MAP562 Optimal design of structures

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Exercise 1

In this exercise we optimize the thickness h of a two-dimensional elastic body in Ω. As in the other exercises, we have two boundary parts Γ_D and Γ_N. For an homogeneous elastic body the governing equations satisfy linearized elasticity: Find $u \in V$ such that

$$\begin{aligned} -\nabla \cdot (\sigma(u)) &= f \quad \text{in } \Omega, \\ \sigma(u) \cdot n &= g_N \quad \text{on } \Gamma_N, \\ u &= u_D \quad \text{on } \Gamma_D, \end{aligned}$$

where the stress is given by the classical law:

$$\sigma(u) = h(2\mu e(u) + \lambda tr(e(u))I)$$

with the linearized strain tensor $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$, I is the identity matrix, and μ and λ are the Lamé coefficients. Furthermore, $f = 0$ and $u_D = 0$. The specific domains Ω and the traction force vector g_N will be discussed in the TP on the blackboard.

We have two goals in mind:

- Compliance minimization;
- Matching a given displacment.

The compliance $J(h)$ of the structure is defined as:

$$J_C(h) = \int f \cdot u(h)\, dx + \int_{\Gamma_N} g \cdot u(h)\, ds.$$

The displacement-matching problem is given by:

$$J_D(h) = \int_\omega |u - u_0|^2\, dx,$$

where ω is a small region around the point where the forces are applied. Moreover, $u_0 = 0$. The admissible set for both problem configurations is given by

$$h \in U = \{h \in L^\infty(\Omega) : \, h_{min} \le h \le h_{max}\},$$

and with the additional constraint

$$\int_\Omega h(x)\, dx = h_{avg}|\Omega|,$$

where $h_{min} = 0.1$, $h_{max} = 1$ and $h_{avg} = 0.3$.

1. Implement $\min_{h\in U} J_C(h)$ in FreeFem using the optimality criteria method (introduced as discussed in the 4th lecture).
   Hint: Recapitulate the lecture notes why the optimality criterion should be chosen as algorithm rather than gradient.
2. Implement $\min_{h\in U} J_D(h)$ in FreeFem using a gradient algorithm.
   Hint 1: Introduce the Lagrangian and derive the state and adjoint equations and proceed as in the previous homework.
   Hint 2: Play a bit with mesh refinement and also the average value h_{avg}.

3. The domain and the boundary conditions will be specified for each group in the lecture/TP.

Remark:

Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`