Optimal investment under multiple defaults: a BSDE-decomposition approach

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The basic problem

- Assets portfolio subject to **multiple defaults risk**
  - In addition to the default-free assets model (e.g. diffusion model with Brownian W), introduce jumps at random times modelled by a marked point process $(\tau_i, \zeta_i)_i \leftrightarrow$ random measure $\mu(dt, de)$.
- Optimal investment by classical stochastic control methods:
  - (Quadratic) BSDEs with jumps: this relies on martingale representation in the global filtration generated by W and μ.

Optimal investment problem with defaults revisited

- Approach by using:

- Point of view of global filtration as **progressive enlargement of filtration** of the default-free filtration

- **Decomposition in the default-free filtration**

▶ Backward system of BSDEs in Brownian filtration

- Get rid of the jump terms and overcome the technical difficulties in BSDEs with jumps
- Existence and uniqueness results in a general formulation under weaker conditions

Multiple defaults times and marks

On a probability space $(\Omega, \mathcal{G}, \mathbb{P})$:

• **Reference filtration** $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$: default-free information

Progressive information provided, when they occur, by:

• a family of n **random times** $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n)$ associated to a family of n **random marks** $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_n)$.

- τ_i default time of name $i \in \mathbb{I}_n = \{1, \ldots, n\}$.
- The mark ζ_i, valued in E Borel set of $\mathbb{R}^p$, represents a jump size at τ_i, which cannot be predicted from the reference filtration, e.g. the loss given default.

Progressive enlargement of filtrations

The **global market information** is defined by:

$$\mathbb{G} \;=\; \mathbb{F} \vee \mathbb{D}^1 \vee \ldots \vee \mathbb{D}^n,$$

where $\mathbb{D}^i$ is the default filtration generated by the observation of τ_i and ζ_i when they occur, i.e.

$$\mathbb{D}^i \;=\; (\mathcal{D}^i_t)_{t\geq 0}, \qquad \mathcal{D}^i_t \;=\; \sigma\{\mathbf{1}_{\tau_i \leq s}, s \leq t\} \vee \sigma\{\zeta_i \mathbf{1}_{\tau_i \leq s}, s \leq t\}.$$

$\rightarrow$ $\mathbb{G} = \mathbb{F} \vee \mathbb{F}^\mu$, where $\mathbb{F}^\mu$ is the filtration generated by the jump random measure $\mu(dt, de)$ associated to (τ_i, ζ_i).

Successive defaults

For simplicity of presentation, we assume that

$$\tau_1 \;\leq\; \ldots \leq\; \tau_n$$

Remarks.

• This means that we do not distinguish specific credit names, and only observe the ordered defaults: relevant for classical portfolio derivatives, e.g. basket default swaps.

• The general multiple random times case for $(\tau_1, \ldots, \tau_n)$ can be derived from the ordered case by considering the filtration generated by the corresponding ranked times $(\hat{\tau}_1, \ldots, \hat{\tau}_n)$ and the index marks ι_i, $i = 1, \ldots, n$ so that $(\hat{\tau}_1, \ldots, \hat{\tau}_n) = (\tau_{\iota_1}, \ldots, \tau_{\iota_n})$.

Notations

- For any $k = 0, \ldots, n$:

$$\begin{aligned} \Omega_t^k &= \{\tau_k \leq t < \tau_{k+1}\}, \\ \Omega_{t^-}^k &= \{\tau_k < t \leq \tau_{k+1}\}, \end{aligned}$$

with the convention $\Omega_t^0 = \{t < \tau_1\}$, $\Omega_t^n = \{\tau_n \leq t\}$.

$\rightarrow$ Scenario of k defaults before time t, the other names having not yet defaulted.

Ω_t^k: k-**default scenario** at time t, $(\Omega_t^k)_{k=0,\ldots,n}$ partition of Ω.

- For $k = 0, \ldots, n$,

$$\boldsymbol{\tau}_k = (\tau_1, \ldots, \tau_k), \qquad \boldsymbol{\zeta}_k = (\zeta_1, \ldots, \zeta_k),$$

with the convention $\boldsymbol{\tau}_0 = \emptyset$, $\boldsymbol{\zeta}_0 = \emptyset$.

Decomposition of $\mathbb{G}$-adapted and predictable processes

Lemma

Any $\mathbb{G}$-adapted process Y is represented as:

$$Y_t = \sum_{k=0}^{n} 1_{\Omega_t^k} Y_t^k(\tau_k, \zeta_k), \quad (1)$$

where Y_t^k is $\mathcal{F}_t \otimes \mathcal{B}(\mathbb{R}_+^k) \otimes \mathcal{B}(E^k)$-measurable.

Remarks. • A similar decomposition result holds for $\mathbb{G}$-predictable processes: $\Omega_t^k \leftrightarrow \Omega_{t-}^k$, and $Y^k \in \mathcal{P}_{\mathbb{F}}(\mathbb{R}_+^k, E^k)$-measurable in (1).

• Extension of Jeulin-Yor result (case of single random time without mark).

• We identify Y with the $n+1$-tuple $(Y^0, \ldots, Y^n)$.

- **Portfolio of N assets** with $\mathbb{G}$-adapted value process S:

$$S_t = \sum_{k=0}^{n} 1_{\Omega_t^k} S_t^k(\boldsymbol{\tau}_k, \boldsymbol{\zeta}_k),$$

where $S^k(\boldsymbol{\theta}_k, \mathbf{e}_k)$, $\theta_k = (\theta_1, \ldots, \theta_k) \in \mathbb{R}_+^k$, $\mathbf{e}_k = (e_1, \ldots, e_k) \in E^k$, indexed $\mathbb{F}$-adapted process valued in $\mathbb{R}_+^N$, represents the assets value given the past default events $\boldsymbol{\tau}_k = \boldsymbol{\theta}_k$ and marks at default $\boldsymbol{\zeta}_k = \mathbf{e}_k$.

Change of regimes with jumps at defaults

- Dynamics of $S = S^k$ between $\tau_k = \theta_k$ and $\tau_{k+1} = \theta_{k+1}$:

$$dS_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) \quad = \quad S_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) * \left(b_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)dt + \sigma_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)dW_t \right),$$

where W is a m-dimensional $(\mathbb{P}, \mathbb{F})$-Brownian motion, $m \geq N$.

- Jumps at $\tau_{k+1} = \theta_{k+1}$:

$$S_{\theta_{k+1}}^{k+1}(\boldsymbol{\theta}_{k+1}, \mathbf{e}_{k+1}) \quad = \quad S_{\theta_{k+1}^-}^k(\boldsymbol{\theta}_k, \mathbf{e}_k) * \left(\mathbf{1}_N + \gamma_{\theta_{k+1}}^k(\boldsymbol{\theta}_k, \mathbf{e}_k, e_{k+1}) \right),$$

Credit derivative

- A credit derivative of maturity T is represented by a $\mathcal{G}_T$-measurable random variable H_T:

$$H_T = \sum_{k=0}^{n} 1_{\Omega_T^k} H_T^k(\boldsymbol{\tau}_k, \zeta_k),$$

where $H_T^k(.,.)$ is $\mathcal{F}_T \otimes \mathcal{B}(\mathbb{R}_+^k) \otimes \mathcal{B}(E^k)$-measurable, and represents the option payoff in the k-default scenario.

Exogenous counterparty default

- One default time τ ($n = 1$) inducing jumps in the price process S of N-assets portfolio:

$$S_t = S_t^0 1_{t<\tau} + S_t^1(\tau,\zeta)1_{t\geq\tau},$$

where S^0 is the price process before default, governed by

$$dS_t^0 = S_t^0 * (b_t^0 dt + \sigma_t^0 dW_t)$$

and $S^1(\theta, e)$, $(\theta, e) \in \mathbb{R}_+ \times E$, is the indexed price process after default at time θ and with mark e:

$$\begin{aligned} dS_t^1(\theta,e) &= S_t^1(\theta,e) * (b_t^1(\theta,e)dt + \sigma_t^1(\theta,e)dW_t), \quad t \geq \theta, \\ S_\theta^1(\theta,e) &= S_\theta^0 * (\mathbf{1}_N + \gamma_\theta(e)). \end{aligned}$$

Multilateral counterparty risk

• Assets family (e.g. portfolio of defaultable bonds) in which each underlying name is subject to its own default but also to the defaults of the other names (contagion effect).

- number of defaults $n = N$ number of assets $S = (P^1, \ldots, P^n)$
  - τ_i default time of name P^i, and ζ_i its (random) recovery rate (P^i is not traded anymore after τ_i)
  - τ_i induces jump on P^j, $j \neq i$.

Admissible control strategies

- A trading strategy in the N-assets portfolio is a $\mathbb{G}$-predictable process $\pi = (\pi^0, \ldots, \pi^n)$:

$$\pi^k(\boldsymbol{\theta}_k, \mathbf{e}_k) \quad \text{is valued in} \quad A^k \quad \text{closed convex set of } \mathbb{R}^N,$$

denoted $\pi^k \in \mathcal{P}_{\mathbb{F}}(\mathbb{R}_+^k, E^k; A^k)$, and representing the amount invested given the past default events $(\boldsymbol{\tau}_k, \boldsymbol{\zeta}_k) = (\boldsymbol{\theta}_k, \mathbf{e}_k)$, $k = 0, \ldots, n$, and until the next default time.

▶ The set of *admissible controls*: $\mathcal{A}_{\mathbb{G}} = \mathcal{A}_{\mathbb{F}}^0 \times \ldots \times \mathcal{A}_{\mathbb{F}}^n$, where $\mathcal{A}_{\mathbb{F}}^k$ includes some integrability conditions

Remark on the control set

• In this modelling, we allow the control set A^k to vary after each default time. This means that we allow the investor to update her portfolio constraint after each default time.

$\longrightarrow$ More general than standard formulation where the control set A is invariant in time.

Wealth process

- Given an admissible trading strategy $\pi = (\pi^k)_{k=0,\ldots,n}$, the controlled wealth process is given by:

$$X_t = \sum_{k=0}^{n} 1_{\Omega_t^k} X_t^k(\boldsymbol{\tau}_k, \boldsymbol{\zeta}_k), \quad t \geq 0,$$

where X^k is the wealth process with an investment π^k in the assets of price S^k given the past defaults events $(\boldsymbol{\tau}_k, \boldsymbol{\zeta}_k)$.

▶ Dynamics between $\tau_k = \theta_k$ and $\tau_{k+1} = \theta_{k+1}$:

$$dX_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) = \pi_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)'\big(b_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)dt + \sigma_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)dW_t\big).$$

▶ Jumps at default time $\tau_{k+1} = \theta_{k+1}$:

$$X_{\theta_{k+1}}^{k+1}(\boldsymbol{\theta}_{k+1}, \mathbf{e}_{k+1}) = X_{\theta_{k+1}^-}^k(\boldsymbol{\theta}_k, \mathbf{e}_k) + \pi_{\theta_{k+1}}^k(\boldsymbol{\theta}_k, \mathbf{e}_k)'\gamma_{\theta_{k+1}}^k(\boldsymbol{\theta}_k, \mathbf{e}_k, e_{k+1}).$$

Random terminal utility function

- A nonnegative map G_T on $\Omega \times \mathbb{R}$ such that $(\omega, x) \mapsto G_T(\omega, x)$ is $\mathcal{G}_T \otimes \mathcal{B}(\mathbb{R})$-measurable

$$G_T(x) \quad = \quad \sum_{k=0}^{n} 1_{\Omega_T^k} G_T^k(x, \boldsymbol{\tau}_k, \zeta_k)$$

where G_T^k is $\mathcal{F}_T \otimes \mathcal{B}(\mathbb{R}) \otimes \mathcal{B}(\mathbb{R}^k) \otimes \mathcal{B}(E^k)$-measurable.

Remarks. 1) Interpretation: there is a change of regimes in the utility after each default time (state-dependent utility functions)
2) Other example: utility function U with option payoff H_T,

$$G_T(x) \quad = \quad U(x - H_T) \quad = \quad \sum_{k=0}^{n} 1_{\Omega_T^k} U(x - H_T^k(\boldsymbol{\tau}_k, \zeta_k)).$$

Value function

- **Value function** of the optimal investment problem:

$$V_0(x) \quad = \quad \sup_{\pi \in \mathcal{A}_{\mathbb{G}}} \mathbb{E}\Big[G_T(X_T^{x,\pi}) \Big], \quad x \in \mathbb{R}.$$

Remark. One can also deal with running gain function, involving e.g. utility from consumption.

▶ How to solve this problem?

Global approach

• Write the dynamics of assets and wealth process in the global filtration $\mathbb{G}$
$\longrightarrow$ Jump-Itô controlled process under $\mathbb{G}$ in terms of W and μ (random measure associated to $(\tau_k, \zeta_k)_k$).

• Use a martingale representation theorem for (W, μ) w.r.t. $\mathbb{G}$ under intensity hypothesis on the default times

▶ Derive the dynamic programming Bellman equation in the $\mathbb{G}$ filtration
$\longrightarrow$ BSDE with jumps or Integro-Partial-differential equations

Global approach: some references

- Single default time: Ankirchner, Blanchet-Scalliet, and Eyraud-Loisel (09), Lim and Quenez (09)
- Multiple default times: Jeanblanc, Matoussi, Ngoupeyou (10)
  - BSDE with jumps and quadratic generators
  - Existence and uniqueness under a boundedness condition on portfolio strategies
  - This approach does not allow to change the control portfolio set after default: π_t valued in A for all t

Our solutions approach

• By relying on the $\mathbb{F}$-decomposition of $\mathbb{G}$-processes,

and

• density hypothesis on the defaults: El Karoui, Jeanblanc, Jiao (09,10)

▶ find a suitable decomposition of the $\mathbb{G}$-control problem on each default scenario $\rightarrow$ sub-control problems in the $\mathbb{F}$-filtration

Density approach

• Conditional density hypothesis on the joint distribution of default times and marks: There exists a map $(t, \omega, \boldsymbol{\theta}, \mathbf{e}) \mapsto \alpha_t(\omega, \boldsymbol{\theta}, \mathbf{e})$, $\mathcal{O}(\mathbb{F}) \otimes \mathcal{B}(\mathbb{R}^n_+) \otimes \mathcal{B}(E^n)$-measurable s.t. for all $t \geq 0$,

$$\textbf{(DH)} \qquad \mathbb{P}\Big[(\boldsymbol{\tau}, \boldsymbol{\zeta}) \in d\boldsymbol{\theta} d\mathbf{e} \Big| \mathcal{F}_t\Big] \;=\; \alpha_t(\boldsymbol{\theta}, \mathbf{e}) d\boldsymbol{\theta} \eta(d\mathbf{e})$$

where $d\boldsymbol{\theta} = d\theta_1 \ldots d\theta_n$ is the Lebesgue measure on $\mathbb{R}^n$, and $\eta(d\mathbf{e}) = \eta_1(de_1) \ldots \eta_n(de_n)$, with $\eta_i(de_i)$ nonnegative Borel measure on E.

Comments on density hypothesis

• Under **(DH)**, $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n)$ admits a $\mathbb{F}$-conditional density w.r.t. the Lebesgue measure:

- τ_i totally inacessible: default events arrive by surprise
- $\tau_i \neq \tau_j$ a.s. for $i \neq j$: non simultaneous default times

• By considering a density process $\alpha_t(.)$, one can take into account some dependence between default times and basic assets price information $\mathbb{F}$

• More general setting than intensity approach: one can express the intensity of each default time in terms of the density. Immersion hypothesis **(H)** (martingale invariance property) is not required.

Conditional survival density processes

- Under the density hypothesis, let us define the indexed survival density processes α^k, $k = 0, \ldots, n-1$, by:

$$\alpha_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) = \int_{[t,\infty)^{n-k} \times E^{n-k}} \alpha_t(\boldsymbol{\theta}, \mathbf{e}) d\boldsymbol{\theta}_{n-k} \eta(d\mathbf{e}_{n-k}), \; t \geq 0,$$

where $d\boldsymbol{\theta}_{n-k} = \prod_{j=k+1}^n d\theta_j$, $\eta(d\mathbf{e}_{n-k}) = \prod_{j=k+1}^n \eta_j(de_j)$.

- $$\mathbb{P}\big[\tau_{k+1} > t | \mathcal{F}_t\big] = \int_{\mathbb{R}_+^k \times E^k} \alpha_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) d\boldsymbol{\theta}_k \eta(d\mathbf{e}_k).$$

Decomposition result

The value function V_0 is obtained by backward induction from the optimization problems in the $\mathbb{F}$-filtration:

$$
\begin{aligned}
V_n(x,\boldsymbol{\theta},\mathbf{e}) &= \operatorname*{ess\,sup}_{\pi^n\in\mathcal{A}^n_{\mathbb{F}}} \mathbb{E}\Big[G^n_T(X^{n,x}_T,\boldsymbol{\theta},\mathbf{e})\alpha_T(\boldsymbol{\theta},\mathbf{e})\Big|\mathcal{F}_{\theta_n}\Big]\\
V_k(x,\boldsymbol{\theta}_k,\mathbf{e}_k) &= \operatorname*{ess\,sup}_{\pi^k\in\mathcal{A}^k_{\mathbb{F}}} \mathbb{E}\Big[G^k_T(X^{k,x}_T,\boldsymbol{\theta}_k,\mathbf{e}_k)\alpha^k_T(\boldsymbol{\theta}_k,\mathbf{e}_k)\\
&\quad + \int_{\theta_k}^T\int_E V_{k+1}\big(X^{k,x}_{\theta_{k+1}}+\pi^k_{\theta_{k+1}}.\gamma^k_{\theta_{k+1}}(e_{k+1}),\boldsymbol{\theta}_{k+1},\mathbf{e}_{k+1}\big)\\
&\qquad \eta_{k+1}(de_{k+1})d\theta_{k+1}\Big|\mathcal{F}_{\theta_k}\Big].
\end{aligned}
$$

Comments

• This recursive decomposition can be viewed as a dynamic programming relation by considering value functions between two consecutive default times: V_k interpreted as the value function after k defaults.

• This $\mathbb{F}$-decomposition of the $\mathbb{G}$-control problem can be viewed as a **nonlinear extension of Dellacherie-Meyer and Jeulin-Yor formula**, which relates linear expectation under $\mathbb{G}$ in terms of linear expectation under $\mathbb{F}$, and is used in option pricing for credit derivatives.

Remarks

• Each step in the backward induction $\longleftrightarrow$ stochastic control problem in the $\mathbb{F}$-filtration (solved e.g. by dynamic programming and BSDE)

• In the particular case where all A^k are identical, our method provides an alternative to the dynamic programming method in the $\mathbb{G}$-filtration, by "getting rid of" the jump terms.

Exponential utility

- Consider the indifference pricing problem of (bounded) defaultable claim:

$$G_T(x) \quad = \quad U(x - H_T) \; = \; \sum_{k=0}^{n} 1_{\Omega_T^k} U(x - H_T^k(\boldsymbol{\tau}_k, \zeta_k)),$$

with an exponential utility function

$$U(x) \quad = \quad -\exp(-px), \quad p > 0, \;\; x \in \mathbb{R}.$$

- Assume that $\mathbb{F} = \mathbb{F}^W$ Brownian filtration generated by W.

BSDEs formulation

- Then, the value functions V_k, $k = 0, \ldots, n$, are given by

$$V_k(x, \boldsymbol{\theta}_k, \mathbf{e}_k) \quad = \quad U\big(x - Y^k_{\theta_k}(\boldsymbol{\theta}_k, \mathbf{e}_k)\big),$$

where Y^k, $k = 0, \ldots, n$, are characterized by means of a recursive system of (indexed) BSDEs, derived from dynamic programming arguments in the $\mathbb{F}$-filtration.

BSDE after n defaults

$$
\begin{aligned}
Y_t^n(\boldsymbol{\theta}, \mathbf{e}) &= H_T^n(\boldsymbol{\theta}, \mathbf{e}) + \frac{1}{p} \ln \alpha_T(\boldsymbol{\theta}, \mathbf{e}) + \int_t^T f^n(r, Z_r^n, \boldsymbol{\theta}, \mathbf{e}) dr \\
& \quad - \int_t^T Z_r^n . dW_r, \quad t \geq \theta_n,
\end{aligned}
$$

with a (quadratic) generator f^n:

$$
f^n(t, z, \boldsymbol{\theta}, \mathbf{e}) = \inf_{\pi \in A^n} \Big\{ \frac{p}{2} \big| z - \sigma_t^n(\boldsymbol{\theta}, \mathbf{e})' \pi \big|^2 - b^n(\boldsymbol{\theta}, \mathbf{e}) . \pi \Big\}.
$$

Remark. Similar BSDE as in El Karoui, Rouge (00), Hu, Imkeller, Müller (04), Sekine (06), for default-free market

BSDE after k defaults, $k = 0, \ldots, n-1$

$$
\begin{aligned}
Y_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) &= H_T^k(\boldsymbol{\theta}_k, \mathbf{e}_k) + \frac{1}{p} \ln \alpha_T^k(\boldsymbol{\theta}_k, \mathbf{e}_k) \\
&\quad + \int_t^T f^k(r, Y_r^k, Z_r^k, \boldsymbol{\theta}_k, \mathbf{e}_k) dr - \int_t^T Z_r^k . dW_r, \quad t \geq \theta_k
\end{aligned}
$$

with a generator

$$
\begin{aligned}
f^k(t, y, z, \boldsymbol{\theta}_k, \mathbf{e}_k) &= \inf_{\pi \in A^k} \Big\{ \frac{p}{2} \big| z - \sigma_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k)' \pi \big|^2 - b_t^k(\boldsymbol{\theta}_k, \mathbf{e}_k) . \pi \\
&\quad + \frac{1}{p} U(y) \int_E U\big(\pi . \gamma_t^k(e_{k+1}) - Y_t^{k+1}(\boldsymbol{\theta}_k, t, \mathbf{e}_k, e_{k+1})\big) \\
&\qquad \eta_{k+1}(de_{k+1}) \Big\}.
\end{aligned}
$$

BSDE characterization of the optimal investment problem

Theorem. Under standard boundedness conditions on the coefficients of the model $(b, \sigma, \gamma, \alpha, H_T)$, there exists a unique solution $(\mathbf{Y}, \mathbf{Z}) = (Y^0, \ldots, Y^n, Z^0, \ldots, Z^n) \in \mathbf{S}^\infty \times \mathbf{L}^2$ to the recursive system of quadratic BSDEs. The initial value function is

$$V_0(x) \quad = \quad U\big(x - Y_0^0\big),$$

and the optimal strategies between τ_k and τ_{k+1} by

$$\begin{aligned}\pi_t^k \quad \in \quad & \arg\min_{\pi \in A^k} \Big\{ \frac{p}{2} \big| Z_t^k - (\sigma_t^k)' \pi \big|^2 - b_t^k . \pi \\ & + \frac{1}{p} U(Y_t^k) \int_E U\big(\pi . \gamma_t^k(e_{k+1}) - Y_t^{k+1}(t, e_{k+1})\big) \eta_{k+1}(de_{k+1}) \Big\}.\end{aligned}$$

Technical remarks

• Existence for the system of recursive BSDEs: quadratic term in z + exponential term in y:

- Kobylanski techniques + approximating sequence + convergence

• Uniqueness: verification arguments + BMO techniques

• We don't need to assume boundedness condition on the portfolio control set

Practical remarks

- In the particular case where:
  - $(\boldsymbol{\tau}, \zeta)$ independent of $\mathbb{F}$ $\rightarrow$ the density α is deterministic
  - the assets price coefficients are deterministic, and the payoff H_T^k are constants (e.g. for constant recovery rates)

then the BSDEs reduce to a recursive system of ordinary differential equations, which can be easily solved numerically.
$\rightarrow$ Numerical results in Jiao, P (09) illustrating the impact of a single default time w.r.t. Merton problem

- Further practical use
  - explicit models for the default density process
  - numerical resolution of quadratic BSDEs or in a Markovian case (factor models to be specified) of semilinear PDEs

Concluding remarks (I)

• Beyond the optimal investment problem considered here, we provide a general formulation of stochastic control under progressive enlargement of filtration with multiple random times and marks:

- Change of regimes in the state process, control set and gain functional after each random time
- Includes in particular the formulation via jump-diffusion controlled processes

• Recursive decomposition on each default scenario of the $\mathbb{G}$-control problem into $\mathbb{F}$-stochastic control problems by relying on the density hypothesis

Concluding remarks (II)

• Solution characterized by dynamic programming in the $\mathbb{F}$-filtration: BSDE, Bellman PDE, ...

• $\mathbb{F}$-decomposition method $\longrightarrow$ another perspective for the study of (quadratic) BSDEs with (finite number of) jumps
$\longrightarrow$ Get rid of the jump terms $\longrightarrow$ obtain comparison theorems under weaker conditions
$\longrightarrow$ Work in progress by Kharroubi and Lim (10).

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