Optimal investment under multiple defaults and recursive system of BSDEs

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Workshop New advances in Backward SDEs for financial engineering applications

Tamerza, October 25-28, 2010

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Indifference pricing

Single default model

Let $M_t = \mathbb{1}_{\tau < t} - \int_0^t \widehat{\lambda}_s ds$ with $\widehat{\lambda}_s = \mathbb{1}_{s<\tau}\lambda_s$ be the compensated martingale of the default process.

We assume that the price process follows

$$dS_t = S_{t-}(\nu dt + \sigma dW_t + \varphi dM_t), \quad S_0 > 0$$

Let

$$H = H_T^1 \mathbb{1}_{\tau<T} + H_T^0 \mathbb{1}_{T<\tau}$$

where $H_T^1 = H_T^1(\tau)$ with $H_T^1(u) \in \mathcal{F}_T$ and $H^0(T) \in \mathcal{F}_T$.

Exponential models Assuming immersion property.

The value function is $e^{X_t^* - Z_t}$ where

$$dZ_t = \left(\frac{\nu^2}{2\sigma^2} - \frac{\nu}{\sigma} \widehat{z}_t + (e^{\widetilde{z}_t} - \widetilde{z}_t - 1)\widehat{\lambda}_t \right) dt - \widehat{z}_t dW_t - \widetilde{z}_t dM_t,\ Z_T = H$$

and one can solve this equation in two parts

After the default

$$dZ_t^1 = \left(\frac{\nu^2}{2\sigma^2} - \frac{\nu}{\sigma} \widehat{z}_t \right) dt - \widehat{z}_t dW_t, Z_T^1 = H_T^1$$

This leads to a solution $Z_t^1(\tau)$, hence to a family of processes $Z_t^1(u)$. and, since $\widetilde{z}_t = jump = Z_t^1(t) - Z_t^0$

$$dZ_t^0 = \left(\frac{\nu^2}{2\sigma^2} - \frac{\nu}{\sigma} \widehat{z}_t + (e^{(Z_t^0 - Z_t^1(t))} - 1) \right) dt - \widehat{z}_t dW_t, Z_T^0 = H_T^0$$

In the case of deterministic coefficients, deterministic target H^0 and deterministic functions $H_1(\cdot)$, the problem can be explicitly solved.

No density process involved: this is due to immersion property

General case: write W as a semimartingale in the enlarged filtration. This decomposition involves the density process

Mean Variance Hedging

The value function is of the form $V_t x^2 + U_t x + A_t$ where

$$\begin{aligned} dV_t &= \left(\frac{(\nu V_t + \widehat{v}_t \sigma + \widetilde{\lambda}_t \varphi \widetilde{v}_t)^2}{V_t \sigma^2 + \varphi^2 \widetilde{\lambda}_t (\widetilde{v}_t + V_t)} dt + \widehat{v}_t dW_t + \widetilde{v}_t dM_t \right) \\ V_T &= 1. \end{aligned}$$

The solution is

$$V_t = V_t^0 \mathbb{1}_{t<\tau} + V_t^1 \mathbb{1}_{\tau \leq t}$$

where

$$V_t^1 = e^{\frac{\nu}{\sigma^2}(T-t)}$$

and

$$\begin{aligned} dV_t^0 &= \frac{(\nu V_t^0 + \widehat{v}_t \sigma + \lambda_t \varphi (V_t^0 - V_t^1))^2}{V_t^0 \sigma^2 + \varphi^2 \lambda_t V_t^1} dt + \widehat{v}_t dW_t + (V_t^1 - V_t^0) \lambda dt \\ V_T^0 &= 1. \end{aligned}$$

In the case of deterministic coefficients, one has to solve

$$
\begin{aligned}
dV_t^0 &= \left(\frac{(\nu V_t^0 + \lambda_t \varphi (V_t^0 - V_t^1))^2}{V_t^0 \sigma^2 + \varphi^2 \lambda_t V_t^1} + (V_t^1 - V_t^0)\lambda_t \right) dt \\
V_T^0 &= 1.
\end{aligned}
$$