MAP562 Optimal design of structures

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Exercise 1

Group 1.

We consider the minimization problem from exercise 2.4:

$$\min_{v \in H_0^1(\Omega)} E(v) = \frac{1}{2} \int_\Omega a(v(x)) |\nabla v(x)|^2 \, dx - \int_\Omega f(x) \, v(x) \, dx \,. \qquad (1)$$

1. Implement in FreeFem++ the minimization of the functional using a gradient method. Specifically, we use the domain $\Omega = (0, 10)^2$ with boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$, where Γ_D is prescribed on the right and top boundaries and Γ_N on the remaining two parts. Moreover, let $f = 1$ and $a(v) = \frac{v^2+1}{v^2+2}$.
[**Hint:** Compute the first-order optimality condition, which results in a variational formulation. See also exercise sheet No. 2.]
2. Implement the same problem in a second code, but now using Newton's method.

Exercise 2

Group 2.

1. Using a gradient method, implement in FreeFem++ the minimization of the functional (which we discussed in exercise 2.5):

$$J(u) = \int_\Omega |T - T_0|^2 \, dx$$

where

$$\begin{aligned} -\Delta T &= 1_\omega u \quad \text{in } \Omega \\ T &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

and $\Omega = (0, 10)^2$ and ω a ball with radius 1.
2. Test different places for ω and guess which is the optimal place.

Remark:

Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`