# MAP562 Optimal design of structures

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## Exercise 5, Feb 01, 2017

**in amphi Grégory**

## Exercise 1

We consider a structure $\Omega \in \mathbb{R}^d, d = 2, 3$ (open, bounded, and sufficiently regular). Let $x_\Omega$ be its gravitational center and $V(\Omega)$ its volume. We define the admissible set as

$$\mathcal{U}_{ad} = \left\{ \Omega \subset \mathbb{R}^d \text{ such that } x_\Omega = 0 \text{ and } V(\Omega) = V_0 \right\}.$$

The goal is the minimization of the trace of the ineria tensor

$$\inf_{\Omega \in \mathcal{U}_{ad}} \left\{ J(\Omega) = \frac{1}{2} \int_\Omega |x - x_\Omega|^2 dx \right\}. \tag{1}$$

1. What can we say about possible minimia of the above problem?

## Exercise 2

Let $V(x)$ be a vector field, i.e., a smooth function from $\mathbb{R}^d$ into $\mathbb{R}^d$. We define the functional

$$J(\Omega) = \int_{\partial\Omega} V \cdot n \, ds,$$

where $n$ is the unit exterior normal to the domain $\Omega$.

1. Compute the shape derivative of $J(\Omega)$.

## Exercise 3

TD4, Exercise 2 (Eigenvalue problem).

## Exercise 4

TP - practical exercise

We consider again the elasticity system. As objective functional we take

$$J_D(h) = \int_{\Gamma_N} k(x) |u - u_0|^2 \, dx,$$

where $u_0 = (0, 1)^T$ and the admissible $U_{ad}$ set as in Homework 4. The function $k(x)$ is an indicator function with $k(x) = 1$ for $x \in \Omega_2$ and $k(x) = 0$ for $x \in \Omega_1$

as displayed in Figure 1. The domain is fixed by a homogeneous Dirichlet condition on $\Gamma_D$. On $\Gamma_{N1}$ a traction force is prescribed. On the top boundary we prescribe no condition for $u_x$ and $u_y = 0$.

The goal of the exercise is:

$$\inf_{h \in U_{ad}} J(h) \text{ s.t. to the elasticity system.}$$

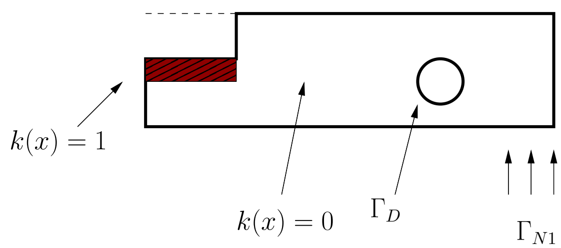


Figure 1: Geometry, boundaries, and forces.