A unified approach to wellposedness of non-Markovian FBSDEs

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Outline

1. The existing methods in the literature
   - Existing methods
   - Examples
2. The decoupling (random) field u
3. Wellposedness of FBSDEs
   - FBSDEs over small time interval
   - Uniform Lipschitz continuity of u
   - The wellposedness result
   - Comparison with the existing methods
4. FBSDEs and Backward SPDEs

Forward Backward SDEs

$$\begin{cases} X_t = x + \int_0^t b(s, X_s, Y_s, Z_s)ds + \int_0^t \sigma(s, X_s, Y_s, Z_s)dB_s; \\ Y_t = g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds - \int_t^T Z_s dB_s. \end{cases} \tag{1}$$

- b, σ, f, g are in general random, and denote $\Theta := (X, Y, Z)$.

Standing Assumptions :

Besides other standard assumptions, the coefficients are

uniformly Lipschitz cont. in (x, y, z) with Lipschitz constant K

For notational simplicity, we assume

the coefficients are differentiable in (x, y, z).

Contraction mapping approach : Antonelli (1993), Pardoux-Tang (1999)

- There exists constants $c_1, c_2 > 0$ such that

$$|\sigma_z| \leq c_1, \quad |g_x| \leq c_2, \quad c_1 c_2 < 1.$$

- One of the following three cases holds :
  - $\diamond$ T small ;
  - $\diamond$ Weak coupling : $b_y, b_z, \sigma_y, \sigma_z$ small or f_x, g_x small ;
  - $\diamond$ Strong decrease : $b_x << 0$ or $f_y << 0$.

Four step scheme : Ma-Protter-Yong (1994), Delarue (2002)

- Markovian setting : all coefficients are deterministic
- $\sigma = \sigma(t, x, y)$ is indepndent of of z
- Uniform nondegeneracy : $\sigma \geq c > 0$
- PDE arguments :

$$\begin{cases} u_t + \frac{1}{2} u_{xx} \sigma^2(t,x,u) + u_x b(t,x,u,u_x\sigma(t,x,u)) \\ \qquad\qquad + f(t,x,u,u_x\sigma(t,x,u)) = 0; \\ u(T,x) = g(x). \end{cases} \tag{2}$$

- Nonlinear Feyman-Kac formula :

$$Y_t = u(t, X_t). \tag{3}$$

Method of continuation : Hu-Peng (1995), Peng-Wu (1999), Yong (1999)

• Monotonicity condition : there exist constants $\beta_1, \beta_2 > 0$ such that, for any $\theta_i := (x_i, y_i, z_i)$, $i = 1, 2$,

$$\begin{aligned}&[b(t,\theta_1) - b(t,\theta_2)]\Delta y + [\sigma(t,\theta_1) - \sigma(t,\theta_2)]\Delta z\\ &\quad -[f(t,\theta_1) - f(t,\theta_2)]\Delta x \le -\beta_1\Big[|\Delta x|^2 + |\Delta y|^2 + |\Delta z|^2\Big];\\ &[g(x_1) - g(x_2)]\Delta x \ge \beta_2|\Delta x|^2.\end{aligned}$$

Great works, but

- The three approaches do not cover each other
  - ◇ We shall provide a unified approach
- Many FBSDEs arising from applications do not fit into any of these frameworks
  - ◇ We shall provide weaker sufficient conditions which solve some FBSDEs from applications,
  - ◇ but not "all".

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Example 1

- Cvitanic-Z. (2005) :

$$\begin{cases} X_t = x + \int_0^t \alpha_s(\omega) Z_s ds + \int_0^t \sigma_s(\omega) B_s; \\ Y_t = h(\omega) X_T - \int_t^T Z_s dB_s, \end{cases}$$

where α, h are bounded.

- None of the above methods works

- Z. (2006), Yong (2006) solved the problem independently

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Example 2

- Jeanblanc : non-uniform monotonicity conditions

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\begin{aligned}
&[b(t,\theta_1) - b(t,\theta_2)]\Delta y + [\sigma(t,\theta_1) - \sigma(t,\theta_2)]\Delta z \\
&\quad -[f(t,\theta_1) - f(t,\theta_2)]\Delta x \le -\beta_1(\omega)\Big[|\Delta x|^2 + |\Delta y|^2 + |\Delta z|^2\Big]; \\
&[g(x_1) - g(x_2)]\Delta x \ge \beta_2(\omega)|\Delta x|^2.
\end{aligned}
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where

$$\beta_1(\omega), \beta_2(\omega) > 0 \quad \text{but without uniform lower bound}$$

- Yu (2010)
- Ma-Wu-Zhang-Z. (2010)

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Example 3

- Linear FBSDE with constant coefficients :

$$\begin{cases} X_t = x + \int_0^t \sigma_3 Z_s dB_s; \\ Y_t = h_0 + h X_T - \int_t^T Z_s dB_s. \end{cases} \tag{4}$$

- The four step scheme does not work : $\sigma = \sigma_3 z$ and is degenerate ;
- The method of continuation does not work : monotonicity condition is violated ;
- The contraction mapping approach :

FBSDE (4) is wellposed if $|\sigma_3 h| < 1$ and T is small enough.

- Ma-Wu-Zhang-Z. (2010) : for arbitrary T

FBSDE (4) is wellposed if and only if $\sigma_3 h \neq 1$.

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Related works to the talk

- Delarue (2002) : PDE arguments

  Markovian, $\sigma = \sigma(t, x, y)$ uniformly nondegenerate

- Z. (2006) : Probabilistic arguments

  Non-Markovian, $\sigma = \sigma(t, x, y)$, possibly degenerate

- Ma-Wu-Zhang-Z. (2010) :

  Non-Markovian, $\sigma = \sigma(t, x, y, z)$, possibly degenerate

- Ma-Yin-Z. (2010) :

  connections with degenerate quasilinear Backward SPDEs

Outline

Some observations

• Assume the general FBSDE (1) is wellposed, by considering it on $[t, T]$ with initial value $X_t = x$, one can define

$$u(t,x) := Y_t^{t,x},$$

which is in general random and is $\mathcal{F}_t$-measurable.

• If u is uniformly continuous in x, one should have

$$Y_t = u(t, X_t).$$

The decoupling field

Definition :

A random field $u(t,x)$ is called a decoupling field of FBSDE (1) if :

- u is $\mathbb{F}$-progressively measurable and $u(T,x) = g(x)$;
- u is uniformly Lipschitz continuous in x
- There $\exists \delta > 0$ s.t., for any $0 \le t_1 < t_2 \le T$ with $t_2 - t_1 < \delta$ and any $\eta \in L^2(\mathcal{F}_{t_1})$, the following FBSDE on $[t_1, t_2]$ is wellposed :

$$\begin{cases} X_t = \eta + \int_{t_1}^{t} b(s,\Theta_s)ds + \int_{t_1}^{t} \sigma(s,\Theta_s)dB_s; \\ Y_t = u(t_2, X_{t_2}) + \int_{t}^{t_2} f(s,\Theta_s)ds - \int_{t}^{t_2} Z_s dB_s. \end{cases}$$

and it holds that $Y_t = u(t, X_t)$, $t \in [t_1, t_2]$.

From the decoupling field to the wellposedness

Theorem 1 :

Assume FBSDE (1) has a decoupling filed u, then FBSDE (1) is wellposed and $Y_t = u(t, X_t)$, $t \in [0, T]$.

Corollary :

FBSDE (1) has at most one decoupling filed u.

- Proof. $u(t,x) = Y_t^{t,x}$.

Strategy for the existence of the decoupling field

- Step 1. Establish wellposedness of FBSDE over small time interval

- Step 2. Define u on $[T-\delta, T]$ and extend it backwardly.

- Step 3. Obtain uniform estimates for u so that its definition can be extended to the whole interval $[0, T]$.

- From now on, we assume all processes are 1-dimensional.

Outline

Linear FBSDE with constant coefficients :

$$\begin{cases} X_t = x + \int_0^t [b_0 + b_1 X_s + b_2 Y_s + b_3 Z_s] ds \\ \qquad + \int_0^t [\sigma_0 + \sigma_1 X_s + \sigma_2 Y_s + \sigma_3 Z_s] dB_s; \\ Y_t = h_0 + h X_T + \int_t^T [f_0 + f_1 X_s + f_2 Y_s + f_3 Z_s] ds \\ \qquad - \int_t^T Z_s dB_s. \end{cases} \tag{5}$$

Proposition.

The FBSDE (5) is wellposed for T small enough if and only if

$$\sigma_3 h \neq 1.$$

Motivation of the conditions

The condition $\sigma_3 h \neq 1$ is equivalent to : there exist

$$c_1 > 0, \quad c_2 > 0, \quad c_1 c_2 < 1$$

such that one of the following three cases holds :

- $|\sigma_3| \leq c_1, \quad |h| \leq c_2$
- $|\sigma_3| \geq c_1^{-1}, |h| \geq c_2^{-1} \quad \Longleftrightarrow$ one of the following cases holds

$$\sigma_3 \geq c_1^{-1}, \quad h \geq c_2^{-1} \quad \text{or} \quad \sigma_3 \leq -c_1^{-1}, \quad h \geq c_2^{-1}$$
$$\text{or} \quad \sigma_3 \geq c_1^{-1}, \quad h \leq -c_2^{-1} \quad \text{or} \quad \sigma_3 \leq -c_1^{-1}, \quad h \leq -c_2^{-1}.$$

- one of the following four cases holds

$$\sigma_3 \leq c_1, \quad 0 \leq h \leq c_2 \quad \text{or} \quad 0 \leq \sigma_3 \leq c_1, \quad h \leq c_2$$
$$\text{or} \quad \sigma_3 \geq -c_1, \quad 0 \geq h \geq -c_2 \quad \text{or} \quad 0 \geq \sigma_3 \geq -c_1, \quad h \geq -c_2.$$

Our conditions

Assumption 1.

Assume there exist $c_1, c_2 > 0$ such that $c_1 c_2 < 1$ and one of the following three cases hold :

- $|\sigma_z| \le c_1, \quad |g_x| \le c_2$
- one of the following four cases holds

$$\sigma_z \ge c_1^{-1}, \quad g_x \ge c_2^{-1} \quad \text{or} \quad \cdots$$

- one of the following four cases holds

$$\sigma_z \le c_1, \quad 0 \le g_x \le c_2 \quad \text{or} \quad \cdots$$

Wellposedness over small time interval

Theorem 2.

Assume Assumption 1 holds. Then there exists a constant $\delta > 0$, depending only on K, c_1, c_2 such that, whenever $T \le \delta$, FBSDE (1) is wellposed.

- Case 1. Antonelli (1993)
- Case 2. Consider transformation :

$$\tilde{X} := Y, \quad \tilde{Y} := X, \quad \tilde{Z} := \sigma(t, X, Y, Z).$$

Then $\tilde{g} = (g)^{-1}, \tilde{\sigma} = (\sigma)^{-1}$ and thus

$$|\tilde{\sigma}_{\tilde{z}}| = \frac{1}{|\sigma_z|} \le c_1, \quad |\tilde{g}_{\tilde{x}}| = \frac{1}{|g_x|} \le c_2.$$

- Case 3. Another transformation.

The linear variational FBSDEs

- Linear variational FBSDE :

$$\begin{cases} \nabla X_t = 1 + \int_0^t [b_x \nabla X_s + b_y \nabla Y_s + b_z \nabla Z_s] ds \\ \qquad + \int_0^t [\sigma_x \nabla X_s + \sigma_y \nabla Y_s + \sigma_z \nabla Z_s] dB_s; \\ \nabla Y_t = g_x \nabla X_T + \int_t^T [f_x \nabla X_s + f_y \nabla Y_s + f_z \nabla Z_s] ds \\ \qquad - \int_t^T \nabla Z_s dB_s. \end{cases} \tag{6}$$

- Since $Y_t^x = u(t, X_t^x)$, then

$$u_x(t, X_t) = \nabla Y_t [\nabla X_t]^{-1} =: \hat{Y}_t.$$

The characteristic BSDE

$$\begin{aligned}\hat{Y}_t &= g_x(X_T) - \int_t^T \hat{Z}_s dB_s \\ &\quad + \int_t^T \left[F(s,\Theta_s,\hat{Y}_s) + G(s,\Theta_s,\hat{Y}_s)\hat{Z}_s + \frac{\sigma_z|\hat{Z}_s|^2}{1-\sigma_z\hat{Y}_s}\right]ds,\end{aligned} \tag{7}$$

where

$$\begin{aligned}F(s,\theta,\hat{y}) &:= \frac{\alpha_0+\alpha_1\hat{y}+\alpha_2\hat{y}^2+\alpha_3\hat{y}^3}{1-\sigma_z(\theta)\hat{y}}, \\ G(s,\theta,\hat{y}) &:= \frac{\beta_0+\beta_1\hat{y}}{1-\sigma_z\hat{y}}\end{aligned}$$

and

$$\alpha_0(t,\theta) := f_x(t,\theta),\cdots.$$

The characteristic BSDE (cont.)

- Quadratic in $\hat{Z}$: Kobylanski (2000), Briand-Hu (2006), Barrieu-El Karoui (2010), $\cdots$

- Backward stochastic Riccati equation : Kohlmann-Tang (2002), Tang (2003), Yong (2006), $\cdots$

- We want the characteristic BSDE (7) has a solution satisfying

$$\text{both } \hat{Y} \text{ and } (1 - \sigma_z(\Theta)\hat{Y})^{-1} \text{ are bounded}$$

Bounds of coefficients

- For a random variable ξ,

$$\operatorname{esssup} \xi := \inf\{x : \xi \le x, \text{a.s.}\}, \quad \operatorname{ess\,inf} \xi := \sup\{x : \xi \ge x, \text{a.s.}\}$$

- Define deterministic functions :

$$\overline{F}(t,y) := \sup_\theta \operatorname{esssup} F(t,\theta,y), \quad \underline{F}(t,y) := \inf_\theta \operatorname{ess\,inf} F(t,\theta,y).$$

Conditions for the desirable u

Assumption 2.

Assume there exist $c_1, c_2, c_3 > 0$ and $\varepsilon > 0$ small enough such that $c_2 < c_3 < c_1^{-1}$ and one of the following three cases hold :

- $|\sigma_z| \le c_1, \quad |g_x| \le c_2, \quad \overline{F}(t, c_3) \le \varepsilon, \quad \underline{F}(t, -c_3) \ge -\varepsilon$
- one of the following four cases holds

$$\sigma_z \ge c_1^{-1}, \quad g_x \ge c_2^{-1} \quad \alpha_3 \ge -\varepsilon, \quad \underline{F}(t, c_3^{-1}) \ge -\varepsilon \quad \text{or} \quad \cdots$$

- one of the following four cases holds

$$\sigma_z \le c_1, \quad 0 \le g_x \le c_2 \quad f_x \ge 0, \quad \overline{F}(t, c_3) \le \varepsilon \quad \text{or} \quad \cdots$$

Main result

Theorem 4.

Let T be arbitrarily given and assume Assumption 2 holds. Then FBSDE (1) has a decoupling field u and thus is wellposed. Moreover,

u_x satisfies the corresponding properties of g_x in Assumption 2 with c_2 replaced by c_3.

- Proof. Choose $\delta > 0$ in Theorem 1 based on (c_1, c_3).

Further results

Theorem 5.

Assume both (b, σ, f, g) and $(b, \sigma, \tilde{f}, \tilde{g})$ satisfy Assumption 2. If

$$f \le \tilde{f}, \quad g \le \tilde{g},$$

then

$$u \le \tilde{u}.$$

- Stability
- L^p-estimates of solutions

Comparison with the method of continuation

- One sufficient condition in Assumption 2 :

$$\sigma_z \le 0, \quad g_x \ge 0 \quad f_x \ge 0, \quad b_z\sigma_y - b_y\sigma_z \le 0 \tag{8}$$

- One case of monotonicity conditions :

$$\Delta b \Delta y + \Delta\sigma\Delta z - \Delta f \Delta x \le -\beta_1|\theta|^2 \le 0, \quad \Delta g \Delta x \ge \beta_2|\Delta x|^2 \ge 0.$$

$\diamond$ 2nd inequality $\Longrightarrow$ $g_x \ge 0$;
$\diamond$ $\Delta y = \Delta x = 0$ $\Longrightarrow$ $\sigma_z \le 0$;
$\diamond$ $\Delta y = \Delta z = 0$ $\Longrightarrow$ $f_x \ge 0$;
$\diamond$ $\Delta x = 0$

$$\Longrightarrow \quad b_y\Delta y^2 + (b_z + \sigma_y)\Delta y \Delta z + \sigma_z \Delta z^2 \le 0, \quad \forall \Delta y, \Delta z$$
$$\Longrightarrow \quad (b_z + \sigma_y)^2 - 4b_y\sigma_z \le 0$$
$$\Longrightarrow \quad 4b_z\sigma_y \le 4b_y\sigma_z \quad \Longrightarrow \quad b_z\sigma_y - b_y\sigma_z \le 0$$

- In particular, this solves Jeanblanc's problem.

Comparison with the contraction mapping approach

- One case in Assumption 2 :

$$|\sigma_z| \le c_1, \quad |g_x| \le c_2, \quad \overline{F}(t, c_3) \le \varepsilon, \quad \underline{F}(t, -c_3) \ge -\varepsilon$$

- For simplicity, assume $\sigma = \sigma(t, x, y)$, then

$$F(t, \theta, \hat{y}) = \alpha_0 + \alpha_1 \hat{y} + \alpha_2 \hat{y}^2 + \alpha_3 \hat{y}^3$$
$$\alpha_0 := f_x, \quad \alpha_1 := f_y + b_x + f_z \sigma_x,$$
$$\alpha_2 := b_y + f_z \sigma_y + b_z \sigma_x, \quad \alpha_3 := b_z \sigma_y.$$

- In the weak coupling case, α_2, α_3 are small ;

- In the strong decrease case, $\alpha_1 << 0$;

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Backward SPDE

- Assume $b = b(t,x,y), \sigma = \sigma(t,x,y)$.
- Assume $du(t,x) = \alpha(t,x)dt + \beta(t,x)dB_t$.
- Apply Ito-Ventzell on $u(t,X_t)$ and compare with dY_t :

$$\begin{aligned} -\alpha(t,x) \quad &:= \quad \frac{1}{2}u_{xx}\sigma^2(t,x,u) + \beta_x\sigma(t,x,u) + u_x b(t,x,u) \\ &\quad\; + f(t,x,u,\beta + u_x\sigma(t,x,u)). \end{aligned}$$

- BSPDE : quasilinear and degenerate

$$\left\{ \begin{aligned} du(t,x) &= -\Big[\frac{1}{2}u_{xx}\sigma^2(t,x,u)] + \beta_x\sigma(t,x,u)) + u_x b(t,x,u) \\ &\quad + f(t,x,u,\beta + u_x\sigma(t,x,u))\Big] dt + \beta(t,x)dB_t; \\ u(T,x) &= g(x). \end{aligned} \right. \tag{9}$$

Characterization of the decoupling field u

Theorem 6.

Under Standing Assumptions,

FBSDE (1) has a decoupling field u

if and only if

BSPDE (9) has a unique Sobolev type weak solution u which is uniformly Lipschitz continuous in x.

Future research

- Wellposedness of multi-dimensional FBSDEs
- Numerical methods for FBSDEs
- Direct proof for the wellposedness of BSPDEs

THANK YOU VERY MUCH !

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