MAP562 Optimal design of structures

by Grégoire Allaire, Thomas Wick

Ecole Polytechnique

Academic year 2016-2017

Homework 1, Jan 4, 2017

Exercise 1

1. Implement in FreeFem++ the Poisson problem with Fourier boundary conditions (related to the theoretical Ex. 1.1). Specifically the problem is given by:

$$\begin{cases} -\Delta u = f \text{ in } \Omega, \\ \alpha u + \partial_n u = g \text{ on } \Gamma_F, \end{cases} \tag{1}$$

   where $\Omega = (100, 100)^2$, $f = xy + 1$, $g = x$, $\alpha = 2$.
   Hint: Write first down the variational formulation.

Exercise 2

1. Implement in FreeFem++ the problem with nonlocal boundary conditions (related to Ex. 1.2).
   Hint: To implement the multiplication of two boundary integrals, you can evaluate the first one and create the corresponding matrix, let us say B. Then you perform $B * B^T$. This matrix you can add to the other parts required for the total stiffness matrix M. Also you create explicitely the right hand side vector L. Finally you solve with

```
set(M,solver=sparsesolver);
u[]=M^-1*L;
```

   in order to obtain the discrete solution u.

Exercise 3

We take the elasticity program `elasticity.edp` and investigate further properties. [**Hint:** Recapitulate first for yourself the variational formulation of elasticity from the lecture notes.]

1. Recapitulate our findings from the exercise session and change the boundary conditions and interpret the results (the specific task will be discussed orally in the session).
2. Change the Lamé coefficients and investigate in particular the case in which Poisson's ratio ν (defined by $\nu := \frac{\lambda}{2(\lambda+\mu)}$) tends to 0.5. What is the physical interpretation of this case? And why does it pose mathematical problems?

Remark:
Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`