# New advances in Backward SDEs for financial engineering applications

## Tamerza, Tunisia

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# Abstracts

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**Vlad BALLY (Université Marne-la-Vallée, France)**

### Density estimates for local-stochastic volatility models

**Abstract :** We consider a stochastic volatility model of Heston type but we allow the coefficients to depend on the state (the volatility and the stock price). This is a generalization of the classical Heston model which seems to be of increasing interest in the financial industry nowadays. Our aim is to give estimates for the tails of the distribution of the stock price and for the density of its law as well. We use these estimates to study the blow up of the moments in long time - which turns out to be similar to the one for constant coefficients. Since the coefficients are not constant we do no more have an explicit expression of the Fourier transform so the approach used for the classical Heston model fails here. Instead we use tubes estimates in order to get lower bounds for the tails of the distribution function and Malliavin calculus in order to obtain similar results for the density.

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**Dirk BECHERER (Humboldt University, Berlin, Germany)**

### From Bounds on Optimal Growth towards a Theory of Good-Deal Hedging

**Abstract :** Good-deal bounds have been introduced as a way to obtain valuation bounds for derivative assets which are tighter than the arbitrage bounds. This is achieved by ruling out not only those prices that violate no-arbitrage restrictions but also trading opportunities that are 'too good'. We study dynamic good-deal valuation bounds that are derived from bounds on optimal expected growth rates, inducing good dynamic properties for good-deal valuation bounds in a general semimartingale setting. When asset prices and state processes evolve as Ito-processes, good-deal bounds can be conveniently described by backward stochastic differential equations. We show that good-deal bounds arise as value functions from an optimal control problem, where a dynamic coherent risk measure is minimized by the choice of a hedging strategy. This demonstrates how the theory of no-good-deal valuation can be associated to a concept of dynamic hedging in continuous time. We discuss variations of the topic, including consequences from uncertainty about excess returns or market prices of risk of tradable assets on the hedging strategy.

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**Imen BEN LATIFA (LANSIN/ENIT (Tunis), Tunisia)**

**Optimal multiple stopping times problems of jump diffusion processes.**

**Abstract :** In this work, we extend the result of Carmona and Touzi for optimal multiple stopping problem to a market where the price process follows a jump diffusion process. We construct a sequence of new reward functions where the pay-off is not a priori right continuous adapted process. We prove the existence of multiple exercise policies under weaker assumptions than in the classical case. Next, we derive a sequence of dynamic programming variational inequality satisfied by the value function in the sense of viscosity solutions. We prove a comparison theorem which will be useful for numerical purpose, postponed in a further study.

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**Bruno BOUCHARD (Université Paris Dauphine, France)**

**Generalized stochastic target problems for the pricing and partial hedging under loss constraints - Application in optimal book liquidation**

**Abstract :** We consider a singular with state constraints version of the stochastic target problems. This provides a general framework for the pricing of contingent claims under risk constraints. Our extended version perfectly suits to market models with proportional transaction costs and to order book liquidation issues. Our main result is a direct PDE characterization of the associated pricing function. As an example of application, we discuss the evaluation of VWAP-guaranteed type book liquidation contracts, for a general class of risk functions.

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**Joceline BION-NADAL (CNRS, CMAP, Ecole Polytechnique, France)**

**Convex Risk Measures under Model Uncertainty**

**Abstract :** Motivated by the general context of model uncertainty, that is when no reference probability measure is given, we study regular convex risk measures on $\mathcal{C}_b(\Omega)$. We prove that every such risk measure on $\mathcal{C}_b(\Omega)$ extends into a convex risk measure on $L^1(c)$ for a certain capacity $c$. Making use of topological results for the dual of $L^1(c)$ that we establish, we prove that every convex risk measure on $L^1(c)$ has a dual representation with a countable set of probability measures.
When the capacity $c$ is $c(f) = \sup_{P\in\mathcal{P}} E_P(f)$ for some weakly compact set $\mathcal{P}$ of probability measures, we prove the existence of an equivalence class of probability measures characterizing the null elements of $L^1(c)_+$.
Making use of our previous results we associate to every regular convex risk measure on $\mathcal{C}_b(\Omega)$ a unique

equivalence class of probability measures on Borel sets, characterizing the riskless non positive elements of $\mathcal{C}_b(\Omega)$. We prove that the convex risk measure has then a dual representation with a countable set of probability measures absolutely continuous with respect to a certain probability measure in this class. We apply our results to the case of $G$-expectations, and also to the case of risk measuring under uncertain volatility. *This is based on joint work with Magali Kervarec (Université d'Evry, France).*

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**Mateo CASSERINI (ETH Zurich, Switzerland)**

### An approach to fully coupled FBSDEs via functional differential equations

**Abstract :** Backward stochastic dynamics have been introduced by Liang, Lyons and Qian as a generalization of Lipschitz BSDEs for non-Brownian filtrations, by relying on a Doob-Meyer decomposition argument. As a consequence, classical BSDEs can be represented in terms of the solution of a functional differential equation, and other classes of equations (not covered by the classical approach) can be treated in this framework.

In our work, this technique is extended to a general class of fully coupled FBSDEs. In particular, the solution of the FBSDE is obtained by solving a system of coupled forward equations, where the driver also depends on the terminal values of the solutions. The conflicting nature between the forward and backward components is thus avoided.

Moreover, we are able to locally treat other types of forward-backward equations not fitting in the classical FBSDE framework. Numerical algorithms based on this approach are also discussed. *This is a joint work with Gechun Liang.*

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**Jean-François CHASSAGNEUX, (Université d'Évry Val d'Essonne, France)**

### A discrete-time approximation for reflected BSDEs related to switching problem

We present a discrete-time approximation for a class of multi-dimensional obliquely reflected BSDEs. In the 2-dimensional case, they were introduced by Hamadène and Jeanblanc and latter generalized by Hu and Tang and Hamadène and Zhang. They are closely related to "switching problem", encountered in real option pricing e.g.. We provide a control on the error of the algorithm by introducing and studying the notion of multidimensional discretely reflected BSDE. In the particular case where the driver of the BSDEs does not depend on the variable $Z$, the error on the approximation grid points between the solution and the numerical scheme is of order $1/2 - \epsilon$, $\epsilon > 0$.

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**Samuel COHEN (Oxford University, United Kingdom)**

**Existence, Uniqueness and Comparisons for BSDEs in general spaces**

**Abstract :** We present a theory of Backward Stochastic Differential Equations in continuous time with an arbitrary filtered probability space. No assumptions are made regarding the left-continuity of the filtration, of the predictable quadratic variations of martingales, or of the measure integrating the driver. We present conditions for existence and uniqueness of square-integrable solutions, using Lipschitz continuity of the driver. These conditions unite the requirements for existence in continuous and discrete time, and allow discrete processes to be embedded with continuous ones. We also present conditions for a comparison theorem, and hence construct time consistent nonlinear expectations in these general spaces. *This is a joint work with Robert J. Elliott (University of Adelaide and University of Calgary).*

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**Stéphane CRÉPEY (Université d'Évry Val d'Essonne, France)**

**Doubly Reflected BSDEs with Call Protection and their Approximation**

**Abstract :** In this work we consider the issue of numerical solution of a doubly reflected backward stochastic differential equation, with an upper barrier which is only active on random time intervals (doubly reflected BSDE with an intermittent upper barrier, RIBSDE for short).
From the point of view of financial interpretation, RIBSDEs arise as pricing equations of game options with call protection, in which the call times of the option's issuer are subject to constraints preventing the issuer from calling the bond on certain random time intervals. Moreover, in the standing example of *convertible bonds*, this protection is typically monitored in a possibly very path-dependent way. Calls may thus be allowed or not at a given time depending on the past values of the underlying asset, which leads, after extension of the state space to markovianize the problem, to highly-dimensional pricing problems. Deterministic pricing schemes are then ruled out by the curse of dimensionality, and simulation methods appear to be the only viable alternative.

The ***purpose of this work*** is to propose a practical and mathematically justified approach to the problem of solving numerically by simulation the RIBSDEs that arise as pricing equations of game options with call protection. Our ***main result*** establishes convergences rates for a discrete time approximation scheme by simulation to an RIBSDE.

The practical value of this scheme is then thoroughly assessed. For problems in dimension up to 30, the accuracy of the simulation scheme, in cases where alternative PDE results are available and can be used as a benchmark (problems with a high 'nominal' dimension, but endowed with a specific structure allowing one to reduce them to a low 'effective dimension'), typically lies in the range of one bp ($10^{-2}$%) to 1% of relative error.

One thus gets a practical and mathematically justified approach to the problem of pricing by simulation convertible bonds with highly path-dependent call protection. More generally, this work is an illustration of the real abilities of simulation/regression numerical schemes for high to very high-dimensional pricing problems. *The talk is based on a joint work with Jean-François Chassagneux (Université d'Évry Val d'Essonne, France) and Abdallah Rahal (Université Libanaise, Liban).*

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**Boualem DJEHICHE (KTH, Stockholm, Sweden)**

### Optimal stopping of expected profit and cost yields in an investment under uncertainty

**Abstract :** We consider a finite horizon optimal stopping problem related to trade-off strategies between expected profit and cost cash- flows of an investment under uncertainty. The optimal problem is first formulated in terms of a system of Snell envelopes for the profit and cost yields which act as obstacles to each other. We then construct both a minimal and a maximal solutions using an approximation scheme of the associated system of respected backward SDEs. We also address the question of uniqueness of solutions of this system of SDEs. When the dependence of the cash-flows on the sources of uncertainty, such as fluctuation market prices, assumed to evolve according to a diffusion process, is made explicit, we also obtain a connection between these solutions and viscosity solutions of a system of variational inequalities (VI) with interconnected obstacles. *The talk is based on a joint work with Said Hamadène and Marie-Amélie Morlais (Université du Maine, France).*

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**Joscha DIEHL (Humboldt University of Berlin, Germany)**

### BSDEs with rough drivers

**Abstract :** Classically, the driver of a backward stochastic differential equation (BSDE) is a Lipschitz continuous function that is integrated with respect to Lebesgue measure. We introduce a new class of BSDEs where the driver consists of an additional integral that can posses much less regularity in time. We show existence, uniqueness and stability results for these equations and establish the connection to backward doubly stochastic differential equations (BDSDEs). Our interest in these equations has been partly motivated by their connection to partial differential equations driven by rough paths, for which we establish a Feynman-Kac type formula. *This is joint work with Peter Friz (TU Berlin, Germany).*

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**M'hamed EDDAHBI (University of Cadi Ayyad, Marrakesh, Morocco)**

### Limit theorems for BSDE with local time applications to nonlinear PDE

**Abstract :** Given a $d$-dimensional Wiener process $W$, with its natural filtration $(\mathcal{F}_t)$, a $\mathcal{F}_T$-measurable real random variable $\xi$, a bounded real measure $\nu$, and an adapted process $(s, y, z) \longmapsto h(s, y, z)$, we consider the following BSDE

$$Y_t = \xi + \int_t^T h(s, Y_s, Z_s) ds + \int_{\mathbb{R}} (L_T^a(Y) - L_t^a(Y))\nu(da) - \int_t^T Z_s dW s$$

for $0 \leq t \leq T$. Here $L_t^a(Y)$ stands for the local time of $Y$ at level $a$. For $h = 0$, we have the existence and the uniqueness of the processes $(Y, Z)$, and if $h$ is continuous with linear growth we have the existence of a solution. We prove limit theorems for solutions of backward stochastic differential equations of the above form. Those limit theorems will permit us to deduce that any solution of that equation is the limit in a strong sense of a sequence of semimartingales which are solutions of ordinary BSDE of the form

$$Y_t = \xi + \int_t^T f(Y_s) Z_s^2 ds - \int_t^T Z_s dW_s.$$

A comparison theorem for BSDE involving measures is discussed. As an application we obtain, with the help of the connection between BSDE and PDE, some corresponding limit theorems for a class of singular nonlinear PDE and a new probabilistic proof of the comparison theorem for PDE. *This is work in progress with Youssef Ouknine (University of Cadi Ayyad, Marrakesh, Morocco).*

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**Nicole EL KAROUI (LPMA/University Pierre and Marie Curie and CMAP/Ecole Polytechnique, France)**

### Quadratic BSDEs revisited, a forward point of view

**Abstract :** We propose a new approach to Quadratic BSDE's based on a forward point of view. More precisely, on a probability space where all martingales are continuous, we first study the semimartingales $Y$ satisfying a "*quadratic structure condition*", $Y_. = Y_0 - V_. + M_.$, where the total variation of the predictable finite variation $V$ satisfies:

$$d|V_t| \ll \frac{1}{\delta} d\Lambda_t + |Y_t|\, dC_t + \frac{\delta}{2} d < M >_t, \quad d\mathbb{P}\text{-a.s..}$$

Such processes $Y$ are called a quadratic semimartingales, when if $\Lambda, C$ identically equal to 0, and $\delta = 1$, $Y$ are called quadratic quasimartingales, or $\mathcal{Q}$-quasimartingales. Typical examples of $\mathcal{Q}$-quasimartingales are the logarithm and the opposite of the logarithm of exponential martingales. More

generally, $\mathcal{Q}$-quasimartingale $X$ may be characterized as an optional process such that both processes $\exp(X_.)$ and $\exp(-X_.)$ are submartingales. As a consequence, $X$ is continuous.
Moreover, if $\mathbb{E}[\exp(|X_T|)] < \infty$, and $\exp(|X_.|)$ in the class $(\mathcal{D})$, then for $\forall p \in [1, p^\eta[$, where $p^\eta = \sup\{p;\ \mathbb{E}\big[\exp(p|X_T|)\big] < +\infty\}$

$$\mathbb{E}[< M >_T^{\frac{p}{2}}] \leq (2p)^p\, \mathbb{E}\big[\exp(p|X_T|)\big].$$

In the general case, the same characterization is obtained by replacing the exponential by the transformation:

$$U_t^{\Lambda,C}(e^Y) = e^{Y_t} + \int_0^t e^{Y_s} d\Lambda_s + \int_0^t e^{Y_s}|Y_s| dC_s,$$

Following Briand and Hu, put $y_{t,T} = e^{C_{t,T}}|y_T| + \int_t^T e^{C_{t,u}} d\Lambda_u$ and $\Phi_t = \mathbb{E}[\exp(y_{t,T})|\mathcal{F}_t]$. If $\mathbb{E}[e^{(C_T|y_T| + \int_0^T e^{C_u} d\Lambda_u)}] = \mathbb{E}[e^{|y_{0,T}|}] < +\infty$, and $e^{|Y_.|} \leq \Phi_.$, the same estimate holds true,

$$\mathbb{E}[< M >_T^{\frac{p}{2}}] \leq (2p)^p\, \mathbb{E}\big[\exp(p|y_{0,T}|)\big].$$

There is also a BMO-version of this estimate.
As a consequence, the class of quadratic semimartingales satisfying the previous assumptions is closed with respect to the a.s monotone convergence, with strongly convergence of the martingale parts in the appropriate spaces.

These stability results allows us to show existence results for BSDEs with quadratic growth, whose the coefficient satisfies

$$|\, g(.,t,y,z)| \leq Q(t,y,z) \equiv \frac{1}{\delta}|l_t| + c_t|y| + \frac{\delta}{2}|z|^2 \quad d\mathbb{P} \otimes dt\text{-a.s.},$$

Special attention is given to strongly quadratic coefficient $g$ with the decomposition $g(.,t,y,z) = f(.,t,y,z) + \frac{\delta}{2}|z|^2$ where $f(.,t,y,z)$ is a Lipschitz coefficient with linear growth. On the assumption that $\mathbb{E}[e^{\delta p(C_T|Y_T| + \int_0^T e^{C_u} d\Lambda_u)}] < +\infty$, for some $p > 1$, there exists a solution $Y$ belonging to the previous class. Then, by classical regularization techniques as in Lepeltier and San Martin, we obtain the existence of the solution for general quadratic BSDEs with $\delta = 1$, dominated by the logarithm of $\Phi$. *This a joint work with Pauline Barrieu.*to the previous class. Then, by classical regularization techniques as in Lepeltier and San Martin, we obtain the existence of the solution for general quadratic BSDEs with $\delta = 1$, dominated by the logarithm of $\Phi$. *This a joint work with Pauline Barrieu.*

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**Emmanuel GOBET (CMAP, Ecole Polytechnique, Palaiseau, France)**

**Generalized fractional smoothness and Lp-variation of BSDEs with non-Lipschitz terminal condition**

**Abstract :** We relate the $L_p$-variation, $2 \leq p < \infty$, of a solution of a backward stochastic differential equation with a path-dependent terminal condition to a generalized notion of fractional smoothness in the sense of the Wiener space. This concept of fractional smoothness takes into account the quantitative propagation of singularities in time. *This talk is based on the joint work with Christel Geiss and Stefan Geiss.*

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**Saïd HAMADÈNE (University of Maine, Le Mans, France)**

**The Risk-Sensitive Switching Problem under Knightian Uncertainty**

**Abstract :** In this talk we discuss the switching problem in the case when the controller integrates her/his sensibility with respect to risk in the payoff and when the probability of the future is not fixed and could change among a family of probabilities $P^u$, $u \in \mathcal{U}$. This problem is tackled in using BSDEs with non standard coefficients and systems of reflected BSDEs with inter-connected obstacles. Actually with the help of those equations we characterize the quantity

$$J^* = \sup_{\delta} \inf_{u} J(\delta, u)$$

where $\delta$ is a switching strategy, $P^u$ a possible probability of the future and $J(\delta, u)$ the payoff, under $P^u$, when $\delta$ is implemented. In the energy market, $J^*$ is related to the price of a power plant. We also show the existence of a pair $(\delta^*, u^*)$ such that $J^* = J(u^*, \delta^*)$. Finally at the end of the talk, when randomness stems from a Markov diffusion process, we discuss the connection of this problem with systems of variational inequalities with inter-connected obstacles. *This is a work in progress with Hao Wang.*

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**Ying HU (IRMAR, University of Rennes 1, France)**

### Ergodic BSDEs under weak dissipative assumptions and application to ergodic control

**Abstract :** In this talk, we first introduce the notion of ergodic BSDE which arises naturally in the study of ergodic control. The ergodic BSDE is a class of infinite-horizon BSDEs:

$$Y_t^x = Y_T^x + \int_t^T [\psi(X_\sigma^x, Z_\sigma^x) - \lambda] d\sigma - \int_t^T Z_\sigma^x dB_\sigma, \quad \mathbb{P} - \text{a.s.}, \quad \forall\, 0 \leq t \leq T < \infty,$$

where $X^x$ is a diffusion process. We underline that the unknowns in the above equation is the triple $(Y, Z, \lambda)$, where $Y, Z$ are adapted processes and $\lambda$ is a real number. We review the existence and uniqueness result for ergodic BSDE under strict dissipative assumptions.

Then we study ergodic BSDEs under weak dissipative assumptions. On the one hand, we show the existence of solution to the ergodic BSDE by use of coupling estimates for perturbed forward stochastic differential equations. On the other hand, we show the uniqueness of solution to the associated Hamilton-Jacobi-Bellman equation by use of the recurrence for perturbed forward stochastic differential equations.

Finally, applications are given to the optimal ergodic control of stochastic differential equations to illustrate our results. We give also the connections with ergodic PDEs.

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**Peter IMKELLER (Humboldt University of Berlin, Germany)**

### Utility maximization and systems of forward-backward SDE

**Abstract :** In this talk we re-address the problem of utility maximization for general utility functions. For utility functions defined on the whole real line we show that if an optimal strategy exists then it is given in terms of the solution $(X, Y, Z)$ of a fully coupled FBSDE. Conversely if the FBSDE admits a solution $(X, Y, Z)$ then an optimal strategy can be obtained. In the complete market case, an assumption on the risk aversion guarantees that the FBSDE admits a solution for any finite time horizon. As a particular example of our approach we recover the BSDE obtained by Hu et al. (2005) for exponential utility, and are able to treat non-classical utility functions like the sum of exponential ones. For utility functions defined on the half line we also reduce the maximization problem to the solution of FBSDE connected with the ones obtained by Peng (1993). In complete markets, once again we provide conditions for solvability that are applicable to the power, the logarithmic and some non-classical utilities. In our approach we propose an alternative form of the maximum principle for which the Hamiltonian is reflected in a remarkable martingale. *This is joint work with U. Horst, Y. Hu, A. Réveillac, and J. Zhang.*

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**Azmi MAKHLOUF (Osaka University, Osaka, Japan)**

### $\mathbf{L}_2$-time regularity of BSDEs with irregular terminal functions

**Abstract:** We study the $\mathbf{L}_2$-time regularity of the $Z$-component of a Markovian BSDE, whose terminal condition is a function $g$ of a forward SDE $(X_t)_{0\le t\le T}$. When $g$ is Lipschitz continuous, Jianfeng Zhang (2004) proved that the related squared $\mathbf{L}_2$-time regularity is of order one with respect to the size of the time mesh. We extend this type of result to any function $g$, including irregular functions such as indicator functions for instance. We show that the order of convergence is explicitly connected to the rate of decreasing of the expected conditional variance of $g(X_T)$ given $X_t$ as $t$ goes to $T$. This holds true for any Lipschitz continuous generator. The results are optimal. *This talk is based on a joint work Emmanuel Gobet (Ecole Polytechnique, Palaiseau, France).*

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**Konstantinos MANOLARAKIS (Imperial College London, United Kingdom)**

### Second order discretization and efficient simulation of Markovian BSDEs

**Abstract :** In this talk, we present an extension to the Euler type discretization of Bouchard-Touzi-Zhang for BSDEs, to the second order. Utilizing on the fact that the $Y$ part of the solution can be expressed as an integral against the law of the forward diffusion, we can achieve the second order by using the trapezoid method for the driver part and predictor-corrector style techniques for the $Z$-part of the solution.

We then suggest a fully implementable scheme, by evaluating the associated conditional expectations by means of the cubature method. Numerical examples are also presented, that validate the claimed estimates. T*his is joint work with Dan Crisan (Imperial College London, United Kingdom).*

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**Ludovic MOREAU (CEREMADE, Université Paris-Dauphine, Paris 9, France)**

### Stochastic Target Problems with Controlled Loss in Jump Diffusion Models

**Abstract :** For $0 \le t \le T$, we are given two controlled diffusion processes $\left\{X^{\nu}_{t,x}(s), t \le s \le T\right\}$ and $\left\{Y^{\nu}_{t,x,y}(s), t \le s \le T\right\}$ with values respectively in $\mathbb{R}^d$ and $\mathbb{R}$, starting at time $t$ in $(x, y)$. The aim of the controller is to find the minimal initial condition $y$ for which it is possible to find a control $\nu$ satisfying $\mathbb{E}\left[\Psi\left(X^{\nu}_{t,x}(T), Y^{\nu}_{t,x,y}(T)\right)\right] \ge p$ for some given measurable map $\Psi$, non-decreasing in the $y$-variable, and for a level $p$. Namely, he wants to compute:

$$v(t,x,p) := \inf \left\{ y \geq -\kappa : \mathbb{E}\left[ \Psi\left( X^{\nu}_{t,x}(T), Y^{\nu}_{t,x,y}(T) \right) \right] \geq p \text{ for some control } \nu \right\}, \tag{1}$$

where $\kappa \in \mathbb{R}_+$. When $p = 1$ and $\Psi(x,y) = 1_{\{V(x,y) \geq 0\}}$ for some map $V$, then $v(t,x,1)$ coincides with the stochastic target problem studied in Bouchard (2002), and in Soner and Touzi (2002) for Brownian controlled SDEs. In the above mentioned papers, the authors restricted to the setting of controls with values in a compact subset of $\mathbb{R}^d$. Their proofs are heavily relying on this assumption. One may note that for $V(x,y) = y - g(x)$ for some function $g$, $v(t,x,1)$ coincides with the superreplication price of the claim $g\left(X^{\nu}_{t,x}\right)$.

Such problems have already been discussed by Follmer and Leukert (1999) in the context of financial mathematics. However, they used a duality argument which does not extend to general non linear controlled diffusion cases.

In order to deal with the problem 1, Bouchard, Elie and Touzi (2009) introduced an additional controlled diffusion process $P^{\alpha}_{t,p}$, which appears to (essentially) correspond to the conditional probability of reaching the target $V\left(X^{\nu}_{t,x}(T), Y^{\nu}_{t,x,y}(T)\right) \geq 0$. This allowed them to reformulate the problem and to rewrite the problem into a classical stochastic target problem already studied by Soner and Touzi. However, the new control $\alpha$ appearing in the diffusion $P^{\alpha}$ can no longer be assumed to take values in a compact set, as it is given by the martingale representation theorem.

The aim of this presentation is to extend the work of Bouchard, Elie and Touzi (2009) to the setting of jump diffusions, in order to obtain a PDE characterization for the value function 1. This approach allows us to deal with general problem, such as the utility indifference pricing, quantile hedging, loss functions among others.

We follow the key idea of Bouchard, Elie and Touzi (2009) so as to convert the problem $v$ into a singular stochastic target problem. This is done by diffusing the conditional expectation $\mathbb{E}\left[\Psi\left(X^{\nu}_{t,x}(T), Y^{\nu}_{t,x,y}(T)\right) \middle| \mathcal{F}_s\right]$ for $s \in [t,T]$, and considering it as an additional controlled state variable $P^{\alpha,\chi}_{t,p}$, where the additional control $\chi$ comes from the jump part of the martingale representation.

The main new technical difficulty is due to the presence of jumps and of the new control $\chi$. First, it leads to an additional (non-local) term in the PDE characterization. Moreover, part of the control now takes values in an unbounded set of measurable maps. This also leads to a new (non-trivial) relaxation of the non-local part of the associated operator, in comparison to Bouchard, Elie and Touzi (2009). We shall see that the convex face-lifting phenomenon in the $p$-variable observed by Bouchard, Elie and Touzi (2009) at the terminal date extends to a much more general context.

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**Mohamed MRAD (University of Paris 13, Villetaneuse, France)**

**An Exact Connection between two Solvable SDEs and a Non Linear Utility Stochastic PDEs**

**Abstract :** The paper proposes a new approach to consistent stochastic utilities, also called forward dynamic utility, recently introduced by M. Musiela and T. Zariphopoulou. These utilities satisfy a property of consistency with a given incomplete financial market which gives them properties similar to the function values of classical portfolio optimization. First, we derive a non linear stochastic PDEs that satisfy consistent stochastic utilities processes of Itô type and their dual convex conjugates. Then, under some assumptions of regularity and monotony on the stochastic flow associated with the optimal wealth as function of the initial capital, and on the optimal state price dual process, we characterize all consistent utilities for a given increasing optimal wealth process from the composition of the dual optimal process and the inverse of the optimal wealth. This allows us to reduce the resolution of fully nonlinear second order utility SPDE to the existence of monotone solutions of two stochastic differential equations. We also, express the volatility of consistent utilities as an operator of the first and the second order derivatives of the utility in terms of the optimal primal and dual policies. *This a joint work work with Nicole El Karoui (University Pierre and Marie Curie and Ecole Polytechnique, France).*

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**Marcel NUTZ (ETH, Zürich, Switzerland)**

**Random $G$-Expectations**

**Abstract :** We construct a time-consistent sublinear expectation (i.e. risk measure) in the setting of volatility uncertainty. This mapping extends Peng's $G$-expectation by allowing the range of the volatility uncertainty to be stochastic. Our construction is purely probabilistic and based on an optimal control formulation with path-dependent control sets.

In the second part of the talk, we consider a general class of sublinear expectations in this setting and discuss the associated nonlinear martingales and superhedging problems. *This is a joint work with Mete Soner (ETH, Zürich, Switzerland).*

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**Bernt OKSENDAL, Oslo University, Norway**

**Optimal control of stochastic delay equations and time-advanced backward stochastic differential equations**

**Abstract :** We consider the problem of controlling optimally a delay jump diffusion, i.e. a system described by a stochastic differential equation with delay, driven by Brownian motions and compensated Poisson random measures. Such delay systems may occur in several situations, e.g. in finance and biology where the growth of the state depends not only on the current value of the state but also on previous state values.

We give both a sufficient and a necessary maximum principle for such control problems. These maximum principles involve backward stochastic differential equations (BSDEs) which are "anticipative", in the sense that they have a time-advanced drift coefficient. We prove existence and uniqueness theorems for such time-advanced BSDEs. The results are illustrated by examples. *The presentation is based on recent joint work with Agnès Sulem (INRIA Rocquencourt, Paris, France) and Tusheng Zhang (University of Manchester).*

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**Gilles PAGÈS (LPAM/University Pierre and Marie Curie, France)**

## Dual quantization methods and application to Finance

**Abstract :** Regular quantization tree methods are based on the mapping of a discrete time Feller Markov chain at each time step on a grid following a nearest neighbor projection. When these grids are optimized with respect to the marginal distributions of the chain, the resulting backward dynamic programming computation method (the so-called quantization tree) has shown its efficiency in the solving of many non-linear problems arising in Finance like multi-asset American option pricing and hedging, stochastic control like swing options, or non linear filtering (see *e.g.* [2]).

We recently developed (see [3,4,5]) a new approach to quantization, called *dual quantization*, see [4,5], based on the mapping of the Markovian dynamics at each time step onto the vertices of a Delaunay triangulation spanned by a grid (the Delaunay triangulation is the dual geometrical object of the Voronoi tessellation involved in regular vector quantization). The aim of this talk is to present this new approach as well as its first applications to finance.

For a static $\mathbb{R}^d$-valued $L^p$-integrable random vector, say $X$, we prove the existence of an optimal dual quantizer of size at most $N$ (see [4]) and, under a slightly stronger moment assumption, we establish the sharp asymptotics for the resulting optimal dual quantization error as $N$ goes to infinity in the expected $N^{-\frac{1}{d}}$-scale (see [5]). The constant in this asymptotics depends on the distribution of $X$ (through its density) and on a universal constant $C_{p,d}^{dual}$ (made explicit in one dimension). This result

is the exact counterpart for dual quantization of Zador's Theorem (see [1]). New simulation based stochastic optimization procedures have been derived to produce optimal dual quantization grids for any simulatable distribution.

However, by contrast with optimal regular quantization, dual quantization grids all share a stationarity property, regardless of their optimality. As a first consequence, this induces a significant improvement of the performances of *dual quantization trees*, especially for smaller grid sizes. But the most striking consequence is the resulting robustness and flexibility of such trees with respect to the possible parameter variations when dealing with a parametrized underlying dynamics. So, it should become an efficient tool for perform calibrations or to solve multi-dimensional stochastic control problems when the underlying dynamics depends on the control.

First applications to credit derivatives (see [3]) and to multi-asset option pricing will be presented as well as first connections with the finite element methods. *This presentation is based on a several joint works with Benedikt Wilbertz (UPMC).*

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**Huyên PHAM (LPMA, Université de Paris 7, France)**

**Optimal investment under multiple defaults and recursive systems of BSDEs**

**Abstract :** We study an optimal investment problem under contagion risk in a financial market model subject to multiple jumps and defaults. The global market information is formulated as progressive enlargement of filtrations, and the dependence of default times is modeled dynamically by a conditional density hypothesis. In this general framework, we give a decomposition of the corresponding stochastic control problem into stochastic control problems in the default-free filtration, which are determined in a recursive induction. The dynamic programming method leads then to a characterization of the solution to the optimal investment problem through a recursive system of quadratic BSDEs, whose existence and uniqueness is proved under usual conditions. Beyond the financial problem, our decomposition approach provides another perspective for solving quadratic BSDEs with finite number of jumps.

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**Dylan POSSAMAI, (CMAP, Ecole Polytechnique, Palaiseau, France)**

**Second Order Backward Stochastic Differential Equations with Continuous Coefficients**

**Abstract :** We prove the existence and uniqueness of a solution for one-dimensional second order backward stochastic differential equations introduced by Soner, Touzi and Zhang (2010) with a generator which is Lipschitz in $z$ and continuous with linear growth with a monotonicity condition in $y$. We also prove existence of a minimal and maximal solution when the generator is only continuous with linear growth in $y$ and $z$ and when the family of mutually singular probability measures under which the problem is written is weakly compact.

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**Marina SANTACROCE (Department of Mathematics of Politecnico in Turin, Italy)**

**Power utility maximization under partial information: some convergence results**

**Abstract :** We consider the power utility maximization problem under partial information in a continuous semimartingale setting. Investors construct their strategies using the available information, which eventually may not even include the observations of the asset prices.
Resorting to stochastic filtering, the problem is transformed into an equivalent one, which is formulated in terms of observable processes. The value process, related to the equivalent optimization problem, is then characterized as the unique bounded solution of a semimartingale backward stochastic differential

equation (BSDE). This yields a unified characterization for the value process related to the power and exponential utility maximization problems, the latter arising as a particular case. The convergence of the corresponding optimal strategies is obtained by means of BSDEs.

In a Brownian model with constant correlation, we get the expression of the value process and of the optimal strategy, directly solving the BSDE. When $\rho$ is stochastic and not constant in time, we can still express the value process in a form which resembles the one obtained for constant $\rho$. In this case, the goal is reached getting lower and upper bounds for the value process and then by interpolation. *This is a joint work with D. Covello and M. Mania.*

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**Peter TANKOV (Ecole Polytechnique, Palaiseau, France)**

### A finite dimensional approximation for pricing American options on moving average

**Abstract :** We introduce a new method to value American options whose payoff depends on the moving average of the underlying asset price $S$:

$$X_t = \frac{1}{\delta}\int_{t-\delta}^{t} S_u du, \quad \forall t \geq \delta$$

American-style options on moving average are principally used in corporate finance and in energy markets. In gas markets, these contracts typically allow the holder to purchase an amount of gas at a strike price, which is indexed on moving averages of various oil-prices.

Pricing moving window options with early exercise is challenging both from the theoretical and the numerical viewpoint. In a continuous time framework, as the cashflow from exercise depends on the path(s) of the underlying price(s) over the averaging window, the problem is infinite dimensional. In a discrete time framework (pricing of a Bermudean option instead of an American option) there is a computational challenge, due to high dimensionality: the dimension is equal to the number of time steps within the averaging window. This in particular makes it difficult to compute the conditional expectations involved in the optimal exercise rule.

In this talk, we propose a method for pricing moving average American options based on a finite dimensional approximation of the infinite-dimensional dynamics of the moving average process. Consider a general moving average process of the form

$$M_t = \int_0^\infty S_{t-u} h(u) du$$

The moving average American option pricing problem is:

$$\sup_{\tau \in \mathcal{T}_{[\delta,T]}} \mathbb{E}\left[\phi\left(S_\tau, M_\tau\right)\right], \tag{2}$$

where $\phi : \mathbb{R}_+^2 \to \mathbb{R}$ is the pay-off function and $\mathcal{T}_{[\delta,T]}$ is the set of $\mathcal{F}$-stopping times valued in $[\delta, T]$. We propose a finite-dimensional approximation to $M$ which can be written as

$$M_t^n = HS_t + \int_0^\infty S_{t-u} h_n(u) du, \tag{3}$$

where the function $h_n$ is of the form

$$h_n(t) = \sum_{k=0}^{n-1} a_k^p L_k^p(t).$$

Here, $H$ and $a_k^p$ are coefficients which we determine and $(L_k^p)_{k\geq 0}$ are the scaled Laguerre functions defined on $[0, +\infty)$ by

$$L_k^p(x) = \sqrt{2p}\; P_k(2px)e^{-px}, \quad \forall k \geq 0$$

where $(P_k)_{k\geq 0}$ is the family of Laguerre polynomials.

Approximation (3) can also be written as

$$M_t^p = HS_t + \sum_{k=0}^{n-1} a_k^p X_t^{p,k}, \quad \forall t \geq 0$$

where $(X^{p,k})_{k=0,\dots,n-1}$ are $n$ *Laguerre processes* defined by:

$$X_t^{p,k} = \int_0^{+\infty} L_k^p(v) S_{t-v} dv, \forall t \geq 0, \forall k = 0, \dots, n-1$$

whose joint dynamics is Markovian. The American option price (2) can then be approximated by

$$\sup_{\tau \in \mathcal{T}_{[\delta,T]}} \mathbb{E}\left[\phi\left(S_\tau, M_\tau^n\right)\right]. \tag{4}$$

The resulting problem is then a finite-dimensional optimal stopping problem, which we propose to solve with a Monte Carlo Longstaff and Schwartz-type [2] approach. We use the improved technique of adaptative local basis proposed by Bouchard and Warin [1].

We compute the rate of convergence of our method as function of the number of terms in the series. If the stock price is a continuous Itô process satisfying suitable integrability conditions, the pricing error admits the bound

$$\mathcal{E}^\delta(n,p) := \left| \sup_{\delta \leq \tau \leq T} \mathbb{E}\left[\phi\left(S_\tau, M_\tau\right)\right] - \sup_{\delta \leq \tau \leq T} \mathbb{E}\left[\phi\left(S_\tau, M_\tau^{n,p}\right)\right] \right| \leq C\varepsilon(n^{-\frac{3}{4}}).$$

where $C > 0$ is a constant independent of $n$ and $\varepsilon(h) := \sqrt{h \ln\left(\frac{2T}{h}\right)}$.

Numerical results are presented for the moving average options commonly encountered in electricity and gas markets. *This is a joint work with Marie Bernhart and Xavier Warin .*

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**Nizar TOUZI (Ecole Polytechnique, Palaiseau, France)**

**Wellposedness of Second Order Backward SDEs**

**Abstract :** We provide an existence and uniqueness theory for an extension of backward SDEs to the second order. While standard Backward SDEs are naturally connected to semilinear PDEs, our second order extension is connected to fully nonlinear PDEs. In particular, we provide a fully nonlinear extension of the Feynman-Kac formula. The formulation of this paper insists that the equation must hold under a non-dominated family of mutually singular probability measures.

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**Plamen TURKEDJIEV (Humboldt University of Berlin, Germany)**

**Convergence of the Bender/Denk algorithm**

**Abstract :** In this talk, we present results about the $\mathbf{L^2}$-convergence of the processes generated by the popular algorithm due to Bender and Denk (2007) to the true solution of decoupled FBSDEs. The algorithm is a discretized Picard iterations' method which uses and empirical least squares minimization to estimate projection at each point in the time-grid onto a finite-dimensional vector space spanned by a preselected basis. We have determined, given a fixed basis, number of Picard iterations, and time-grid, that the $\mathbf{L^2}$-error in the algorithm due to empirical estimation is $o(M^{-1/2})$, where $M$ is the number of simulated paths of the forward process. We include a discussion of the effect of parameter selection, particularly the choice of time-grid and number of Picard iterations, on the bounds of the $\mathbf{L^2}$-error. In addition, we include some examples to show the influence of basis selection on the performance of the algorithm.

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**Mingyu XU (Institute of Applied Mathematics, Beijing, China)**

**Backward stochastic differential equation with ratio constraint and its application**

**Abstract :** Non-linear backward stochastic differential equations (BSDEs in short) were first studied by Pardoux and Peng (1990), who proved the existence and uniqueness of the adapted solution, under smooth square integrability assumptions on the coefficient and the terminal condition, and when the coefficient $g(t, \omega, y, z)$ is Lipschitz in $(y, z)$ uniformly in $(t, \omega)$. From then on, the theory of backward stochastic differential equations (BSDE) has been widely and rapidly developed. And many problems in mathematical finance can be treated as BSDEs. The natural connection between BSDE and partial differential equations (PDE) of parabolic and elliptic types is also important applications. In this talk, we study a new development of BSDE, BSDE with ratio constraint, i.e. portfolio process is controlled by a function of wealth process. The existence and uniqueness results are presented and we will give some application of this kind of BSDE at last.

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**Jianfeng ZHANG (University of Southern California (LA), USA)**

**A unified approach to wellposedness of non-Markovian FBSDEs**

**Abstract :** Compared to BSDEs, the theory of FBSDEs is much less mature. There are typically three methods in the literature: (i) the contraction mapping approach, (ii) the four step scheme, and (iii) the method of continuation. The three methods do not cover each other, and many FBSDEs arising from applications do not fit into any of them. We shall proposes a new approach which essentially covers all the existing methods and provides weaker sufficient conditions. Our main device is a decoupling random filed, which extends the four step scheme to non-Markovian cases. It turns out that the uniform Lipschitz continuity of this decoupling random field plays the central role in our approach. Our results are also closely related to Backward Stochastic Riccati equations with quadratic growth in $Z$, and Quasilinear Backward Stochastic PDEs. As a by product, we obtain the comparison principle for this random decoupling field.

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**Qi ZHANG (Fudan University, Shanghai, China)**

### Probabilistic Representation of Weak Solutions of PDEs with Polynomial Growth Coefficients via Corresponding Backward SDEs

**Abstract :** We develop a new weak convergence and compact embedding method to study the existence and uniqueness of the weighted-$L^{2p}(dx) \otimes L^2(dx)$ valued solutions of backward SDEs with polynomial growth coefficients. Then we give the probabilistic representation of the weak solutions of PDEs with polynomial growth coefficients via corresponding backward SDEs. This connection has independent interests in the related problems of both PDEs and backward SDEs.

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