MAP562 Optimal design of structures

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Exercise 1

(Exercise 3 from TD 6)

We consider again heat optimization, which we have addressed in the optimal section some weeks ago. Let Ω be a domain standing for a room. In this room we have a heat source and its shape is given by $\omega \subset \Omega$. We assume that the temperature outside the domain is zero. In the heater we are given a constant temperature $T_1 > 0$. The state equation that describes the temperature distribution is given by:

$$\begin{aligned} -\Delta T + u \cdot \nabla T &= 0 \quad \text{in } \Omega \setminus \omega, \\ T &= 0 \quad \text{on } \partial\Omega, \\ T &= T_1 \quad \text{in } \partial\omega. \end{aligned}$$

The goal will be to achieve a constant temperature T_0 in the entire room Ω. To this end, we introduce the cost functional:

$$J(\omega) = \int_{\Omega\setminus\omega} |T(\Omega) - T_0|^2 \, dx,$$

where $T(\Omega)$ is the solution of the state equation. Note that ω is the variable of optimization.

1. Determine the standard variational formulation of the state equation.
2. Recast the variational problem by expressing the constraint $T(\omega) = T_1$ on $\partial\omega$ (namely a Dirichlet boundary condition on the inner boundary) by introducing a Lagrange multiplier $\lambda(\omega)$.
3. Determine the Lagrangian associated to the minimization problem where we do not use the standard variational formulation of the state equation. Hint: write both the strong forms of the equation and of the boundary condition on ω as contraints. We assume that both $T(\omega)$ and $\lambda(\omega)$ are differentiable with respect to ω. Compute the shape derivative $\langle \frac{\partial J}{\partial \omega}, \theta \rangle$ for all vector fields θ such that $\theta \cdot n = 0$ on $\partial\Omega$.