Discussion on the talk by B.Bouchard "Stochastic target problems and pricing under risk constraints"

The starting point of the paper is:

For $(t, x, y) \in [0, T] \times R^{k+1}$ and α a control process valued in U, let $(X_s^{\alpha,t,x})_{s\in[t,T]}$ and $(Y_s^{\alpha,t,y})_{s\in[t,T]}$ be diffusion processes which take their value in R^{k+1} such that $X_t^\alpha = x$ and $Y_t^\alpha = y$. For example X^α (resp. Y^α) is the stock price in the market (resp. the value of a portfolio).

Let $v(t, x)$ be the following function:

$$v(t,x) = \inf\{y, \exists\alpha \in \mathcal{A} \text{ such that } Y_T^{\alpha,t,x,y} \geq g(X_T^{\alpha,t,x,y})\}.$$

This is a stochastic target problem (STP).

In the case when U is bounded this problem has been considered by by Soner-Touzi (00) and $v(t, x)$ is characterized as a unique solution of its associated HJB equation in viscosity sense.

The drawback of this point of view is that it gives high hedging prices in the market. Therefore another point of view is introduced by Follmer-Leukert (00) and developed by Bouchard-Elie-Touzi (09) (in a general case) which is the following:

Instead of v one considers:

$$w(t,x,p) = \inf\{y, \exists \alpha \in \mathcal{A} \text{ such that } \\ P[Y_T^{\alpha,t,x,y} \geq g(X_T^{\alpha,t,x,y})] \geq p\}$$

where $p \in (0,1)$.

In relaxing the probability p and adding a component to X^α they have been able to write $w(t,x,p)$ in the same way as $u(t,x)$ but in this case U is no more bounded. Therefore the feeling of the need to consider the STP for unbounded domains of controls U. This is done in BET (09). Actually they obtain a DPP and show that the value function is a viscosity solution of its associated HJB equation. The uniqueness of the solution in the general case is left aside.

The aim of the authors in this paper is to go beyond the framework of BET and to consider dynamics which can integrate e.g. markets with transaction costs. This is done in the first part of the paper where they obtain a DPP for their problem and then show that the value function is solution in viscosity sense for its associated HJB equation. In the second part of the paper they specify the model to *optimal book liquidation (VWAP)* and they show that the value function of the problem is also unique.

My questions are:

(i) what about the prices provided by those methods of quantiles? Are they reasonable?

(ii) what about numerics of the problem and subsequently the question of uniqueness of the viscosity solution in the first part.

(iii) does it make sense to consider in the model bounded variation processes and not only non-decreasing process?

(iv) what about the BSDE point of view involved at the end of the talk?

(v) what about the method which consist to invert $p(t,x,y)$ such that

$$p(t,x,y) = \max_{\alpha} P[Y^{\alpha}_{t,x,y} \geq g(X^{\alpha}_{t,x,y})]$$

and then to deduce $v(t,x,p)$ such that

$$p(t,x,v(t,x,p)) = p.$$