric coefficient $\varepsilon^\delta$ satisfies

$$\varepsilon^\delta=\begin{cases}\varepsilon_1 \quad \text{in } \Omega_1^\delta,\\ \varepsilon_2 \quad \text{in } \Omega_2^\delta,\end{cases} \tag{26}$$

and the chirality admittance $\beta^\delta$ satisfies

$$\beta^\delta=\begin{cases}\beta_1\,=\,\beta \quad \text{in } \Omega_1^\delta,\\ \beta_2\,=\,0 \quad \text{in } \Omega_2^\delta,\end{cases} \tag{27}$$

where $\mu_1,\mu_2,\varepsilon_1,\varepsilon_2$ are positive constants such that $\mu_1\neq\mu_2,\varepsilon_1\neq\varepsilon_2$ and $\beta$ a real-constant.
Setting

$$E_j^\delta\,=\,E^\delta|_{\Omega_j^\delta},\quad (j=1,2),$$

the equations (8) yield

$$\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}E_1^\delta-2\gamma_1^2\,\beta\,\overrightarrow{\mathrm{curl}}\,E_1^\delta-\gamma_1^2\,E_1^\delta\,=\,0 \quad \text{in } \Omega_1^\delta, \tag{28}$$

and

$$\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}E_2^\delta - \omega^2\,\varepsilon_2\,\mu_2\,E_2^\delta \;=\; 0 \quad \text{in } \Omega_2^\delta. \tag{29}$$

The weak form of the Maxwell equations in a neighborhood of $\Gamma^\delta$ gives the following jump relations on $\Gamma^\delta$ :

$$\begin{cases} \left(E_1^\delta \;-\; E_2^\delta\right) \wedge n \;=\; 0, \\ \left(\overrightarrow{\mathrm{curl}}\,E_1^\delta - \dfrac{1}{\mu_c}\overrightarrow{\mathrm{curl}}\,E_2^\delta\right) \wedge n \;=\; \beta\,(\gamma_1)^2\,E_1^\delta \wedge n, \end{cases} \tag{30}$$

where $n$ is the normal to $\Gamma^\delta$ and $\mu_c$ defined by $\mu_c \;=\; \dfrac{\mu_2\,(1-\omega^2\varepsilon_1\mu_1\beta^2)}{\mu_1}$.

The purpose of this section is to derive approximative impedance conditions useful to perform numerical calculation of the scattered field by the chiral layer. Our program is as follows. First, we derive these approximative impedance conditions. Then, we establish the existence and uniqueness of a solution $\mathcal{E}^\delta$ to the Maxwell equations in $\Omega_2^\delta$ satisfying the radiation condition with these new boundary conditions on $\Gamma^\delta$. Finally, by using a simple continuity argument, error estimates between $E_2^\delta - E^{in}$ and $\mathcal{E}^\delta$ are proved.

Using Lemma 2.1, it follows that

$$\overrightarrow{\mathrm{curl}}\,E_1^\delta \wedge n \;=\; \frac{\partial}{\partial s}E_{\Gamma_s^\delta}^\delta - \overrightarrow{\mathrm{grad}}_{\Gamma_s^\delta}(E_1^\delta \,.\, n) + \mathcal{R}(E_{\Gamma_s^\delta}^\delta). \tag{31}$$

The remarkable identity (1) yields

$$\overrightarrow{\mathrm{curl}}\,H_1^\delta \,.\, n \;=\; curl_{\Gamma_s^\delta}\,H_{\Gamma_s^\delta}^\delta \;=\; \mathrm{div}_{\Gamma_s^\delta}(H_1^\delta \wedge n). \tag{32}$$

Since $E_1^\delta \wedge n = 0$ on $\Gamma^\delta$, by integrating the identity (31), we obtain

$$E_{\Gamma^\delta}^\delta(x_{\Gamma^\delta}) \;=\; \int_{-\delta}^{0} \overrightarrow{\mathrm{curl}}\,E_1^\delta \wedge n\;ds + \int_{-\delta}^{0} \overrightarrow{\mathrm{grad}}_{\Gamma_s^\delta}(E_1^\delta \,.\, n)\;ds - \int_{-\delta}^{0} \mathcal{R}(E_{\Gamma_s^\delta}^\delta)\;ds. \tag{33}$$

(8) gives

$$\overrightarrow{\mathrm{curl}}\,E_1^\delta \wedge n \;=\; (\gamma_1)^2\,\beta\,E_1^\delta \wedge n + i\omega\mu_1(\frac{\gamma_1}{k_1})^2\,H_1^\delta \wedge n. \tag{34}$$

Combining (33) to (34) gives

$$\begin{aligned} E_{\Gamma^\delta}^\delta(x_{\Gamma^\delta}) \;\;=\;\; & \delta\,(\gamma_1)^2\,\beta\,E_1^\delta \wedge n + i\delta\,\omega\mu_1(\frac{\gamma_1}{k_1})^2\,H_1^\delta \wedge n \\ & +\delta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}(E_1^\delta(x_{\Gamma^\delta})\,.\,n) + 0(\delta^2). \end{aligned} \tag{35}$$

Now, from

$$\mathrm{div}_{\Gamma^\delta}\,(H_1^\delta \wedge n) \;=\; (\gamma_1)^2\beta\,H_1^\delta \,.\, n - i\omega\varepsilon_1(\frac{\gamma_1}{k_1})^2\,E_1^\delta \,.\, n, \tag{36}$$

and

$$\mathrm{div}_{\Gamma^\delta}\,(E_1^\delta \wedge n) \;=\; (\gamma_1)^2\beta\,E_1^\delta \,.\, n + i\omega\mu_1(\frac{\gamma_1}{k_1})^2\,H_1^\delta \,.\, n, \tag{37}$$

it is easily seen that

$$E_1^\delta \,.\, n \;=\; \frac{i}{\omega\varepsilon_1}\;\mathrm{div}_{\Gamma^\delta}\,(H_1^\delta \wedge n) - \beta\;\mathrm{div}_{\Gamma^\delta}\,(E_1^\delta \wedge n). \tag{38}$$

It follows that

$$(39)\qquad \begin{aligned} -n\wedge(n\wedge E_2^\delta) &= i\delta\omega\mu_1(\frac{\gamma_1}{k_1})^2\,H_2^\delta\wedge n+\frac{i\delta}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(H_2^\delta\wedge n)\\ &\quad -\delta\beta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(E_2^\delta\wedge n)+0(\delta^2). \end{aligned}$$

In the case of achiral thin layer of dielectric material, (39) is reduced to the impedance boundary condition obtained by Engquist and Nédélec [13] :

$$(40)\qquad -n\wedge(n\wedge E_2^\delta)=i\delta\omega\mu_1\,H_2^\delta\wedge n+\frac{i\delta}{\omega\varepsilon_1}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(H_2^\delta\wedge n)+0(\delta^2).$$

Our aim is to prove the existence and uniqueness of $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ solutions to the Maxwell equations :

$$(41)\qquad \begin{cases} \overrightarrow{\mathrm{curl}}\,\mathcal{E}^\delta = i\omega\,\mu_2\,\mathcal{H}^\delta & \text{in } \Omega_2^\delta,\\ \overrightarrow{\mathrm{curl}}\,\mathcal{H}^\delta = -i\omega\,\varepsilon_2\,\mathcal{E}^\delta & \text{in } \Omega_2^\delta, \end{cases}$$

with the new boundary conditions given as :

$$(42)\qquad \begin{aligned} &n\wedge(n\wedge\mathcal{E}^\delta)+i\delta\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\,\mathcal{H}^\delta\wedge n+\frac{i\delta}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(\mathcal{H}^\delta\wedge n)\\ &-\delta\beta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(\mathcal{E}^\delta\wedge n)=g^\delta \quad \text{on } \Gamma^\delta. \end{aligned}$$

where $g^\delta$ is a known smooth vector field and $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ which satisfy the radiation condition at infinity. First, we prove the following uniqueness result.

LEMMA 4.1. *Assume $\beta=0$. Then, the Maxwell system* (41) *with the boundary conditions* (42) *has at most one solution.*

*Proof.* Let $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ satisfy (41) with the following boundary conditions on $\Gamma^\delta$ :

$$(43)\quad n\wedge(n\wedge\mathcal{E}^\delta)+i\delta\omega\mu_1\,\mathcal{H}^\delta\wedge n+\frac{i\delta}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(\mathcal{H}^\delta\wedge n)=0 \quad \text{on } \Gamma^\delta,$$

and the radiation condition at infinity. Direct computation yields

$$\begin{aligned} &\int_{\Omega^\delta_{2,R}}|\overrightarrow{\mathrm{curl}}\,\mathcal{E}^\delta|^2-\omega^2\,\varepsilon_2\mu_2\int_{\Omega^\delta_{2,R}}|\mathcal{E}^\delta|^2+\delta\omega^2\mu_1\,\mu_2\int_{\Gamma^\delta}|\mathcal{H}^\delta\wedge n|^2\\ &-\frac{\delta\,\mu_2}{\varepsilon_1}\int_{\Gamma^\delta}|\mathrm{div}_{\Gamma^\delta}\,(\mathcal{H}^\delta\wedge n)|^2+i\omega\,\mu_2\int_{\Sigma_R}(\mathcal{H}^\delta\wedge n).\,\overline{\mathcal{E}}^\delta=0. \end{aligned}$$

By taking the imaginary part in the above expression, we obtain

$$(44)\qquad \Re e\Big(\int_{\Sigma_R}(\mathcal{H}^\delta\wedge n).\,\overline{\mathcal{E}}^\delta\Big)=0.$$

Classical arguments, based on the uniqueness theorem of Cauchy-Kowaleska and the well-known properties of the pseudo-differential operator $T_R(\omega)$ (*cf.* Appendix A), insure that $\mathcal{E}^\delta=0$ and $\mathcal{H}^\delta=0$ in $\Omega^\delta_{2,R}$ and the proof of the uniqueness of a solution to (41) with the boundary conditions (43) is complete. □ The Lemma 4.1 provides a proof of the uniqueness of the solution to the Maxwell system (41) with the approximative impedance condition written by Engquist and Nédélec.

LEMMA 4.2. *The Maxwell system* (41) *with the boundary conditions*

$$
\begin{aligned}
& n \wedge (n \wedge \mathcal{E}^\delta) + i\delta\omega\mu_1 (\frac{\gamma_1}{k_1})^2 \mathcal{H}^\delta \wedge n + \frac{i\delta}{\omega\varepsilon_1} \overrightarrow{\mathrm{grad}}_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} (\mathcal{H}^\delta \wedge n) \\
& -\delta\beta \overrightarrow{\mathrm{grad}}_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} (\mathcal{E}^\delta \wedge n) = 0 \quad \text{on } \Gamma^\delta.
\end{aligned} \tag{45}
$$

*has non trivial solutions only if $\mu_1$ belongs to a countable set of exceptional values.*

*Proof.* We introduce the scalar three-dimensional fundamental solution corresponding to $\omega$ as

$$
\Phi(x,y) = \frac{e^{i\omega\sqrt{\varepsilon_2\mu_2}|x-y|}}{4\pi|x-y|}, \quad x \neq y.
$$

Let

$$
u^\delta = \mathcal{H}^\delta \wedge n.
$$

The electric field $\mathcal{E}^\delta$ has the following integral representation :

$$
\begin{aligned}
\mathcal{E}^\delta(x) &= -\frac{i}{\omega\varepsilon_2} \overrightarrow{\mathrm{grad}} \int_{\Gamma^\delta} \Phi(x-y)\, \mathrm{div}_{\Gamma^\delta} u^\delta(y)\, d\gamma(y) \\
&\quad -i\omega\mu_2 \int_{\Gamma^\delta} \Phi(x-y)\, u^\delta(y)\, d\gamma(y) \\
&\quad + \overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} n(y) \wedge \mathcal{E}^\delta(y)\, \Phi(x-y)\, d\gamma(y).
\end{aligned} \tag{46}
$$

From

$$
\begin{aligned}
\overrightarrow{\mathrm{curl}}\, \mathcal{E}^\delta \,.\, n &= -\mathrm{curl}_{\Gamma^\delta} \Big( n \wedge (n \wedge \mathcal{E}^\delta) \Big), \\
&= i\delta\, \omega\, \mu_1 (\frac{\gamma_1}{k_1})^2 \mathrm{curl}_{\Gamma^\delta} u^\delta,
\end{aligned}
$$

we deduce that

$$
\begin{aligned}
n \wedge \mathcal{E}^\delta &= i\delta\omega\mu_1 (\frac{\gamma_1}{k_1})^2 n \wedge u^\delta + \frac{i\delta}{\omega\varepsilon_1} \overrightarrow{\mathrm{curl}}_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} u^\delta \\
&\quad + i\delta^2\, \beta\, \omega\, \mu_1 (\frac{\gamma_1}{k_1})^2 \overrightarrow{\mathrm{curl}}_{\Gamma^\delta} \mathrm{curl}_{\Gamma^\delta} u^\delta,
\end{aligned} \tag{47}
$$

and then

$$
\mathrm{div}_{\Gamma^\delta} (n \wedge \mathcal{E}^\delta) = i\delta\omega\mu_1 (\frac{\gamma_1}{k_1})^2 \mathrm{div}_{\Gamma^\delta} (n \wedge u^\delta).
$$

We rewrite the approximative impedance conditions (45) as

$$
\begin{aligned}
& n \wedge (n \wedge \mathcal{E}^\delta) + i\delta\, \omega\, \mu_1 (\frac{\gamma_1}{k_1})^2 u^\delta + \frac{i\delta}{\omega\varepsilon_1} \overrightarrow{\mathrm{grad}}_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} (u^\delta) \\
& -i\delta^2\omega\mu_1\beta (\frac{\gamma_1}{k_1})^2 \overrightarrow{\mathrm{grad}}_{\Gamma^\delta} (\mathrm{div}_{\Gamma^\delta} (u^\delta \wedge n)) = 0.
\end{aligned} \tag{48}
$$

Now, multiplying (48) by $v^\delta$ and integrating by parts on $\Gamma^\delta$ yields

$$
\begin{aligned}
& \int_{\Gamma^\delta} \mathcal{E}^\delta(x) \,.\, v^\delta(x)\, d\gamma(x) = i\delta\omega\mu_1 (\frac{\gamma_1}{k_1})^2 \int_{\Gamma^\delta} u^\delta(x) \,.\, v^\delta(x)\, d\gamma(x) \\
& -\frac{i\delta}{\omega\varepsilon_1} \int_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} u^\delta(x)\, \mathrm{div}_{\Gamma^\delta} v^\delta(x)\, d\gamma(x) \\
& +i\delta^2\, \omega\mu_1 (\frac{\gamma_1}{k_1})^2 \beta \int_{\Gamma^\delta} \mathrm{div}_{\Gamma^\delta} v^\delta(x)\, \mathrm{div}_{\Gamma^\delta} (u^\delta(x) \wedge n(x))\, d\gamma(x).
\end{aligned}
$$

From the integral representation (46) it is easily seen that

$$\begin{aligned}
&\frac{1}{2}\int_{\Gamma^\delta}\mathcal{E}^\delta(x)\,.\,v^\delta(x)\,d\gamma(x) \;=\; \frac{i}{\omega\varepsilon_2}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\mathrm{div}_{\Gamma^\delta}\,u^\delta(y)\,\mathrm{div}_{\Gamma^\delta}\,v^\delta(x)\,d\gamma(y)\,d\gamma(x)\\
&-i\omega\mu_2\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,u^\delta(y)\,.\,v^\delta(x)\,d\gamma(y)\,d\gamma(x)\\
&+\int_{\Gamma^\delta}\Big(\overrightarrow{\mathrm{curl}}\int_{\Gamma^\delta}n(y)\,\wedge\,\mathcal{E}^\delta(y)\,\Phi(x-y)\,d\gamma(y)\Big)\,.\,v^\delta(x)\,d\gamma(x).
\end{aligned}\tag{49}$$

Then, we obtain that

$$\begin{aligned}
&\frac{2i}{\omega\varepsilon_2}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\mathrm{div}_{\Gamma^\delta}\,u^\delta(y)\,\mathrm{div}_{\Gamma^\delta}\,v^\delta(x)\,d\gamma(y)\,d\gamma(x)\\
&-2i\omega\mu_2\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,u^\delta(y)\,.\,v^\delta(x)\,d\gamma(y)\,d\gamma(x)\\
&+2\int_{\Gamma^\delta}\Big(\overrightarrow{\mathrm{curl}}\int_{\Gamma^\delta}n(y)\,\wedge\,\mathcal{E}^\delta(y)\,\Phi(x-y)\,d\gamma(y)\Big)\,.\,v^\delta(x)\,d\gamma(x)\\
&=\; i\delta\omega\mu_1(\frac{\gamma_1}{k_1})^2\int_{\Gamma^\delta}u^\delta(x)\,.\,v^\delta(x)\,d\gamma(x)-\frac{i\delta}{\omega\varepsilon_1}\int_{\Gamma^\delta}\mathrm{div}_{\Gamma^\delta}u^\delta(x)\,\mathrm{div}_{\Gamma^\delta}\,v^\delta(x)\,d\gamma(x)\\
&+i\delta^2\,\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\mathrm{div}_{\Gamma^\delta}\,v^\delta(x)\,\mathrm{div}_{\Gamma^\delta}\,(u^\delta(x)\,\wedge\,n(x))\,d\gamma(x).
\end{aligned}\tag{50}$$

We seek $u^\delta$ in $TH^1(\Gamma^\delta) \;=\Big\{c\,\in\,(H^1(\Gamma^\delta))^3, c\,.\,n\,=\,0\Big\}$, such that (50) holds for all tangential field $v^\delta\,\in\,TH^1(\Gamma^\delta)$. We use the following decomposition of the space $TH^1(\Gamma^\delta)$.

LEMMA 4.3. $TH^1(\Gamma^\delta) \;=\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,H^2(\Gamma^\delta)\,\oplus\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,H^2(\Gamma^\delta)$.

The proof of this lemma is exactly the same as in [19] for the space

$$TH^{-1/2}(\mathrm{div},\Gamma^\delta) \;=\Big\{c\in\,(H^{-1/2}(\Gamma^\delta))^3, c\,.\,n=0, \mathrm{div}_{\Gamma^\delta}\,c\in\,H^{-1/2}(\Gamma^\delta)\Big\}.$$

We decompose the tangential fields $u^\delta$ and $v^\delta$ as follows

$$\left\{\begin{array}{rcl} u^\delta &=& \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta\,+\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi^\delta,\\ v^\delta &=& \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta_t+\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi^\delta_t. \end{array}\right.$$

(50) leads to the following equation :

$$\begin{aligned}
&\frac{i\delta}{\omega\varepsilon_1}\int_{\Gamma^\delta}\Delta_{\Gamma^\delta}\,\psi^\delta(x)\,\Delta_{\Gamma^\delta}\,\psi^\delta_t(x)\,d\gamma(x)\\
&+\frac{2i}{\omega\varepsilon_2}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\,\Delta_{\Gamma^\delta}\,\psi^\delta_t(x)\,d\gamma(y)\,d\gamma(x)\\
&-2i\omega\mu_2\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta(y)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta_t(x)\,d\gamma(y)\,d\gamma(x)\\
&-2i\omega\mu_2\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi^\delta(y)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta_t(x)\,d\gamma(y)\,d\gamma(x)\\
&+2\int_{\Gamma^\delta}(\mathcal{R}(x)-2c(x))\Big\{\int_{\Gamma^\delta}\Big(i\delta\omega\mu_1\,(-\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi^\delta(y)+\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi^\delta(y))
\end{aligned}$$

$$+\frac{i\delta}{\omega\varepsilon_1}\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\Big)\,\Phi(x-y)\,d\gamma(y)\,\wedge\,n(x)\Big\}\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x)$$

$$-i\delta^2\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}(\mathcal{R}(x)-2c(x))\,\Big(\int_{\Gamma^\delta}\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\varphi^\delta(y)$$

$$\Phi(x-y)\,d\gamma(y)\,\wedge\,n(x)\Big)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x)$$

$$+i\delta\omega\mu_1\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,(-\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi^\delta(y)+\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi^\delta(y))\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(y)\,d\gamma(x)$$

$$+\frac{i\delta}{\omega\varepsilon_1}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(y)\,d\gamma(x)$$

$$-i\delta^2\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\varphi^\delta(y)\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(y)\,d\gamma(x)$$

$$+i\delta^2\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\Delta_{\Gamma^\delta}\,\varphi^\delta(x)\,\Delta_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x)$$

$$-i\delta\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\int_{\Gamma^\delta}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta(x)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x)\;=\;0.$$

By multiplying (47) by $\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi_t^\delta$, integrating by parts and using (49), we obtain

$$i\delta^2\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\,\beta\int_{\Gamma^\delta}\Delta_{\Gamma^\delta}\,\varphi^\delta(x)\,.\,\Delta_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)$$

$$-i\delta\omega\mu_1\int_{\Gamma^\delta}\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi^\delta(x)\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)$$

$$=\frac{2i}{\omega\varepsilon_2}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\,\Delta_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)\,d\gamma(y)$$

$$-2i\omega\mu_2\int_{\Gamma^\delta}\int_{\Gamma^\delta}\Phi(x-y)\,(\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi^\delta(y)+\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi^\delta(y))\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)\,d\gamma(y)$$

$$+2\int_{\Gamma^\delta}(\mathcal{R}(x)-2c(x))\,\Big\{\int_{\Gamma^\delta}\Big(i\delta\omega\mu_1\,(-\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi^\delta(y)+\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi^\delta(y))$$

$$+\frac{i\delta}{\omega\varepsilon_1}\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\Big)\,\Phi(x-y)\,d\gamma(y)\,\wedge\,n(x)\Big\}\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)$$

$$-i\delta^2\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}(\mathcal{R}(x)-2c(x))\,\Big(\int_{\Gamma^\delta}\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\varphi^\delta(y)$$

$$\Phi(x-y)\,d\gamma(y)\,\wedge\,n(x)\Big)\,.\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x)$$

$$-i\delta\omega\mu_1\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,(-\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\psi^\delta(y)+\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi^\delta(y))\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(y)\,d\gamma(x)$$

$$-\frac{i\delta}{\omega\varepsilon_1}\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\psi^\delta(y)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(y)\,d\gamma(x)$$

$$+i\delta^2\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\int_{\Gamma^\delta}\frac{\partial\Phi}{\partial n_x}(x-y)\,\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\Delta_{\Gamma^\delta}\,\varphi^\delta(y)\,.\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(y)\,d\gamma(x).$$

Straightforward computations gives that the system of the above two equations leads

to a compact perturbation of the coercive form

$$\begin{pmatrix} \frac{1}{\omega\varepsilon_1}\int_{\Gamma^\delta} \Delta_{\Gamma^\delta}\,\psi^\delta(x)\,\Delta_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x) & \delta\omega\mu_1(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\Delta_{\Gamma^\delta}\,\varphi^\delta(x)\,\Delta_{\Gamma^\delta}\,\psi_t^\delta(x)\,d\gamma(x) \\ 0 & \omega\mu_1\delta(\frac{\gamma_1}{k_1})^2\beta\int_{\Gamma^\delta}\Delta_{\Gamma^\delta}\,\varphi^\delta(x)\,\Delta_{\Gamma^\delta}\,\varphi_t^\delta(x)\,d\gamma(x) \end{pmatrix},$$

on $H^2(\Gamma^\delta)/\mathbb{C}\times \mathrm{H}^2(\Gamma^\delta)/\mathbb{C}$.
Now, we write the approximative impedance conditions (48) in the following form

$$\begin{aligned} & n\wedge(n\wedge\mathcal{E}^\delta)+i\delta\,\omega\,u^\delta+\frac{i\delta}{\omega\varepsilon_1}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(u^\delta) \\ & -\delta\beta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,(\overrightarrow{\mathrm{curl}}\,\mathcal{E}^\delta\,.\,n)\;=-i\delta\,\omega\,(\mu_1\,(\frac{\gamma_1}{k_1})^2-1)\,u^\delta. \end{aligned} \tag{51}$$

The above computations show that $u^\delta$ appears in (51) as an eigenvector of a compact perturbation of an invertible operator and

$$-i\delta\,\omega\,(\mu_1\,(\frac{\gamma_1}{k_1})^2-1)$$

is its eigenvalue. Since the operator

$$\begin{aligned} & u^\delta\longmapsto n\wedge(n\wedge\mathcal{E}^\delta)+i\delta\,\omega\,u^\delta+\frac{i\delta}{\omega\varepsilon_1}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(u^\delta) \\ & -\delta\beta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,(\overrightarrow{\mathrm{curl}}\,\mathcal{E}^\delta\,.\,n) \end{aligned}$$

is independent of $\mu_1$, the proof of the uniqueness Lemma 4.2 is then over. □ The proof of Lemma 4.2 also shows the existence of solutions to our boundary value problem. Now, we use the boundary integral equation method to obtain optimal error estimates. We make an ansatz for $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ in the form

$$\begin{cases} \mathcal{E}^\delta(x)\;=\;\omega\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{curl}}\displaystyle\int_{\Gamma^\delta}a(y)\,\Phi(x,y)\,ds(y) & \text{in }\Omega_2^\delta, \\ \mathcal{H}^\delta(x)\;=\;\dfrac{1}{i\omega\mu_2}\,\overrightarrow{\mathrm{curl}}\,\mathcal{E}^\delta(x) & \text{in }\Omega_2^\delta, \end{cases} \tag{52}$$

where $a$ is a vector function in $TH^2(\Gamma^\delta)\;=\Big\{c\;\in\;(H^2(\Gamma^\delta)^3, c\,.\,n\;=\;0\Big\}$. From the properties of $\Phi$, we see that $(\mathcal{E}^\delta,\mathcal{H}^\delta)$ satisfy the Maxwell equations (41) and the radiation condition. The tangential components of $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ on $\Gamma^\delta$ take the form :

$$\begin{aligned} n\wedge\mathcal{E}^\delta(x) &= \omega\sqrt{\varepsilon_2\mu_2}\,n\wedge\int_{\Gamma^\delta}\overrightarrow{\mathrm{curl}}_x(a(y)\,\Phi(x,y))\,ds(y) \\ &\quad +\tfrac{\omega}{2}\sqrt{\varepsilon_2\mu_2}\,a(x),\quad x\in\Gamma^\delta, \\ n\wedge\mathcal{H}^\delta(x) &= -i\sqrt{\frac{\varepsilon_2}{\mu_2}}\,n\wedge\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\int_{\Gamma^\delta}a(y)\,\Phi(x,y)\,ds(y),\quad x\in\Gamma^\delta. \end{aligned} \tag{53}$$

The boundary conditions (53) for $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ will lead to an integral equation on $\Gamma^\delta$ for the unknown vector $a$. In order to write it, we introduce the following boundary

operators :

$$
\begin{aligned}
Q(a)(x) &= n \wedge a(x), \quad x \in \Gamma^\delta, \\
M(a)(x) &= n \wedge \overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a(y)\, \Phi(x,y)\, ds(y), \quad x \in \Gamma^\delta, \\
N(a)(x) &= n \wedge \int_{\Gamma^\delta} a(y)\, \Phi(x,y)\, ds(y), \quad x \in \Gamma^\delta, \\
P(a)(x) &= n \wedge \overrightarrow{\mathrm{curl}}\, \overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a(y)\, \Phi(x,y)\, ds(y), \quad x \in \Gamma^\delta.
\end{aligned}
$$

Then, (42) leads to

$$
\begin{aligned}
&\frac{1}{2}\,\delta\,\beta\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,a - \frac{\delta}{\omega}\sqrt{\frac{\varepsilon_2}{\mu_2}}\,\frac{1}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,Pa \\
&+\delta\,\beta\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,Ma + \delta\,\mu_1\,(\frac{\gamma_1}{k_1})^2\,\sqrt{\frac{\varepsilon_2}{\mu_2}}\,Pa \\
&+\frac{\sqrt{\varepsilon_2\mu_2}}{2}\,Qa + \sqrt{\varepsilon_2\mu_2}\,QMa \;=\; g^\delta \in TL^2(\mathrm{curl}_{\Gamma^\delta}, \Gamma^\delta),
\end{aligned}
\tag{54}
$$

where $TL^2(\mathrm{curl}_{\Gamma^\delta}, \Gamma^\delta) = \Big\{ c \in (L^2(\Gamma^\delta))^3, c\,.\,n = 0, \mathrm{curl}_{\Gamma^\delta}\, c \in H^1(\Gamma^\delta) \Big\}$. Thus we have the following theorem.

THEOREM 4.4. *The vector function $a \in TH^2(\Gamma^\delta)$ is a solution of the integral equation* (54) *if and only if the fields $(\mathcal{E}^\delta, \mathcal{H}^\delta)$ satisfy the boundary conditions* (42).

In order to discuss the solvability of the integral equation (54), we compute $\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,P\,a$, $\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,M\,a$, $\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,N\,a$, $\mathrm{curl}_{\Gamma^\delta}\,P\,a$ and $\mathrm{curl}_{\Gamma^\delta}\,QM\,a$. Direct computation yields

$$
\begin{aligned}
\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,P\,a &= \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(a \wedge n) - \omega^2\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,N\,a, \\
&= \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{curl}_{\Gamma^\delta}\,a - \omega^2\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,N\,a,
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,M\,a &= -\frac{\partial}{\partial s}\Big(n \wedge (n \wedge \overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a\,\Phi)\Big) + \omega^2\,\varepsilon_2\,\mu_2\; n \wedge \overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a\,\Phi \\
&\quad -n \wedge \overrightarrow{\mathrm{curl}}\, a, \\
&= -i\,\sqrt{\frac{\mu_2}{\varepsilon_2}}\,\frac{\partial}{\partial s}\Big(n \wedge (n \wedge \mathcal{H}^\delta)\Big) + \omega\,\sqrt{\varepsilon_2\,\mu_2}\; n \wedge \mathcal{E}^\delta - n \wedge \overrightarrow{\mathrm{curl}}\, a,
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,N\,a &= \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\Big(\overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a\,\Phi\,.\,n\Big), \\
&= \frac{1}{\omega\,\sqrt{\varepsilon_2\mu_2}}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}(\mathcal{E}^\delta\,.\,n),
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\mathrm{curl}_{\Gamma^\delta}\,P\,a &= \mathrm{curl}_{\Gamma^\delta}\Big(\, n \wedge (\mathcal{R} - 2c - \frac{\partial}{\partial n}) \\
&\quad \Big\{ -\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}(\int_{\Gamma^\delta} a(y).(n(x) - n(y))\Phi(x-y)\,ds(y)) \\
&\quad -\mathcal{R}(n \wedge (n \wedge \int_{\Gamma^\delta} a\Phi)) + \frac{\partial}{\partial n}(n \wedge (n \wedge \int_{\Gamma^\delta} a\Phi))\Big\}
\end{aligned}
\tag{58}
$$

$$\text{(59)}\quad \mathrm{curl}_{\Gamma^\delta}\, QM\, a \;=\; \mathrm{curl}_{\Gamma^\delta}\Big(n\wedge\int_{\Gamma^\delta}(\mathcal{R}-2c)(a\wedge n)\Phi-\int_{\Gamma^\delta} a\wedge n\frac{\partial\Phi}{\partial n_x}\Big).$$

Now, we write the integral equation (54) as follows :

$$\text{(60)}\qquad \Big(L^\delta+K^\delta\Big)a\;=\;g^\delta,$$

where

$$\text{(61)}\qquad L^\delta\, a\;=\;\frac{1}{2}\delta\,\beta\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\, a-\frac{\delta}{\omega}\sqrt{\frac{\varepsilon_2}{\mu_2}}\,\frac{1}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(a\wedge n),$$

and

$$\text{(62)}\qquad \begin{aligned} K^\delta\, a \;=\;& \delta\,\frac{\varepsilon_2}{\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\, N\, a+\delta\,\beta\,\sqrt{\varepsilon_2\mu_2}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\, M\, a\\ &+\sqrt{\varepsilon_2\mu_2}\,QM\, a+\delta\,\mu_1\,(\frac{\gamma_1}{k_1})^2\sqrt{\frac{\varepsilon_2}{\mu_2}}\,P\, a+\frac{\sqrt{\varepsilon_2\mu_2}}{2}\,Q\, a. \end{aligned}$$

We treat this equation in the spaces

$$L^\delta+K^\delta\;:TH^2(\Gamma^\delta)\longmapsto TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta).$$

From (55) – (59), it is clear that $\mathrm{curl}_{\Gamma^\delta}\, K^\delta\, a\in\, H^2(\Gamma^\delta)$ and

$$K^\delta\, a\in TH^1(\Gamma^\delta)=\Big\{c\in\,(H^1(\Gamma^\delta))^3, c.n=0\Big\},$$

for $a\in TH^2(\Gamma^\delta)$. The operator $K^\delta$ is then compact between these spaces. We show that the operator $L^\delta$ is an isomorphism from $TH^2(\Gamma^\delta)$ onto $TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)$. By using the decomposition

$$a\;=\;\overrightarrow{\mathrm{curl}}_{\Gamma^\delta}\,\varphi+\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\psi,$$

the integral equation

$$L^\delta\, a\;=\;g^\delta$$

is equivalent to the following elliptic system

$$\begin{cases} \dfrac{\sqrt{\varepsilon_2\mu_2}}{2}\,\Delta_{\Gamma^\delta}\,\psi\;=\;-\mathrm{curl}_{\Gamma^\delta}\, g^\delta\;\in H^1(\Gamma^\delta),\\ \sqrt{\dfrac{\varepsilon_2}{\mu_2}}\,\dfrac{\delta}{\omega^2\varepsilon_1}\,\Delta^2_{\Gamma^\delta}\,\varphi\;=\;-\dfrac{1}{2}\delta\,\beta\,\sqrt{\varepsilon_2\mu_2}\,\Delta^2_{\Gamma^\delta}\,\psi+\mathrm{div}_{\Gamma^\delta}\, g^\delta\;\in H^{-1}(\Gamma^\delta), \end{cases}$$

which has a unique solution $(\varphi,\psi)\in(H^3(\Gamma^\delta)/\mathbb{C})^2$. The integral equation $L^\delta\, a=g^\delta$ has then a unique solution in $TH^2(\Gamma^\delta)$.
Applying these results to (54) yields

$$\text{(63)}\qquad a+(L^\delta)^{-1}\,K^\delta a\;=\;(L^\delta)^{-1}\,g^\delta,$$

which is a Fredholm equation of the second kind in $TH^2(\Gamma^\delta)$. Now, under the assumption that $\mu$ is not is the set of exceptional values introduced in Lemma 4.2, we can prove the following theorem.

THEOREM 4.5. *There exists a unique solution to the integral equation* (54) *in* $TH^2(\Gamma^\delta)$.

From Lemma 4.2 the boundary value problem (41) – (42) itself has at most one solution. To show the uniqueness of the solution to the integral equation (54), let us consider $a \in TH^2(\Gamma^\delta)$ a solution of (54) and define

$$\mathcal{E}^\delta(x) = \omega \sqrt{\varepsilon_2 \mu_2}\, \overrightarrow{\mathrm{curl}} \int_{\Gamma^\delta} a(y)\, \Phi(x,y)\, ds(y) \quad \text{in } \Omega_2^\delta,$$

$$\mathcal{H}^\delta(x) = \frac{1}{i\omega\mu_2}\, \overrightarrow{\mathrm{curl}}\, \mathcal{E}^\delta(x) \quad \text{in } \Omega_2^\delta.$$

Using standard arguments gives that $\mathcal{E}^\delta$ and $\mathcal{H}^\delta$ are solutions of the boundary value problem (41) – (42). Hence, there exists a unique solution to the integral equation (54) in $TH^2(\Gamma^\delta)$.

The uniform continuity of the operator $\left( (L^\delta)^{-1} K^\delta + I \right)^{-1} (L^\delta)^{-1}$ from $TL^2(\mathrm{curl}_{\Gamma^\delta}, \Gamma^\delta)$ onto $TH^2(\Gamma^\delta)$ with respect to $\delta$ gives the existence of a strictly positive constant $C$ independent of $\delta$ such that the unique solution of the equation (63) satisfies the following estimate :

$$||a||_{TH^2(\Gamma^\delta)} \leq C\, ||g^\delta||_{TL^2(\mathrm{curl}_{\Gamma^\delta}, \Gamma^\delta)}$$

From the integral representations (52), it is easily established that for each $O \subset\subset \Omega_2^\delta$, there exists a strictly positive constant $C$ independent of $\delta$ such that :

$$||\mathcal{E}^\delta||_{H(\overrightarrow{\mathrm{curl}}, O)} + ||\mathcal{H}^\delta||_{H(\overrightarrow{\mathrm{curl}}, O)} \leq C\, ||a||_{TH^2(\Gamma^\delta)}.$$

Therefore, we obtain the following result.

THEOREM 4.6. *For each $O \subset\subset \Omega_2^\delta$, there exists a positive constant $C$ independent of $\delta$ such that :*

$$||\mathcal{E}^\delta||_{H(\overrightarrow{\mathrm{curl}}, O)} + ||\mathcal{H}^\delta||_{H(\overrightarrow{\mathrm{curl}}, O)} \leq C\, ||g^\delta||_{TL^2(\mathrm{curl}_{\Gamma^\delta}, \Gamma^\delta)}. \tag{64}$$

Now, let $\mathcal{E}^\delta, \mathcal{H}^\delta$ be the solution of the Maxwell equations with the new boundary conditions corresponding to

$$\begin{aligned} g^\delta = \; & n \wedge (n \wedge E^{in}) + i\delta\omega\mu_1 \left(\frac{\gamma_1}{k_1}\right)^2 H^{in} \wedge n + \frac{i\delta}{\omega\varepsilon_1}\, \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\, \mathrm{div}_{\Gamma^\delta}\, (H^{in} \wedge n) \\ & - \delta\beta\, \overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\, \mathrm{div}_{\Gamma^\delta}\, (E^{in} \wedge n) \end{aligned}$$

in (42) ($H^{in} = -i\omega\, \varepsilon_2\, \overrightarrow{\mathrm{curl}}\, E^{in}$). By Theorem 4.3, we obtain the following error estimate.

THEOREM 4.7. *For each $O \subset\subset \Omega_2^\delta$, there exists a positive constant $C$ independent of $\delta$ such that :*

$$||\mathcal{E}^\delta - E_2^{sc,\delta}||_{H(\overrightarrow{\mathrm{curl}}, O)} + ||\mathcal{H}^\delta - H_2^{sc,\delta}||_{H(\overrightarrow{\mathrm{curl}}, O)} \leq C\, \delta^2, \tag{65}$$

*where $E_2^{sc,\delta} = E_2^\delta - E^{in}$ and $H_2^{sc,\delta} = H_2^\delta - H^{in}$ are the unique solutions of (8) in $\Omega_2^\delta$ satisfying the radiation condition at infinity.*

*Proof.* We first recall that the tangential component of the electric field on $\Gamma^\delta$ is given by

$$\begin{aligned} E^\delta_{\Gamma^\delta}(x_{\Gamma^\delta}) &= (\gamma_1)^2\,\beta\int_{-\delta}^{0} E_1^\delta \wedge n\,ds + i\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\int_{-\delta}^{0} H_1^\delta \wedge n\,ds \\ &+\frac{i}{\omega\varepsilon_1}\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(H_1^\delta\wedge n)\,ds - i\beta\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(E_1^\delta\wedge n)\,ds \\ &-\int_{-\delta}^{0}\mathcal{R}(E^\delta_{\Gamma^\delta_s})\,ds. \end{aligned}$$

Let us define

$$\begin{cases} e^\delta = \mathcal{E}^\delta - E_2^{sc,\delta}, \\ h^\delta = \mathcal{H}^\delta - H_2^{sc,\delta}. \end{cases}$$

It is clear that $(e^\delta, h^\delta)$ satisfy the Maxwell equations

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,e^\delta = i\omega\,\mu_2\,h^\delta \quad \text{in } \Omega_2^\delta, \\ \overrightarrow{\mathrm{curl}}\,h^\delta = -i\omega\,\varepsilon_2\,e^\delta \quad \text{in } \Omega_2^\delta, \end{cases}$$

with the boundary conditions given by :

$$\begin{aligned} &n\wedge(n\wedge e^\delta) + i\delta\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\,h^\delta\wedge n + \frac{i\delta}{\omega\varepsilon_1}\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(h^\delta\wedge n) \\ &-\delta\beta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(e^\delta\wedge n) = w^\delta \quad \text{on } \Gamma^\delta, \end{aligned}$$

where

$$\begin{aligned} w^\delta &= \frac{i}{\omega\varepsilon_1}\Big(\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(H_1^\delta\wedge n)\,ds - \delta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(H_1^\delta(x_{\Gamma^\delta})\wedge n)\Big) \\ &-i\beta\Big(\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(E_1^\delta\wedge n)\,ds - \delta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(E_1^\delta(x_{\Gamma^\delta})\wedge n)\Big) \\ &-\int_{-\delta}^{0}\mathcal{R}(E^\delta_{\Gamma^\delta_s})\,ds + (\gamma_1)^2\,\beta\int_{-\delta}^{0}E_1^\delta\wedge n\,ds \\ &+i\omega\mu_1\,(\frac{\gamma_1}{k_1})^2\Big(\int_{-\delta}^{0}H_1^\delta\wedge n\,ds - \delta\,H_1^\delta\wedge n\Big). \end{aligned}$$

From Theorem 4.3, it follows that for each $O\subset\subset\Omega_2^\delta$, there exists a positive constant $C$ independent of $\delta$ such that :

$$||e^\delta||_{H(\overrightarrow{\mathrm{curl}},O)} + ||h^\delta||_{H(\overrightarrow{\mathrm{curl}},O)} \le C\,||w^\delta||_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)}. \tag{66}$$

Now, in order to prove the optimal error estimate (65), we should estimate the following quantities :

$$|\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(H_1^\delta\wedge n)\,ds - \delta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(H_1^\delta(x_{\Gamma^\delta})\wedge n)\,|_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)},$$

$$|\int_{-\delta}^{0}\overrightarrow{\mathrm{grad}}_{\Gamma^\delta_s}\,\mathrm{div}_{\Gamma^\delta_s}\,(E_1^\delta\wedge n)\,ds - \delta\,\overrightarrow{\mathrm{grad}}_{\Gamma^\delta}\,\mathrm{div}_{\Gamma^\delta}\,(E_1^\delta(x_{\Gamma^\delta})\wedge n)\,|_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)},$$

$$| \int_{-\delta}^{0} \mathcal{R}(E^{\delta}_{\Gamma^{\delta}_s})\, ds\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)}, | \int_{-\delta}^{0} E^{\delta}_1 \wedge n\, ds\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)},$$

and

$$| \int_{-\delta}^{0} H^{\delta}_1 \wedge n\, ds - \delta\, H^{\delta}_1 \wedge n\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)}.$$

We need the following lemma.

LEMMA 4.8. *Let $u$ be a smooth vectorial function defined in $\Omega^{\delta}_1$. We have*

$$| \int_{-\delta}^{0} \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}_s}\, \mathrm{div}_{\Gamma^{\delta}_s}\, (u \wedge n)\, ds - \delta\, \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}}\, \mathrm{div}_{\Gamma^{\delta}}\, (u(x_{\Gamma^\delta}) \wedge n)\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2,$$

*where $C$ is a strictly positive constant independent of $\delta$.*

Formally, since the derivatives of the tangential components of the magnetic and the electric field are bounded uniformly with respect to $\delta$, it follows that there exists a strictly positive constant $C$ such that

$$| \int_{-\delta}^{0} \mathcal{R}(E^{\delta}_{\Gamma^{\delta}_s})\, ds\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2,$$

$$| \int_{-\delta}^{0} E^{\delta}_1 \wedge n\, ds\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2,$$

and

$$| \int_{-\delta}^{0} H^{\delta}_1 \wedge n\, ds - \delta\, H^{\delta}_1 \wedge n\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2.$$

Lemma 4.2 yields

$$| \int_{-\delta}^{0} \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}_s}\, \mathrm{div}_{\Gamma^{\delta}_s}\, (H^{\delta}_1 \wedge n)\, ds - \delta\, \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}}\, \mathrm{div}_{\Gamma^{\delta}}\, (H^{\delta}_1(x_{\Gamma^\delta}) \wedge n)\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2,$$

$$| \int_{-\delta}^{0} \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}_s}\, \mathrm{div}_{\Gamma^{\delta}_s}\, (E^{\delta}_1 \wedge n)\, ds - \delta\, \overrightarrow{\mathrm{grad}}_{\Gamma^{\delta}}\, \mathrm{div}_{\Gamma^{\delta}}\, (E^{\delta}_1(x_{\Gamma^\delta}) \wedge n)\, |_{TL^2(\mathrm{curl}_{\Gamma^\delta},\Gamma^\delta)} \leq C\, \delta^2.$$

The proof of the error estimate is then complete by using (66). □

Finally, we note that the present method for obtaining the approximative impedance conditions is quite general, and it is applicable to coatings with arbitrary constitutive relations, such as dielectric layers with tensor permittivity, and non-reciprocal coatings. Multiple layers may also be handled in a systematic manner.

**5. Conclusions.** In this paper, the diffraction problem by a chiral layer is formulated. The existence and the uniqueness of a solution to this problem is proved except for a possible discrete set of frequencies. In the case of a thin chiral layer, approximative impedance conditions are derived. The well-posedness of the new boundary value problem obtained from these equivalent boundary conditions is established and optimal error estimates between the unique solution of this new boundary value problem and the solution of the diffraction problem by the chiral layer are obtained.

Appendix A Let $(Y^m_l)_{-l\leq m\leq l}$ be an orthonormal sequence of spherical harmonics of order $l$ on the unit sphere $\Sigma$, normalized such that

$$\int_{\Sigma} Y^m_l\, .\, \overline{Y}^{m'}_{l'} = \delta^{m,m'}_{l,l'}.$$

The basis functions for tangential fields on $\Sigma_R$ are then

$$G_l^m \;=\; \frac{1}{\sqrt{l\,(l+1)}}\,\overrightarrow{\mathrm{grad}}_\Sigma\, Y_l^m \text{ and } R_l^m \;=\; n \wedge G_l^m, \quad \text{for } -\,l \le m \le l,\, l \ge 1.$$

The tangential vector fields $G_l^m$ and $G_l^m$ are an orthonormal basis on the unit sphere (in the $L^2$ inner product). The multipolar vector basis for Maxwell's equations

$$\left\{ \begin{array}{l} \overrightarrow{\mathrm{curl}}\, E \;=\; i\omega\,\mu_2\, H, \\ \overrightarrow{\mathrm{curl}}\, H \;=\; -i\omega\,\varepsilon_2\, E, \end{array} \right. \tag{67}$$

are given by

$$\left\{ \begin{array}{l} M_{j,l}^m(x) \;=\; \overrightarrow{\mathrm{curl}}\Big( x\, h_l^{(j)}(k|x|)\, Y_l^m(\dfrac{x}{|x|})\Big), \\ \\ N_{j,l}^m(x) \;=\; \dfrac{1}{ik}\,\overrightarrow{\mathrm{curl}}\, M_{j,l}^m(x), \end{array} \right. \tag{68}$$

where $j \in \{1,2\}, m = -l, \ldots, l, l \ge 1, k = \omega\sqrt{\varepsilon_2\mu_2}$. $h_l^{(j)}$ denotes the spherical Hankel functions of the $j$ kind and order $l$. It is well-known that any solution of Maxwell's equations (67) satisfying the outgoing radiation condition can be written for $|x| > R$ in the form

$$E(x) \;=\; \sum_{l\ge 1}\sum_{m=-l}^{l} \Big(\alpha_l^m\, N_{1,l}^m(x) + \beta_l^m\, M_{1,l}^m(x)\Big). \tag{69}$$

In this appendix, we shall prove that if $\Im m \int_{\Sigma_R} \Big(\overrightarrow{\mathrm{curl}}\, E \wedge n\Big)\,.\,\overline{E} \;=\; 0$ then $E \equiv 0$ in $|x| > R$. We need to express

$$\Im m \int_{\Sigma_R} \Big(\overrightarrow{\mathrm{curl}}\, E \wedge n\Big)\,.\,\overline{E},$$

where $E$ is a solution to Maxwell's equations (67) satisfying the outgoing radiation condition at infinity in terms of the coefficients of the expansions for $E$. Using the expansion (69) we obtain

$$\begin{aligned} \overrightarrow{\mathrm{curl}}\, E \wedge n \;\; = \;\; & \sum_{l\ge 1}\sum_{m=-l}^{l} \Big(\sqrt{l(l+1)}\,\beta_l^m\, h_l^{(1)}(kR)\, G_l^m(\frac{x}{R})\Big) \\ & + \frac{i}{kR}\sum_{l\ge 1}\sum_{m=-l}^{l} \Big(\sqrt{l(l+1)}\,\alpha_l^m \Big\{ h_l^{(1)}(kR) + kR(h_l^{(1)})'(kR) \Big\}\, R_l^m(\frac{x}{R})\Big), \end{aligned}$$

on $\Sigma_R$. Now, from the definitions of $G_l^m$ and $R_l^m$, we show that

$$-n\wedge\Big(n \wedge N_{1,l}^m(x)\Big) \;=\; -\frac{i}{kR}\sqrt{l(l+1)}\,\Big(h_l^{(1)}(kR) + kR(h_l^{(1)})'(kR)\Big)\, G_l^m(\frac{x}{R}), \quad \text{on } \Sigma_R,$$

and

$$-n\wedge\Big(n \wedge M_{1,l}^m(x)\Big) \;=\; -\sqrt{l(l+1)}\, h_l^{(1)}(kR)\, R_l^m(\frac{x}{R}), \quad \text{on } \Sigma_R.$$

Direct computation yields

$$\int_{\Sigma_R}\Big(\overrightarrow{\mathrm{curl}}\, E \wedge n\Big)\,.\,\overline{E} = \sum_{l\ge 1}\sum_{m=-l}^{l} l\,(l+1)\,\Big(|\alpha_l^m|^2 + |\beta_l^m|^2\Big)\,\Im m\,\Big(h_l^{(1)}(kR)\,(h_l^{(1)})'(kR)\Big).$$

From

$$\Im m\ h_l^{(1)}(kR)\,\overline{(h_l^{(1)})'}(kR) < 0, \quad \forall l \geq 1,$$

proved in [2], we obtain that

$$\Im m \int_{\Sigma_R} \Big(\overrightarrow{\mathrm{curl}}\, E \wedge n\Big) \,.\, \overline{E} = 0 \implies E \equiv 0 \text{ in } |x| > R.$$

Appendix B For the convenience of the reader we summarize in this appendix the vector identities used in this paper. Let $\Omega$ be a bounded domain with a smooth boundary $\partial\Omega$. $n$ denotes its outward normal. Let $u$ and $v$ be two sufficiently smooth vectorial functions defined in $\Omega$ and $w$ on $\partial\Omega$ such that $n \,.\, w = 0$. We have

$$\int_\Omega \overrightarrow{\mathrm{curl}}\, u \,.\, v = \int_\Omega u \,.\, \overrightarrow{\mathrm{curl}}\, v - \int_{\partial\Omega} \Big(u \wedge n\Big) \,.\, \Big(n \wedge (n \wedge v)\Big).$$

$$\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\, u = \overrightarrow{\mathrm{grad}}\,\mathrm{div}\, u - \Delta u.$$

$$\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{grad}}\, u = 0.$$

$$\mathrm{div}\,\overrightarrow{\mathrm{curl}}\, u = 0.$$

$$\mathrm{div}_{\partial\Omega}(u \wedge n) = n \,.\, \overrightarrow{\mathrm{curl}}\, u|_{\partial\Omega}.$$

$$\int_{\partial\Omega} \overrightarrow{\mathrm{grad}}_{\partial\Omega}\,\mathrm{div}_{\partial\Omega}(u \wedge n) \,.\, (v \wedge n) = -\int_{\partial\Omega} \mathrm{div}_{\partial\Omega}(u \wedge n)\,\mathrm{div}_{\partial\Omega}\,(v \wedge n).$$

$$\mathrm{div}_{\partial\Omega}\,\overrightarrow{\mathrm{curl}}_{\partial\Omega}\, w = 0.$$

$$-\mathrm{curl}_{\partial\Omega}\,\overrightarrow{\mathrm{curl}}_{\partial\Omega}\, w = \Delta_{\partial\Omega}\, w = \mathrm{div}_{\partial\Omega}\,\overrightarrow{\mathrm{grad}}_{\partial\Omega}\, w.$$

$$\mathrm{curl}_{\partial\Omega}\,\overrightarrow{\mathrm{grad}}_{\partial\Omega}\, w = 0.$$

$$\mathrm{div}_{\partial\Omega}\,(w \wedge n) = \mathrm{curl}_{\partial\Omega}\, w.$$

$$\overrightarrow{\mathrm{curl}}\, u = \mathrm{curl}_{\partial\Omega}\, u_{\partial\Omega}\, n + \overrightarrow{\mathrm{curl}}_{\partial\Omega}(u \,.\, n) + (\mathcal{R} - 2c)(u \wedge n) - \frac{\partial}{\partial s}(u \wedge n),$$

where

$$u_{\partial\Omega} = -n \wedge (n \wedge u).$$

$$\begin{aligned}\overrightarrow{\mathrm{curl}}\,\overrightarrow{\mathrm{curl}}\, u \;=\; & (\mathrm{curl}_{\partial\Omega}\,\overrightarrow{\mathrm{curl}}_{\partial\Omega}\,(u \,.\, n))\, n + \mathrm{div}_{\partial\Omega}\,(\mathcal{R}\, u_{\partial\Omega})\, n + \mathrm{div}_{\partial\Omega}\,(\frac{\partial}{\partial n}\, u_{\partial\Omega}) \\ & + \overrightarrow{\mathrm{curl}}_{\partial\Omega}\,\mathrm{curl}_{\partial\Omega}\, u_{\partial\Omega} + (\mathcal{R} - 2c - \frac{\partial}{\partial s})\,(-\overrightarrow{\mathrm{grad}}_{\partial\Omega}(u \,.\, n) \\ & + \mathcal{R}\, u_{\partial\Omega} + \frac{\partial}{\partial s}\, u_{\partial\Omega}).\end{aligned}$$

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